

CONEM2024-1494 ANALYSIS OF SHEAR STRESS IN WELDED T JOINTS

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Abstract: During the design phase of metallic connections, the analysis of T-joints in welded steel structural elements is carried out by calculating engineers in order to establish the minimum dimensions of the weld beads or to determine the strength capacity of the joints against the occurrence of limit states. To this end, the mathematical calculation procedures available in the technical literature, based on classical mechanics, should provide practical conditions for determining these design parameters. In this sense, these analytical models can propose competent solutions that can be used by engineers in structural design analyses. Normally, in this context, an acceptable kinematic hypothesis is considered, the aim of which is to obtain a suitable mathematical formulation that allows the stresses to be calculated at the point of interest, where the stress value is usually maximum. Based on this premise, this article develops an analytical formulation based on classical mechanics, based on torsion theory, the aim of which is to determine the maximum shear stresses that occur at the ends of weld seams in T-joints, which connect perpendicular steel plates subjected to torsional moment. T-joints are widely used in the shipbuilding and construction industries, providing safety and economic viability as a structural design solution. The results obtained analytically are compared with three-dimensional numerical modeling using the finite element method (FEM), using tetrahedral and hexahedral solid elements. After comparing the stress results obtained for steel plates with different weld lengths, it was possible to conclude that the analytical solution developed in this article is suitable for sizing weld seams and verifying the maximum shear stresses that occur in welded T-joints between perpendicular steel plates subjected to a torsional moment. The percentage differences found between the mathematical methods analyzed can be considered negligible. In Brazil, there are no technical standards for the solution proposed in this article.

Key words: torsion in welded t-joints, analytical solution, computer modeling, finite element method

1. INTRODUCTION

Most books on the strength of materials and solid mechanics deal with the solution for obtaining maximum shear stresses in structural sections subjected to torsional moments, generally by applying conjugates acting on the ends of a circular cross-section bars. The literature shows that such sections subjected to pure torsion rotate like rigid bodies around their longitudinal axis, the radius remain straight, and the cross-sections keeps circular.

This leads to the conclusion that the maximum shear stresses that occur at the periphery of the part section are directly proportional to the applied torsional moment and the radius of the circle, also known as the polar distance of inertia. These maximum shear stresses can be calculated based on classical mechanics by using an acceptable kinematic hypothesis.

Fillet welds in T-type joint, are widely employed in ships, bridge structures and support frames for pressure vessels and pipelines (Teng *et al.*, 2001). Especially the T-joints of large ships, play an important role in obtaining a monolithic structure, offering safety and economy (Gallo *et al.*, 2017). Problems involving T-joints have been investigated in several areas of civil engineering, as can be seen in the works of Kobel'skii *et al.* (1988), Khalili and Ghaznavi (2013) and more recently, in Azam *et al.* (2017).

Academics have carried out various investigations with the aim of evaluating the stresses and strains in T-joints of welded steel sections subjected to tension, compression and bending. Solutions that allow the design of these connections are not easy to find in the literature. Gonçalves and Martins (2006) refer to such gaps, regarding the design of structural metallic joints, reporting the difficulty of finding technical data that control the manufacturing conditions.

In this direction, Huang, Nemat-Nasser and Zarka (2004) emphasize the need to obtain analytical formulations that can be used to support the design of welded steel structures. Additionally, Nash (2014) records the absence of a

mathematical solution for the case of perpendicularly welded plates, proposing an adaptation of the bending solution to solve problems of T-joints subjected to shear stresses. According to Ivan *et al.* (2008), eccentrically loaded joint configurations are inevitable and more complex if compared to concentrically loaded joints, where the welds are generally subjected to a state of stresses, only, of shear in one direction. In face of the exposed, it was developed in this work, a method for the calculation of the properties of T-joints in steel plates welded, providing an analytical formulation that aims to obtain the maximum shear stress induced by a torsional moment in the joint represented by a weld bead.

With the advent of sophisticated structural analysis programs such as SAP, Ansys or Abaqus, various numerical solutions have been obtained using the finite element method, for a range of geometric possibilities. However, the high investment and maintenance costs of such analysis software do not allow that many structural design engineers have access to such computational facilities.

The aim of this study is therefore to evaluate the results of an analytical formulation developed for the design and verification of shear stresses in welded T-joints for steel plates subjected to torsional moments. The results were verified through numerical modeling using the finite element method (FEM). In Brazil, there are no standards for evaluating shear stresses in T-type welded joints subjected to torsional moments.

2. ANALYTICAL DEVELOPMENT

This article proposes a particular method of analysis for shear stresses in welded T-joints using classical mechanics and numerical modeling. The development of a mathematical solution begins with the establishment of an admissible kinematics for the problem. In this context, the design and verification of welded T-joints composed of perpendicular plates subjected to a torsional moment can be accomplished by understanding the shear stresses in the weld bead (Chen, 2005). The necessary relations to calculate the shear stresses related to the acting torsional moment, including the polar moment of inertia, will be defined considering the basic dimensions of the weld bead and the thickness of the sheets to be joined.

