COB-2023-1108

A SLUG FLOW MODEL WITH CONCAVE INTERFACE FOR GAS-LIQUID FLOW IN HORIZONTAL PIPES

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Abstract. Two-phase slug flows are commonly found in a wide range of industrial applications such as in oil and gas extraction, processing and distribution, in power plants, chemical plants, etc. In this work, the mechanistic model of Orell (2005) for gas-liquid slug flow in horizontal pipes has been reformulated. Instead of a flat gas-liquid interface for the film region in the original model, the reformulated model considers the concavity of the interface by a double-circle model. This work investigated the capacity of the proposed model to predict the pressure drop and liquid holdup, employing five correlations for the interfacial friction factor, compared to the original model. It was observed that, particularly for gas-liquid slug flows with higher superficial gas velocities and lower liquid holdups, the proposed model predicted the pressure drop more accurately compared to the conventional flat interface model. The liquid holdup predicted by the model was considerably more accurate compared to the model of Orell (2005) for the air-water system. While, for the air-oil system, the model predicted the liquid holdup more accurately than the model of Orell (2005) only for lower values of gas superficial flow rate.

Keywords: Slug flow, double-circle model, horizontal flow, two-phase pressure drop

1. INTRODUCTION

Slug flows are frequently encountered in various industrial applications, like in all phases of the oil and gas processing, including extraction, refining and distribution. Slug flows are characterized by their intermittent nature and presence of elongated bubbles, separated from one another by aerated liquid slugs.

One of the most utilized simplified models for slug flow is the one of Orell (2005), which is a reformulated version of the model developed by Taitel and Barnea (1990), including additional sources of pressure loss according to Cook and Behnia (2000). The model of Orell (2005) is a one-dimensional and steady state model, which assumes a uniform liquid film thickness and a flat gas-liquid interface in the film region. The model provided accurate predictions of pressure drop and liquid holdup to a wide range of slug flow parameters, but presented a significant discrepancy in some cases, specially for high values of gas superficial velocity.

Zhang et al. (2000) developed a unified model for gas-liquid flows based on slug flow dynamics, since the slug flow region in the flow pattern map shares borders with all the other flow patterns (stratified, wavy, annular and dispersed flow). The authors suggested using a double-circle model for the description of the gas-liquid interface in the film region of the slug, but opted for a flat interface in their unified model since the double circle formulation would increase significantly the complexity of the model. To the authors’ knowledge, the only slug flow model in literature to account for the concavity of the interface in the film region was the one of Liu et al. (2022), where only few experimental data sets were used for validation. Liu et al. (2022) used the correlation of Fan (2005) for the wetted wall fraction and the correlation of Andritsos and Hanratty (1987) for the interfacial friction factor.

In this work, a reformulated version of the model of Orell (2005) was developed to account for the concavity of the gas-liquid interface in the film region. This work aimed to investigate the applicability of the model in predicting the slug flow main thermal-hydraulic parameters, such as the pressure drop and the liquid holdup, and, additionally, to analyze the effect of different interfacial friction factor correlations on the prediction of those parameters. The developed model predicted the pressure gradient with higher accuracy for the air-water system when compared to the model of Orell (2005), specially when the interfacial friction factor correlation of Andritsos and Hanratty (1987) was used. However, for the air-oil system, the correlation of Agrawal et al. (1973) provided more accurate pressure gradient results. The liquid holdup prediction by the model presented a slight improvement for the air-water system when compared to the model of Orell (2005), but, for the air-oil system, the liquid holdup predicted by the proposed model was more accurate than the model...
of Orell (2005) only for lower values of superficial gas velocity $U_{GS}$.