Consider the connection shown in Fig. 1, composed of a base plate *B* welded to a vertical plate *A*, both of length *L*. The width of the weld bead is designated by *a* and the thickness of the vertical plate by *t*. *M* is the torsional moment that requests the connection. When the connection is under the action of a torsional moment acting in its own plane, it is considered that the shear stress distribution obeys the law arising from the strength of materials given in Eq. (1). In this, *M* is the torsional moment acting on the system, $\rho(z)$ is the generic distance from the joint center, dependent on the variable *z*, and *J* is the polar moment of inertia to be determined as described below.

$$\tau(z) = \frac{M \cdot \rho(z)}{J} \quad (1)$$

This implies that one must consider the weld, inscribed in a circle of radius, whose length changes according to the variation of the polar distance. The induced stresses are shear, obtained at each position of the weld bead, represented by the *z*-axis.

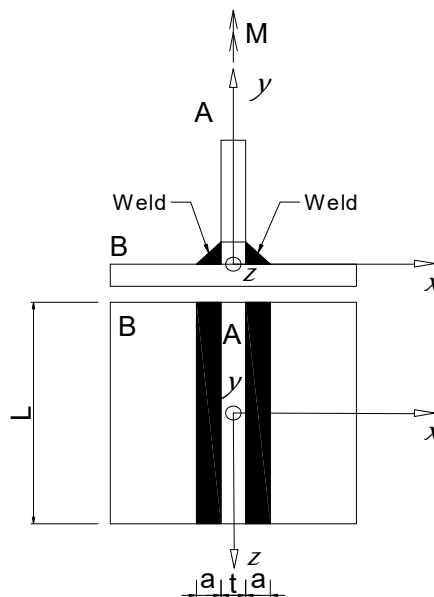


Figure 1. T-joint references.

In Fig. 2, the elementary area dA is equal to $dA = a \cdot dz$, where *a* represents the width of the weld bead.

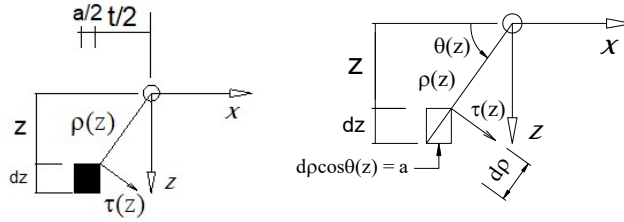


Figure 2. T-joint references for analytical development.

Such an assumption is a formalism that arises because the stress obtained is perpendicular to the lever arm in relation to the geometric center of the weld line along the profile. The distance from the center of the connection is obtained by the hypotenuse of the right triangle, as indicated in Eq. (2) and Fig. 2.

$$\rho(z)^2 = \left(\frac{a}{2} + \frac{t}{2}\right)^2 + z^2 \quad (2)$$

Conceptually, the polar moment of inertia is the integral over the area of the polar distance squared, obtained according to Eq. (3):

$$J(z) = \int \rho(z)^2 \cdot dA \quad (3)$$

Substituting Eq. (2) into Eq. (3), with integration bounds appropriate to the problem, one finds Eq. (4):

$$J = \int_{-\frac{L}{2}}^{+\frac{L}{2}} \left[\left(\frac{a}{2} + \frac{t}{2}\right)^2 + z^2 \right] \cdot a \cdot dz \quad (4)$$

Solving the integral above gives Eq. (5), which provides a way of calculating the polar moment of inertia, depending on the size of the weld and the thickness of the vertical plate:

$$J = \frac{1}{4} \cdot a^3 \cdot L + \frac{1}{2} \cdot a^2 \cdot t \cdot L + \frac{1}{4} \cdot a \cdot t^2 \cdot L + \frac{1}{12} \cdot a \cdot L^3 \quad (5)$$

Substituting the polar moment of inertia obtained from Eq. (5) into Eq. (1), it gives Eq. (6), which gives the shear stress variation in the weld bead.

$$\tau(z) = \frac{M \cdot \rho(z)}{\frac{1}{4} \cdot a^3 \cdot L + \frac{1}{2} \cdot a^2 \cdot t \cdot L + \frac{1}{4} \cdot a \cdot t^2 \cdot L + \frac{1}{12} \cdot a \cdot L^3} \quad (6)$$

Knowing that the stress distribution requires that the maximum stress occurs at the end of the weld, an initial simplification for ρ_{max} equal to $L/2$ is admitted and the equation for maximum shear stress can be written in the form of Eq. (7).

$$\tau_{max} = \frac{M \cdot \left[\left(\frac{a}{2} + \frac{t}{2}\right)^2 + \left(\frac{L}{2}\right)^2 \right]^{\frac{1}{2}}}{\frac{1}{4} \cdot a^3 \cdot L + \frac{1}{2} \cdot a^2 \cdot t \cdot L + \frac{1}{4} \cdot a \cdot t^2 \cdot L + \frac{1}{12} \cdot a \cdot L^3} \quad (7)$$

To evaluate the demonstrated analytical formulations, it is possible to compare the results from the previous development with results obtained by computational modelling using FEM.

3. MODELLING BY FINITE ELEMENT METHOD

The FEM, used to support the results obtained from the proposed analytical formulation, is a numerical method for solving differential equations of continuous systems and has its roots in the variational methods of Rayleigh-Ritz (1870-1909) and the weighted residual methods of Galerkin (1915). The FEM is based on three fundamental principles: (i) a variational formulation (*e.g.* principle of virtual work, principle of minimum total potential energy); (ii) the discretization of the domain of the problem in small parts called elements; (iii) the substitution of the primary variable (*e.g.* displacement, temperature, etc.) into the elements by local interpolation functions (*e.g.* Lagrange interpolation, Hermite interpolation, etc.). The development of the finite element method (FEM) began in 1956 with applications in aeronautical structures and from there it developed in many other areas, currently becoming in the most used computational method in the solution of the engineering problems (Bathe, 2014).

In the area of welding, FEM modeling has been used to analyze problems related to various aspects. Khiabani and Sadrnejad (2009) used it to study residual stresses in cold bending and welding of thick plates. Chen *et al.* (2015) employed finite element analysis to determine the residual stress distribution and distortion field in butt and fillet plates. More recently, Zain-Ul-Abdein *et al.* (2016) used FEM to calculate and compare bulk material properties with experimental results and characterize the effect of grain size on the stress state in a welded aluminum joint.

Considering the ANSI/AISC 360-16 (2016) statements, according to Duncan (2004), a computational modeling can be performed to solve the problems in perpendicularly welded steel plate joints subjected to a torsional moment. For this, the theory of deformable solids can be used for evaluating the stress distribution in the base section of the weld bead.

To achieve the objective of this work, three-dimensional numerical modeling was carried out using Ansys (2023), a commercial software package based on the finite element method. The finite elements adopted in the modelling is the linear tetrahedral and hexahedral solid elements and the displacement field in triaxial stress state considered is made by three displacement components, according to the elasticity theory. Such elements present three degrees of freedom per node, which gives up to twenty-four degrees of freedom per element.

To compare the results between the numerical analysis and those obtained from the analytical formulations, it was necessary to consider the torsional moment applied at the top of the vertical plate (see Fig. 3, left), equal to 5 kNm. The model was restricted in the base plane against all possible movements in that plane (see Fig. 3, right).

3.1. Material Properties

The material properties and parameters considered in the simulation are as follows, applied to the elements plates and the weld:

- ✓ Yield strength: 250 N/mm²
- ✓ Maximum tensile strength: 460 N/mm²
- ✓ Young's modulus: 200,000 N/mm²
- ✓ Transverse modulus of elasticity: 76,923 N/mm²
- ✓ Poisson's ratio: 0.3
- ✓ Density: 7,850 kg/m³

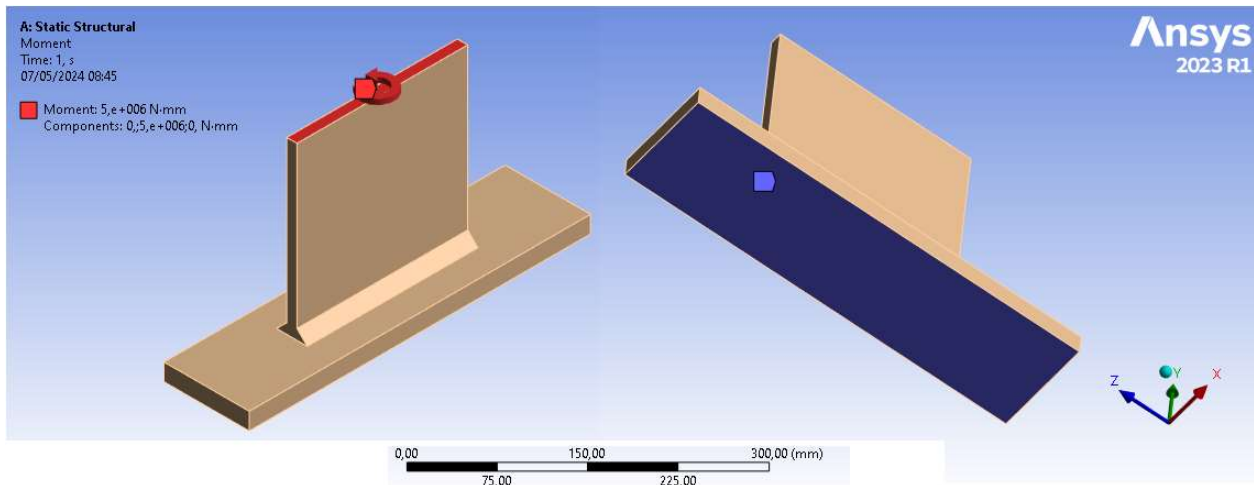


Figure 3. 3D computational model with torsional moment at the top (left) and base fully restricted (right).

The tetrahedral and hexahedral elements chosen, have edges varying between 0.6 mm and 3 mm, defined after observing an adequate tendency for convergence of results. The connections of the nodes to the base plate were determined by restricting translational and rotational movements using a clamp support.

3.2. Mesh Convergency Tests

The mesh convergence method adopted during the refinement analyses used the multizone command, with a sphere of influence located at each vertex of the weld bead base of the weld bead (see Fig. 4). The spheres were parameterized with a radius equal to 18 mm and a mesh converging from 3 mm to 0.6 mm in the vicinity of the point of interest, until convergence (see Fig. 4). After choosing a specific component of the shear stress to start the mesh convergence test, convergence was achieved as shown in Tab. 1.