![Figure 1: Control volume for the slug unit](image)

**2. METHODOLOGY**

The physical problem consists of a gas-liquid slug flow in a horizontal circular pipe. The slug unit of length $\ell_u$ comprises a slug region of length $\ell_s$ and a film region of length $\ell_f$, as shown in Fig. 1. The liquid film thickness was assumed uniform throughout the film region and the liquid droplet entrainment in the gas core within the film region was disregarded. The liquid and gas mass balances over the slug unit are expressed as (Orell, 2005):

$$\ell_u U_{LS} = U_s H_s \ell_s + U_f H_f (\ell_u - \ell_s)$$  \hspace{1cm} (1)

$$\ell_u U_{GS} = U_s (1 - H_s) \ell_s + U_G (1 - H_f) (\ell_u - \ell_s)$$  \hspace{1cm} (2)

where $H_s$ and $H_f$ are, respectively, the liquid holdup in the slug and in the film region, $U_G$ and $U_f$ are, respectively, the actual velocities of the gas and liquid phases.

As suggested by Taitel and Barnea (1990), the aerated liquid slug velocity $U_s$ is given by:

$$U_s = U_m = U_{LS} + U_{GS}$$  \hspace{1cm} (3)

where $U_m$ is the gas–liquid mixture superficial velocity.

The nose of the elongated gas bubble, as well as the slug unit, propagates downstream with a translational velocity $U_t$, defined by (Andreussi et al., 1993):

$$U_t = \begin{cases} 
1.05 U_s + 0.542 \sqrt{gD}, & \text{for } \frac{U_s}{\sqrt{gD}} \leq 3.5 \\
1.2 U_s, & \text{for } \frac{U_s}{\sqrt{gD}} > 3.5 
\end{cases}$$  \hspace{1cm} (4)

As suggested by Taitel and Barnea (1990), the liquid mass balance for a coordinate system moving at the slug unit translational velocity $U_t$ is given by:

$$(U_t - U_f) H_f = (U_t - U_s) H_s$$  \hspace{1cm} (5)

The momentum balance for each phase within the film region is expressed by (Orell, 2005):

$$-A_f \frac{dP}{dz} = \tau_f S_f - \tau_l S_I$$  \hspace{1cm} (6)

$$-A_G \frac{dP}{dz} = \tau_G S_G + \tau_l S_I$$  \hspace{1cm} (7)

where $A_G$ and $A_f$ represent the cross-sectional areas occupied by the gas and liquid phases, respectively. $\tau_G$ and $\tau_f$ denote the shear stresses at the pipe wall for the gas and liquid phases, respectively. $S_G$ and $S_f$ are the wetted perimeters of the gas and liquid phases, respectively. $\tau_I$ corresponds to the interfacial shear stress, $S_I$ is the perimeter of the interface, $P$ represents the pressure, and $z$ denotes the axial coordinate of the pipe. By substituting Eq (6) into Eq (7), the combined momentum equation yields:
The shear stresses $\tau_f$, $\tau_G$, and $\tau_I$ were evaluated as:

$$\tau_f = \frac{1}{2} f_f \rho_L U_f^2, \quad \tau_G = \frac{1}{2} f_G \rho_G U_G^2, \quad \tau_I = \frac{1}{2} f_I \rho_G (U_G - U_f)^2$$

where $f_f$, $f_G$, and $f_I$ represent the Fanning friction factors of the liquid film, gas core, and the gas-liquid interface, respectively, and, considering a smooth pipe, $f_f$ and $f_G$ can be evaluated using the following equations:

$$f_f = \frac{C_f}{Re_f^m}, \quad f_G = \frac{C_G}{Re_G^m}$$

where the Reynolds number of each phase ($Re_f$ and $Re_G$) is calculated by:

$$Re_f = \frac{\rho_L U_f D_{hf}}{\mu_L}, \quad Re_G = \frac{\rho_G U_G D_{hG}}{\mu_G}$$

and the coefficients of Eq. 10 are calculated as:

$$\begin{cases} 
C_f = 16, \quad m = 1, & \text{for } Re_f \leq 2100 \\
C_f = 0.046, \quad m = 0.2, & \text{for } Re_f > 2100 \\
C_G = 16, \quad m = 1, & \text{for } Re_G \leq 2100 \\
C_G = 0.046, \quad m = 0.2, & \text{for } Re_G > 2100 
\end{cases}$$

The hydraulic diameters of the liquid and gas phases in the film region, $D_{hf}$ and $D_{hG}$, respectively, in Eq. 11, were calculated by:

$$D_{hf} = \frac{4 A_f}{S_f}, \quad D_{hG} = \frac{4 A_G}{(S_G + S_I)}$$

So far in this section, all the equations presented constitute the model of Orell (2005), except for Eq. 4, which was reformulated by Andreussi et al. (1993). Since the modified model developed in this work considered a different interface geometry, one of the modifications made are the geometrical equations of the film zone, which are described by a double-circle model.