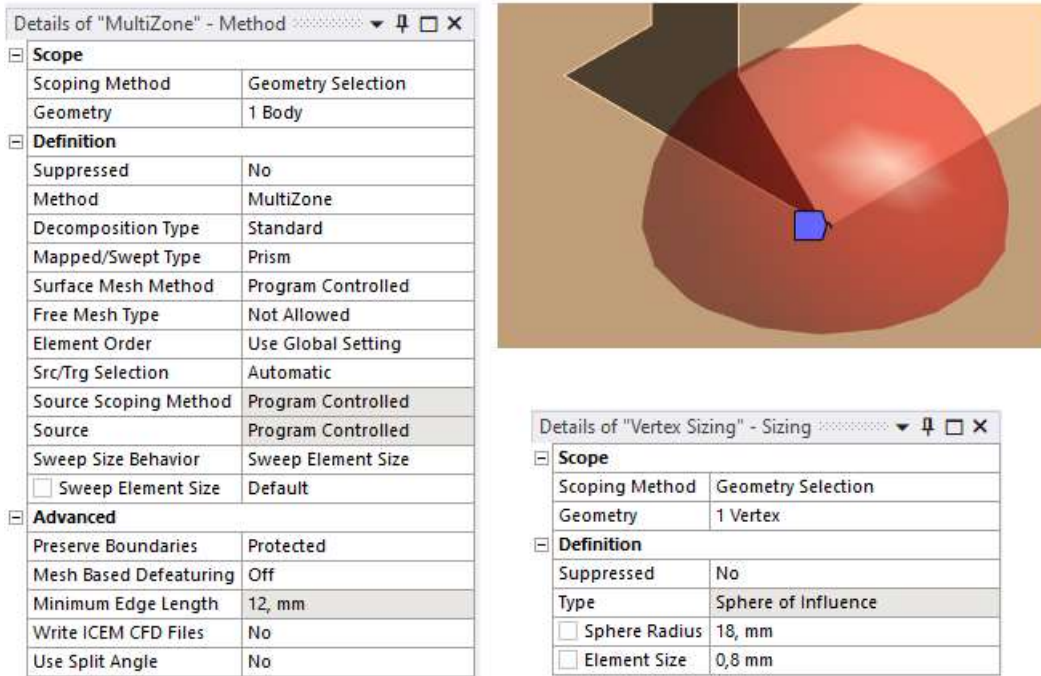


Figure 4. Mesh convergence commands – Ansys FEM software.

Table 1. Mesh convergence results.

Mesh (mm)	Nodal coordinates of the interest point			Number of nodes	Number of elements	Time elapsed	τ_{xz} (MPa)	Tolerance
	x (mm)	y (mm)	z (mm)					
3 – 3	-12.001407	0.071153	-0.001824	552,171	204,250	2' 19.91"	-40.948	-
3 – 2	-12.001408	0.071208	-0.001755	620,996	230,248	2' 36.82"	-45.790	1.06E-01
3 – 1	-12.001409	0.071214	-0.001730	904,987	336,146	4' 29.72"	-51.874	1.17E-01
3 – 0.9	-12.001408	0.071206	-0.001734	979,327	364,018	6' 9.54"	-51.963	1.71E-03
3 – 0.8	-12.001408	0.071229	-0.001736	1,138,375	424,594	7' 54.36"	-52.363	7.64E-03
3 – 0.7	-12.001724	0.012781	0.004954	1,278,040	477,098	9' 15.24"	-52.434	1.35E-03
3 – 0.6	-12.001409	0.071214	-0.001730	1,430,378	533,332	10' 40.97"	-52.782	6.59E-03

4. RESULTS AND DISCUSSIONS

The analysis consisted of numerous models, under the assumption of obtaining stresses in the elastic-linear regime. In this condition, the equilibrium equations are established in the undeformed configuration. During the simulations, the models had to undergo geometry adjustments, allowing the torsional moments applied to the top of the vertical plate to be transferred directly to the base plate, because of the influence of height of the vertical plate.

The length of the horizontal plate was also increased to allow the spreading of the shear stresses at the end of the weld bead. The objective of doing that is to avoid a region of stress concentration which can appears in corners of the computational model. Figure 5 shows models with two types of heigh. It is important to note that the influence of the height of the vertical plate is not considered yet. Because of that, the height of the vertical plate was adjusted aiming to leave the analysis as near as possible from the plane of the weld.

In order to obtain the shear stress components at the point of interest, two limit values were set for the height of the vertical plate. According to the Saint-Venant principle, the stresses at the point of interest will change in the vicinity of the load application region. For this reason, it was necessary to move the point where the shear stress components were obtained in a certain distance from the region of application of the torsional moment. Therefore, the height of the vertical plate was considered to be five times its thickness and equal to 60 mm.

The Eurocode (2005) establishes the slenderness limit for bending as being equal to 200. In the present work, this slenderness will be reached when the height of the vertical plate is close to 350 mm. However, this limitation is not suitable for comparison with torsional slenderness and, as there is no normative prescription on this restriction, it was decided to limit the height of the vertical plate to the value of the weld bead length.

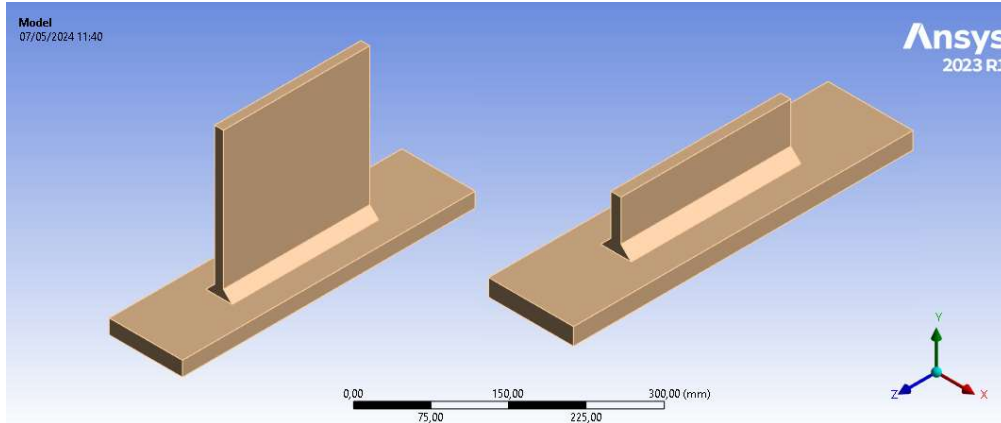


Figure 5. Building the models: initial and final, one example.