### 2.1 Geometric relationships

The gas-liquid interface was described by a double-circle model, as shown in Fig. 2 (Chen et al., 1997). In this model, the geometrical parameters can be expressed as follows:

$$S_G = (\pi - \theta)D, \quad S_f = \theta D, \quad S_I = \theta_1 D_i, \quad \Theta = \frac{\theta}{\pi}$$

$$A_G = (1 - H_f) \frac{\pi D^2}{4}, \quad A_f = H_f \frac{\pi D^2}{4}$$

where $D$ and $D_i$ are the diameters of the pipe and the eccentric circle, respectively, $\theta$ and $\theta_i$ denote the film-wetted angles in the black circle (pipe) and in the blue circle (eccentric circle) of Fig. 2, respectively, while $H_f$ is the film liquid holdup. $\Theta$ is the wetted wall fraction. Additionally, the following geometric relationships are valid:

$$D_i = \frac{D \sin \theta}{\sin \theta_i}$$

$$\theta_i = \left( \frac{\sin \theta_i}{\sin \theta} \right)^2 \left( \theta + \frac{\sin^2 \theta}{\tan \theta_i} - \frac{\sin 2\theta}{2} - \pi H_f \right)$$
Apart from the geometrical relationships, the other modifications made in the original model of Orell (2005) were the different closure relationships, except for the slug liquid holdup $H_s$, where the same relationship presented in Orell (2005) was used. Different closure relationships were introduced for the friction factor $f_I$, the wetted wall fraction $\Theta$ and the slug unit length $\ell_u$.

### 2.2.1 Interfacial friction factor

In the original work of Orell (2005), the interfacial friction factor was calculated by the correlation of Cohen and Hanratty (1968). In this work, five different correlations were considered for the interfacial friction factor $f_I$:

- **Andritsos and Hanratty (1987):**

  \[
  f_I = \begin{cases} 
  f_G, & \text{for } U_{GS} \leq 5 \text{ m/s} \\
  f_G \left[ 1 + 15 \sqrt{\frac{H_I}{D}} \left( \frac{U_{GS}}{5} - 1 \right) \right], & \text{for } U_{SG} > 5 \text{ m/s}
  \end{cases}
  \] (18)

- **Cohen and Hanratty (1968):**

  \[ f_I = 0.0142 \] (19)

- **Hamersma and Hart (1987):**

  \[
  f_I = 0.0625 \left\{ \log \left[ \frac{15}{Re_G} + \frac{2.3H_ID}{4 \arccos \left( 1 - 0.66 \frac{\rho_L}{\rho_L - \rho_G} \frac{U_I^2}{gD} \right)} \right] \right\}^{-2}
  \] (20)

- **Agrawal et al. (1973):**

  \[ f_I = 1.3Re_G^{0.57} \] (21)
• Crowley et al. (1992):

\[ f_I = \begin{cases} 
  f_G, & \text{for } U_{GS} \leq 0.1 \text{m/s} \\
  10 f_G, & \text{for } U_{GS} > 0.1 \text{m/s}
\end{cases} \quad (22) \]

### 2.2.2 Wetted wall fraction

The wetted wall fraction $\Theta$ was calculated by the correlation of Grolman and Fortuin (1997):

\[
\Theta = 0.624 H_f^{0.374} \left( \frac{\sigma_{\text{water}}}{\sigma} \right)^{0.15} + \frac{\rho_G}{\rho_L - \rho_G \cos \theta} \left( \frac{\rho_L U_{SL}^2 D}{\sigma} \right)^{0.25} \left( \frac{U_{SG}^2}{(1 - H_f)^2 gD} \right)^{0.8} \quad (23)
\]

### 2.2.3 Slug liquid holdup

In this study, the correlation originally presented by Andreussi and Bendiksen (1989), and subsequently reformulated by Andreussi et al. (1993), was employed to calculate the slug holdup, as in Orell (2005):

\[
H_s = 1 - \frac{U_m/\sqrt{gD} - F_0}{U_m/\sqrt{gD} + 2400 B_o^{-3/4}} \quad (24)
\]

where $Bo$ is the Bond number, defined by:

\[
B_o = \frac{(\rho_L - \rho_G) g D^2}{\sigma} \quad (25)
\]

and $F_0$ is given by:

\[
F_0 = \begin{cases} 
  2.6 \left[ 1 - 2 \left( \frac{0.025}{D} \right)^2 \right], & \text{for } D \geq 0.0353 \\
  0, & \text{for } D < 0.0353
\end{cases} \quad (26)
\]

### 2.2.4 Slug unit length

In the original model of Orell (2005), the slug length $\ell_s$ is not a variable, since only the relative length of the slug and the film regions is provided ($\ell_s/\ell_u$ and $\ell_f/\ell_u$). According to Gonçalves et al. (2018), the slug length may be furnished by the model of Orell (2005) when an additional closure relationship for the slug frequency $\nu_t$ is added, which is equivalent to fixing the slug unit length $\ell_u$, since the translational velocity $U_t$ is a fixed parameter, and $\nu_t$ and $\ell_u$ are related by the following equation:

\[
\nu_t = \frac{U_t}{\ell_u} \quad (27)
\]

In this work, the slug frequency $\nu_t$ was calculated by the correlation of Zabaras (2000) for horizontal slug flow:

\[
\nu_t = 0.0189 \left( \frac{U_{LS}}{gD} \right)^{1.2} \left( \frac{19.75}{U_s} + U_s \right)^{1.2} \quad (28)
\]

### 2.3 Pressure gradient calculation

The average pressure gradient of the slug unit was determined by (Orell, 2005):
\[
\frac{dP}{dz} = 2f_s \rho_s U_s^2 \ell_s \ell_u + \frac{\tau_f S_f - \tau_i S_i}{A_f} \ell_f \ell_u
\]  
(29)

where \(\rho_s = \rho_L H_s + \rho_G (1 - H_s)\) and the slug friction factor \(f_s\) is calculated by:

\[
f_s = \begin{cases} 
16/Re_s, & \text{for laminar flow} \\
0.046/Re_s^{0.2}, & \text{for turbulent flow}
\end{cases}
\]  
(30)

where the Reynolds number of the slug \(Re_s\) is:

\[
Re_s = \frac{\rho_s U_s D}{\mu_{eff}}
\]  
(31)

and the effective viscosity \(\mu_{eff}\) is defined as:

\[
\mu_{eff} = \mu_L (1 + 2.5(1 - H_s))
\]  
(32)

2.4 Slug unit liquid holdup calculation

The average slug unit liquid holdup \(H_u\) was calculated by:

\[
H_u = H_s \frac{\ell_s}{\ell_u} + H_f \frac{\ell_f}{\ell_u}
\]  
(33)

2.5 SOLUTION METHOD

Equations (1)-(3), (5) and (8) are the main governing equations of this model. Adding the auxiliary Eqs. (9), (10), (14), (15) and (17) into these equations results in a set of nine simultaneous equations that contain nine unknown variables: \(\Theta, U_f, U_G, f_f, f_G, \ell_s, \theta_i, H_f,\) and \(\ell_u\). The resulting system of nonlinear equations was solved numerically using the Newton-Raphson method implemented in the program Wolfram Mathematica v.12.3. The input variables for the code developed were: \(U_{GS}, U_{LS}, D, \rho_L, \rho_G, \mu_L, \mu_G\) and \(\sigma\).

3. RESULTS AND DISCUSSION

In this section, the accuracy of the model was assessed by comparing its pressure gradient and liquid holdup predictions to their respective experimentally measured values. The experimental works cited in Orell (2005) served as data sources for this comparison, where both air-water and air-oil systems were analyzed separately.