4.1. Post Processing Characteristic.

To obtain the maximum shear stresses at the end of the weld bead, it is necessary to analyze the stress components of the solid elements involved. In post-processing of the numerical-computational analysis, the stress components of interest are: τ_{xy} , τ_{yz} , τ_{xz} , σ_{xx} , σ_{yy} , and σ_{zz} , induced by the action of the torsional moment at the top of the vertical plate. Additionally, Fig. 6 shows both local and global reference systems adopted.

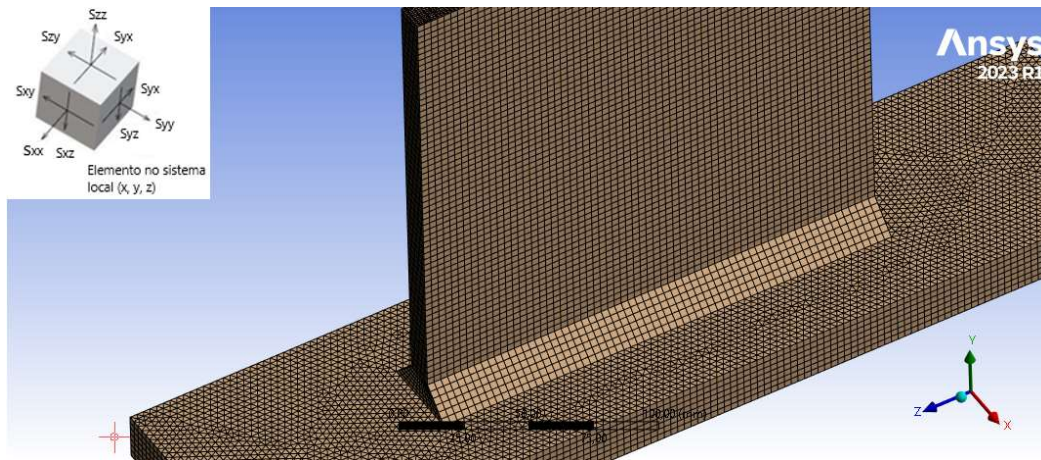


Figure 6. Coordinate systems view and director cosines.

Therefore, the maximum shear stress resultant is localized in the end of the weld bead (see Fig. 7). The stress components are extracted from the computational modelling for each assumed length.

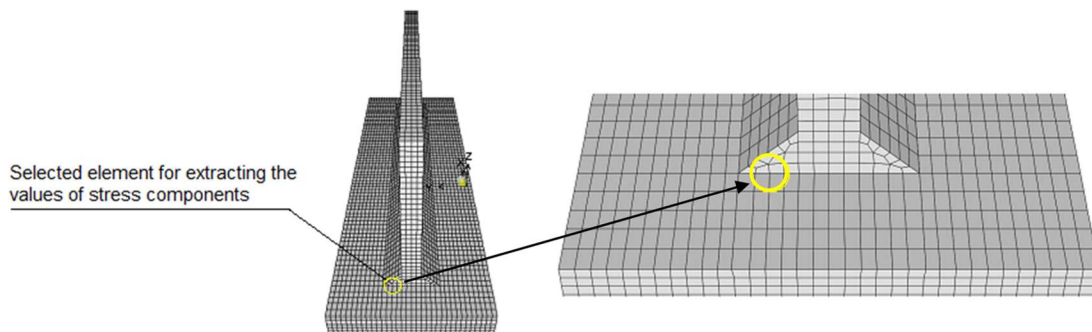


Figure 7. Location of the stress components for calculation maximum shear stress resultant.

4.2 Maximum Shear Stresses Determination

According to Villaça and Taborda (1998), the stress components of each solid element in relation to the local axes must be determined by the system presented in Eq. (8):

$$\begin{aligned}\rho_x &= l \cdot S_{xx} + m \cdot S_{xy} + n \cdot S_{xz} \\ \rho_y &= l \cdot S_{yx} + m \cdot S_{yy} + n \cdot S_{yz} \\ \rho_z &= l \cdot S_{zx} + m \cdot S_{zy} + n \cdot S_{zz}\end{aligned}\tag{8}$$

where:

- ρ_x = component of stress in the direction of the local axis x
- ρ_y = component of stress in the direction of the local axis y
- ρ_z = component of stress in the direction of the local axis z
- S_{ij} = component of stress in the direction of the local axis j acting on the face perpendicular to i ($i, j = x, y, z$) (Fig.6)
- l = cosine director of ρ_x ($\cos \alpha$)
- m = cosine director of ρ_y ($\cos \beta$)
- n = cosine director of ρ_z ($\cos \gamma$)