Figure 3 illustrates a comparison between the pressure gradient calculated by the proposed model, employing five different correlations for the interfacial friction factor, and the pressure gradient predicted by the model of Orell (2005). In Fig. 3a, where the superficial liquid velocity \(U_{LS} = 0.9\) m/s, a satisfactory agreement between the predicted pressure drop and the experimental values of Nadler and Mewes (1995), specially when the correlations of Andritsos and Hanratty (1987) and Crowley et al. (1992) are used. For a superficial liquid velocity \(U_{LS}\) of 1.5 m/s (Fig. 3b), both the proposed model and the model of Orell (2005) significantly underestimated the pressure drop. However, the proposed model predicted the pressure drop more accurately in both conditions, when compared to the model of Orell (2005), and the best performing friction factor correlation was the one of Andritsos and Hanratty (1987).

Figure 4 illustrates a similar comparison, but for the experimental conditions of Kokal and Stanislav (1989): air-oil system with internal pipe diameter \(D = 0.0763\) m and two liquid superficial velocities: \(U_{LS} = 0.9\) m/s (Fig. 4a) and \(U_{LS} = 0.8\) m/s (Fig. 4b). In both cases, it was observed that the proposed model may be more or less accurate than the model of Orell (2005), depending on the interfacial friction factor correlation chosen. The correlation of Agrawal et al. (1973) was the one that provided the most accurate pressure drop prediction. This is presumably explained by the fact that Agrawal et al. (1973) developed their empirical correlation based on experiments with an air-oil system. The same correspondence was obtained for the air-water systems studied (Fig. 3), since the correlation of Andritsos and Hanratty (1987) was developed based on experimental results for a air-water system.

Figure 5 presents the slug unit liquid holdup \(H_s\) as a function of the superficial gas velocity \(U_{GS}\) for the same system parameters of Heywood and Richardson (1979) (air-water system, \(D = 0.042\) m and two different superficial liquid velocities: \(U_{LS} = 0.978\) m/s, shown in Fig. 5a, and \(U_{LS} = 1.22\) m/s, shown in Fig. 5b). The liquid holdup prediction was
Figure 3: Pressure gradient predicted by the present model, for five different interfacial friction factor correlations, and by the model of Orell (2005), in comparison with the experimental results of Nadler and Mewes (1995), for an air-water slug flow in a pipe with $D = 0.059$ m for: (a) $U_{LS} = 0.9$ m/s and (b) $U_{LS} = 1.5$ m/s.

Figure 4: Pressure gradient predicted by the present model, for five different interfacial friction factor correlations, and by the model of Orell (2005), in comparison with the experimental results of Kokal and Stanislav (1989), for an air-oil slug flow in a pipe with $D = 0.0763$ m for: (a) $U_{LS} = 0.5$ m/s and (b) $U_{LS} = 0.8$ m/s.

not considerably affected by the interfacial friction factor correlation used, as shown in Fig. 5, where all the curves of the proposed model nearly overlap each other. When compared to the model of Orell (2005), the proposed model provided a considerably more accurate slug liquid holdup prediction, specially for lower gas superficial velocities.

Similarly, in Figure 6, the slug unit liquid holdup $H_s$ is presented as a function of the superficial gas velocity $U_{GS}$ for the same system parameters of Kokal and Stanislav (1989) (air-oil system, $D = 0.0763$ m and two different superficial liquid velocities: $U_{LS} = 0.5$ m/s, shown in Fig. 6a, and $U_{LS} = 0.8$ m/s, shown in Fig. 6b). It was observed that, when $U_{LS} = 0.5$ m/s, the liquid holdup prediction is not significantly affected by the choice of the interfacial friction factor correlation. When $U_{LS} = 0.8$ m/s, the correlation of Cohen and Hanratty (1968) provided significantly different results compared to the other correlations, which nearly overlap one another, as shown in Fig. 6b. The proposed model was capable of predicting the liquid holdup with significant higher accuracy compared to the model of Orell (2005), except for higher values of $U_{GS}$ and when the correlation of Cohen and Hanratty (1968) is used.

A statistical analysis of the results is presented in Figs. 7 and 8, where other experimental datasets were included. Figure 7 illustrates the pressure gradient (Fig. 7a) and the liquid holdup (Fig. 7b) predicted by the proposed model with the correlation of Andritsos and Hanratty (1987) and by the model of Orell (2005) in comparison with the experimental data for the air-water system. In a similar manner, in Fig. 8, the pressure gradient (Fig. 8a) and the liquid holdup (Fig.
A Slug Flow Model with Concave Interface for Gas-liquid Flow in Horizontal Pipes

Figure 5: Slug unit liquid holdup predicted by the present model with five different interfacial friction factor correlations and by Orell (2005), in comparison with the experimental data of Heywood and Richardson (1979) for air–water flow in a pipe with $D = 0.042$ m for: (a): $U_{LS} = 0.978$ m/s and (b): $U_{LS} = 1.22$ m/s.

Figure 6: Slug unit liquid holdup predicted by the present model with five different interfacial friction factor correlations and by Orell (2005), in comparison with the experimental data of Kokal and Stanislav (1989) for air–oil flow in a pipe with $D = 0.0763$ m for: (a): $U_{LS} = 0.5$ m/s and (b): $U_{LS} = 0.8$ m/s.

8b) predictions provided by the proposed model with the correlation of Agrawal et al. (1973) and by the model of Orell (2005) are compared to experimental data for the air-oil system. Additionally, in Tab. 1, the absolute average percent errors of the proposed model and the model of Orell (2005) are presented, considering the friction factor correlation utilized for the air-water and air-oil systems was, respectively, the correlation of Andritsos and Hanratty (1987) and the correlation of Agrawal et al. (1973). It was observed that, for air-oil systems, the proposed model with the correlation of Agrawal et al. (1973) presented consistent improvement of the pressure drop prediction compared to the model of Orell (2005), while for the liquid holdup, both the proposed model and the model of Orell (2005) presented approximately equivalent levels of accuracy. For air-water systems, the opposite behaviour was observed: the proposed model with the correlation of Andritsos and Hanratty (1987) improved the liquid holdup prediction considerably compared to the model of Orell (2005), while the pressure drop calculated by both the proposed model and the model of Orell (2005) presented approximately the same level of accuracy.
Table 1: Average errors of the proposed model and the model of Orell (2005).

<table>
<thead>
<tr>
<th>Predicted variable</th>
<th>System</th>
<th>Error of the proposed model (%)</th>
<th>Error of the model of Orell (2005) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$dP/dz$</td>
<td>air-water</td>
<td>14.0</td>
<td>13.8</td>
</tr>
<tr>
<td>$dP/dz$</td>
<td>air-oil</td>
<td>19.2</td>
<td>24.3</td>
</tr>
<tr>
<td>$H_u$</td>
<td>air-water</td>
<td>7.4</td>
<td>10.5</td>
</tr>
<tr>
<td>$H_u$</td>
<td>air-oil</td>
<td>9.2</td>
<td>8.8</td>
</tr>
</tbody>
</table>

Figure 7: Slug unit liquid holdup $H_u$ and pressure gradient $dP/dz$ predicted by the proposed model with the correlation of Andritsos and Hanratty (1987) and by the model of Orell (2005) compared to experimental data for air-water systems.

Figure 8: Slug unit liquid holdup $H_u$ and pressure gradient $dP/dz$ predicted by the proposed model with the correlation of Agrawal et al. (1973) and by the model of Orell (2005) compared to experimental data for air-oil systems.

4. CONCLUSIONS AND FUTURE WORK

Considering the gas-liquid interface concavity in the slug flow film region, we proposed a reformulated model that showed a significant improvement in the prediction of the pressure gradient and the slug unit liquid holdup when compared to the original model of Orell (2005). With the experimental datasets analyzed, the correlation of Andritsos and Hanratty (1987) was the most accurate for the air-water systems, while the correlation of Agrawal et al. (1973) was the most accurate for the air-oil systems. Future implementations shall analyze different correlations for the wetted wall fraction and assess the suitability of the model for predicting flow pattern transitions (eg. slug to annular flow).
5. ACKNOWLEDGEMENTS

The authors are thankful for the financial support provided by CNODC Brasil Petrôleo e Gás through ANP PD&I program, EMBRAPPII, CNPq, CAPES and FAPERJ.

6. REFERENCES


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