Therefore, the resultant stress vector is given by:

$$\rho_N = \sqrt{\rho_x^2 + \rho_y^2 + \rho_z^2}\tag{9}$$

In the present case, since there is only a torsional moment load at the top of the vertical plate, the director cosines are: $\alpha = 0^\circ$, $\beta = 90^\circ$ and $\gamma = 90^\circ$. The equation for the normal stress resultant can be written as:

$$\sigma_N = S_{xx} \cdot l^2 + S_{yy} \cdot m^2 + S_{zz} \cdot n^2 + 2 \cdot S_{xy} \cdot l \cdot m + 2 \cdot S_{xz} \cdot l \cdot n + 2 \cdot S_{yz} \cdot m \cdot n\tag{10}$$

Then, the shear resultant stress can be calculated by:

$$\tau_N = \sqrt{\rho_N^2 + \sigma_N^2}\tag{11}$$

The results obtained through the Eq. (10) and Eq. (11) allow calculating the normal and shear stress resultants, determined using the stress components of the numerical modeling (see Tab. 2). Allied to the previously calculation, a comparative analysis between the results of Eq. (7), and computational modelling by using FEM, Eq. (11), generated the results depicted in Figure considering a and t equals to 0.012 m and a torsion moment M equal to 5 kNm.

Figure exemplifies the shear stress distribution in the weld bead at the point of extraction of values for the model with weld length equal to 0.20 m. Figures 9 and 10 shows two shear stress components indicated with asterisks in Tab. 2.

Table 2. Results of stresses calculated.

Weld length (m)	FEM													Eq. (7)		
	Element stresses components (MPa)						Director cosines (deg)			ρ components (MPa)				Resultant stresses (MPa)		Stress (MPa)
	τ_{xy}	τ_{yz}	τ_{xz}	σ_{xx}	σ_{yy}	σ_{zz}	α	β	γ	ρ_x	ρ_y	ρ_z	ρ_N	σ_N	τ_N	
0.20	57*	-116	-44*	109	243	160	0	90	90	109	57	-44	131	109	72	60
0.22	50	-100	-34	89	211	141	0	90	90	89	50	-34	108	89	61	50
0.24	41	-90	-31	82	192	122	0	90	90	82	41	-31	97	82	52	42
0.26	37	-74	-24	68	164	106	0	90	90	68	37	-24	81	68	44	36
0.28	31	-64	-21	57	138	91	0	90	90	57	31	-21	68	57	38	31
0.30	28	-54	-17	51	121	82	0	90	90	51	28	-17	61	51	33	27
0.32	23	-48	-15	42	103	66	0	90	90	42	23	-15	50	42	28	24
0.34	20	-42	-14	37	92	59	0	90	90	37	20	-14	44	37	24	21
0.36	18	-38	-12	34	84	54	0	90	90	34	18	-12	40	34	22	19
0.38	16	-30	-11	33	72	58	0	90	90	33	16	-11	38	33	19	17
0.40	15	-29	10	32	70	53	0	90	90	32	15	10	37	32	18	15

*Shown in Figs. 9 and 10.

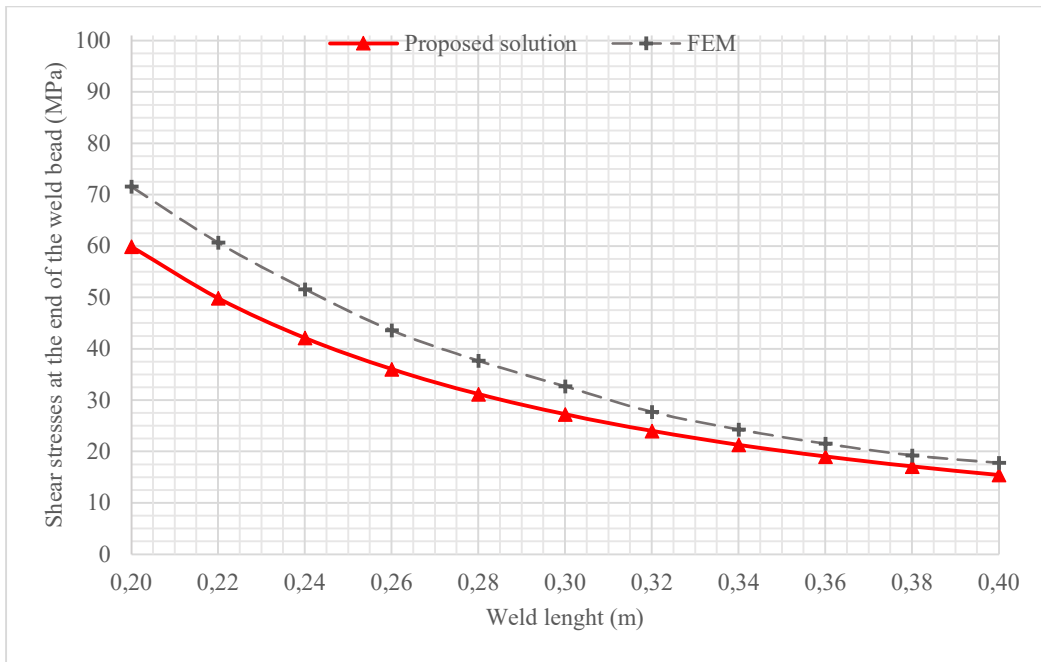


Figure 8. Shear stresses results (FEM = finite element method).

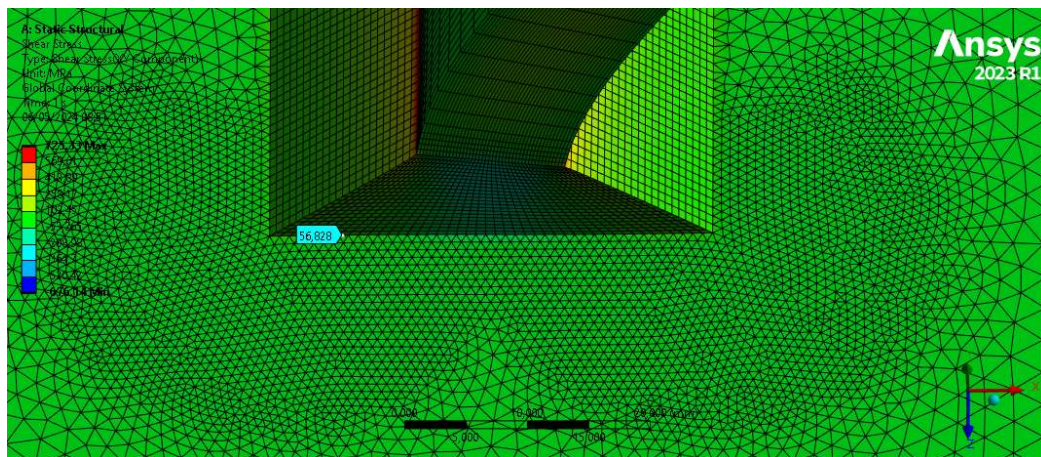


Figure 9. Shear stresses component $\tau_{xy} = 56.828$ MPa for the model with $L = 0.200$ m

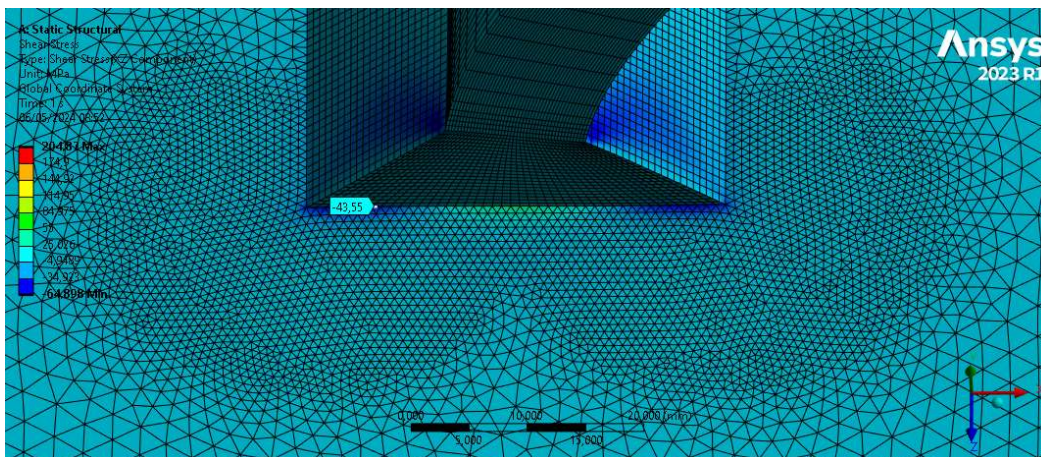


Figure 10. Shear stresses component $\tau_{xz} = -43.55$ MPa for the model with $L = 0.200$ m

The behavior of the shear stresses obtained by Eq. (7), along the length of the weld bead, are shown in Fig. 11, considering various thickness of vertical plates. Figure 12 shows the behavior of the width variation with the weld length.

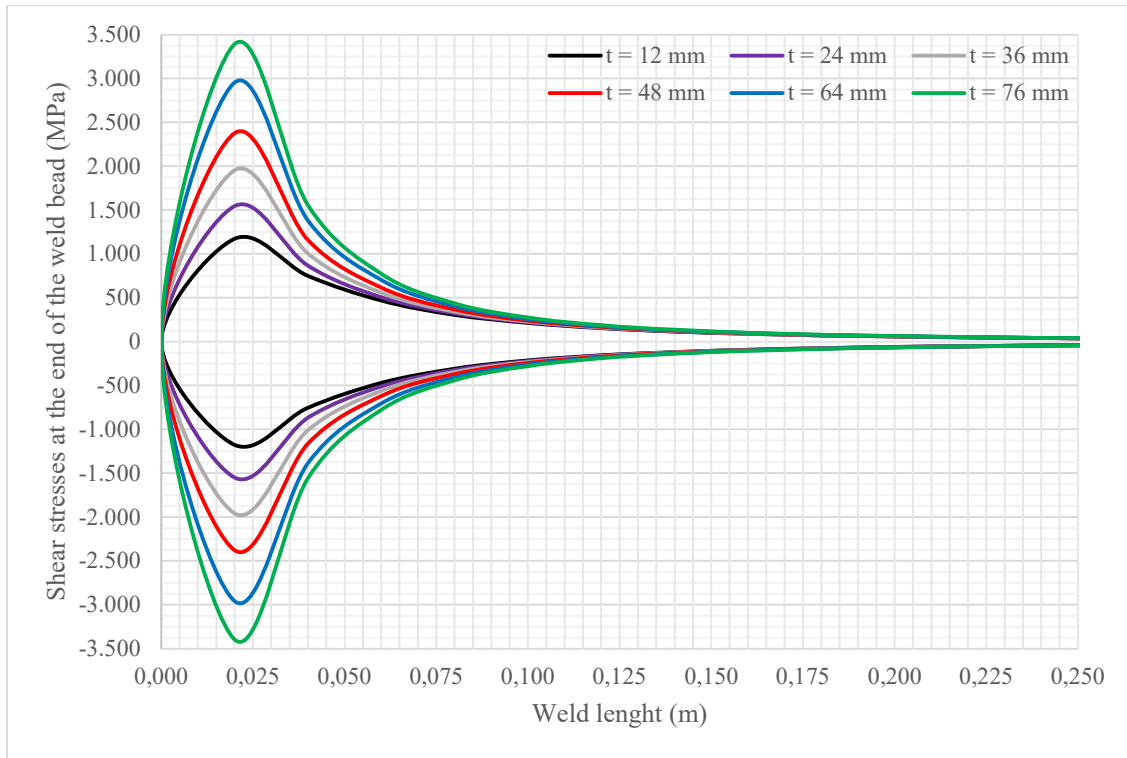


Figure 11. shear stresses along the weld length for various vertical plate thicknesses, Eq. (7).

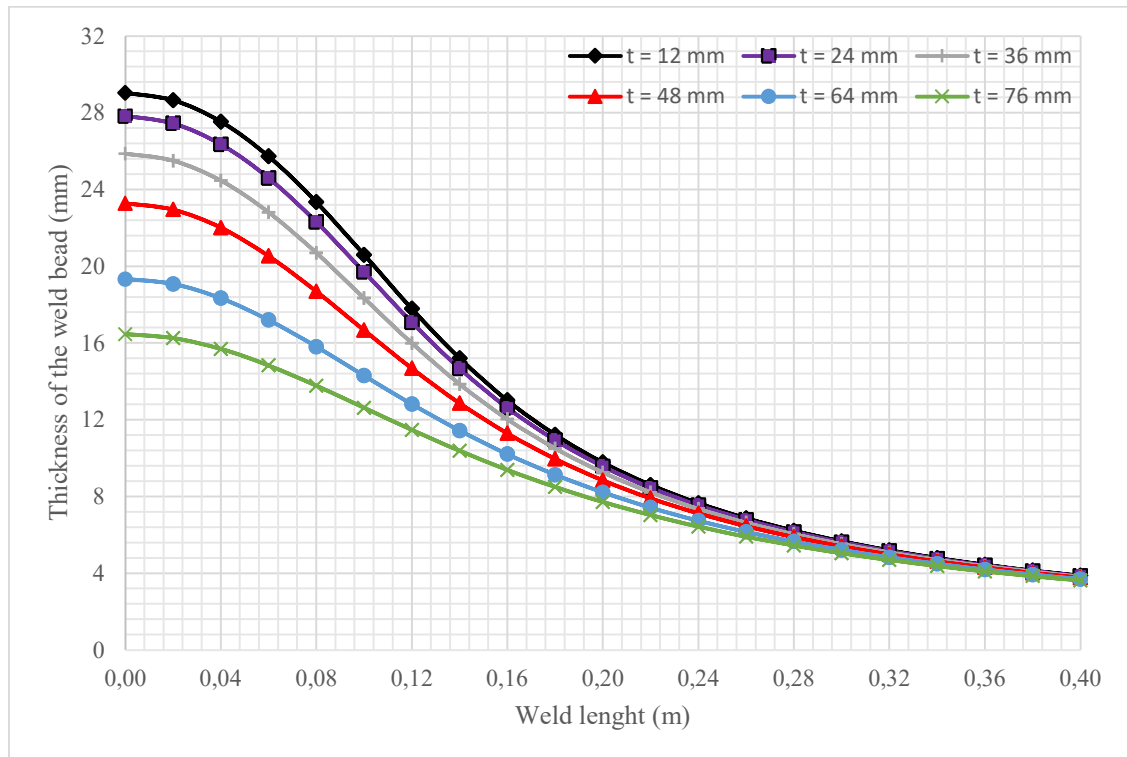


Figure 12. Weld bead thickness variation versus weld length, Eq. (7)

5. CONCLUSIONS

In this work an analytical solution is proposed to determine the ultimate shear stress in a weld line connecting perpendicular plates. A modeling by finite element method was carried out to assess the proposed equations. Therefore, based on the results obtained it is possible to draw the following conclusions.

In comparison with the computational modeling, which is the solution of the linear elasticity problem in three dimensions, it is possible to conclude that the analytical solution developed in this work is an appropriate formulation for

the design and verification of T-joints in perpendicular plates subjected to torsional moment, allowing an evaluation of the shear stresses distribution and the maximum values where really occur.

The results in Tab. 2 compare a pure torsional state in the analytical formulation with the computational modeling by FEM, with three degrees of freedom per node. Residual stresses due to the effect of temperature were not considered in this work and may be evaluated in the future.

The influence of the height of the vertical plate on results also need investigation. It is important also to keep in mind that the solution developed for obtaining the shearing stress can be applied to different kinds of materials.

For future works, other dimensions of the plates and weld beads can be investigated, and experimental activity also can be performed. Simulations based on nonlinear hypothesis can be also object of investigation.

6. ACKNOWLEDGEMENTS

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8. AUTHORIAL RESPONSIBILITY

The authors are solely responsible for the content of this work.