NUMERICAL ANALYSIS OF THERMAL INSULATION PERFORMANCE IN HEATING PIPE SYSTEMS USING THE FINITE VOLUME METHOD

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Abstract. This study aims to investigate the performance of thermal insulation in heating pipe systems by analyzing the thermal and geometric parameters that influence heat flow. The steady-state, one-dimensional heat diffusion equation with constant properties was used to model the problem. Specified temperature and convective boundary conditions were adopted on both sides. To solve the differential equation, the finite volume method was employed, and the Thomas algorithm (TDMA-1D) was used to solve the resulting algebraic equations. The implementation of the numerical code was in the MATLAB R2022a platform. Different heat transfer coefficients, thermal conductivity, and insulation radius ratios were considered to analyze the impact of these parameters on heat flow. The results show that there are configurations where heat flow in the insulation is intensified for low values of the adopted parameters. Variations in the insulation radius ratio also had a significant impact on heat flow. The use of a dimensionless parameter for insulation thickness is a considerably useful tool for comparing different possibilities to be evaluated in thermal insulation pipe projects.

Keywords Thermal insulation, Critical radius, Finite volume method

1. INTRODUCTION

The critical thickness of insulation is a factor of great importance in heat transfer analysis. Adding more insulation to a flat surface reduces the rate of heat transfer. However, adding insulation to a cylindrical piece or spherical shell exhibits a different behavior. There is a tendency for an increase in the conduction resistance in the insulation layer and a reduction in the convection resistance due to the increased external area of the convection surface (Çengel, 2009). Understanding the temperature distribution is useful in the process of optimizing the thickness of an insulating material or determining the compatibility between a coating and a material. It is through the temperature distribution in a medium that it is possible to determine the heat flow.

The application of thermal insulation is a common practice in duct engineering. However, determining the ideal thickness of insulation is a continually evolving challenge. Li et al. (2012) present a finite volume method based on local analytical solution for the analysis of heat conduction in cylinders. Kaynakli (2014) provides a review of studies related to the determination of the ideal thickness of thermal insulation for pipes and ducts, evaluating the operating conditions and parameters adopted in the listed studies. It also presents studies that seek to determine the optimal insulation thickness in pipes and ducts with various geometries to reduce heat transfer by convection and radiation. Zarubin et al. (2016) study heat transfer through a thermal insulation layer in radiation-convection heat transfer on a non-concave surface, developing a qualitative analysis of the heat flow dependency on the determining parameters for the thermal characteristics of the insulation and its thickness. Shi (2020) presents a methodology for calculating the critical thickness of insulation for an elliptical surface and a dimensionless correlation based on the Biot number. The proposed idea can also be adopted for circular sections after adjusting the geometric parameters accordingly. Anisimova (2020) developed a mathematical model for determining the ineffective thickness of a thermal insulation system, adopting the Newton's method algorithm to calculate the solution to the problem. Usman and Kim (2022) employed computational fluid dynamics to evaluate the optimum insulation thickness based on material and digging costs in South Korea, proposing a micro hybrid district heating (DH) system. Powar and Dhamangaonkar (2022) use the academic version of ANSYS Fluent 19.2 software to perform numerical simulations to estimate the critical thickness of thermal insulation in a circular-geometry duct, adopting the Dirichlet boundary condition for the internal region and the Robin boundary condition for the external region.

In this context, the aim of this work is to present the mathematical and numerical modeling for the phenomenon of heat diffusion in a steady state, unidimensional (r-direction) within a thermal insulator. For the boundary condition of the inner region, two approaches are considered: specified temperature and convection. For the outer region, only convection
is considered. Furthermore, the work proposes the implementation of the code using the finite volumes method in the Matlab software. The study also includes the numerical analysis of the heat flow as a function of the dimensionless parameter "radius ratio" \( (\beta) \), considering the influence of the external heat transfer coefficient and thermal conductivity.

2. PROBLEM FORMULATION

In this section, the equations that model the problem and their boundary conditions are presented. The phenomenon under analysis is illustrated in Fig. 1. Due to the tube's small thickness and high thermal conductivity compared to that of the insulation, it was neglected in the mathematical modeling. Starting from the differential equation of heat diffusion in cylindrical coordinates, considering constant thermal conductivity, one-dimensional problem (radial direction), steady-state, and no heat generation, it can be written in the form of Eq. (1).

\[
\frac{d}{dr}\left( r \frac{dT}{dr} \right) = 0, \quad R_{\text{int}} < r < R_{\text{ext}}
\]  

(1)

The boundary conditions considered for the internal surface (specified temperature and convection) yield Eq. (2a.i) and Eq.(2a.ii). On the external surface, the boundary condition is of the convection type, Eq. (2b).

\[
T = cte = 50{\degree}C
\]

\[-k \frac{dT}{dr} = h_{\text{int}}(T_{\infty} - T(R_{\text{int}})) \quad , \quad r = R_{\text{int}}
\]  

(2a.i)

\[-k \frac{dT}{dr} = h_{\text{ext}}(T(R_{\text{ext}}) - T_{\infty}), \quad r = R_{\text{ext}}
\]  

(2a.ii)

(2b)

3. NUMERICAL MODEL

Eq. (1) was numerically solved using the finite volume method (Patankar, 1980; Maliska, 2004; Versteeg and Malalasekera, 2007).

The finite volume method involves first dividing the problem domain into finite volumes, thus forming a computational mesh, as shown in Fig. 2. Then, the integration of the differential equation for the dependent variable is performed over each volume element. This ensures that the conservation of the involved property is satisfied in each volume element of the mesh and, consequently, throughout the solution domain. This procedure results in a system of algebraic equations for each variable of the model. By solving the system of linear algebraic equations, the property distribution of the problem domain is obtained. The system of equations is solved using the Thomas algorithm for one dimension (TDMA-1D), which is considered a direct method for one dimension and an iterative method for two and three dimensions (Maliska, 2004).
Applying the finite volume method to Eq. (1):

\[ \int_{w}^{e} d(r \frac{dT}{dr}) dr = 0 \]  

Integrating Eq. (3) over the limits e-w:

\[ \int_{w}^{e} d(r \frac{dT}{dr}) dr = r \frac{dT}{dr} \bigg|_{e}^{w} = 0 \]  

\[ r_e \left( \frac{T_e - T_p}{\delta r_e} \right) - r_w \left( \frac{T_p - T_w}{\delta r_w} \right) = 0 \]

\[ T_p = \left( \frac{r_e}{\delta r_e} \right) T_E + \left( \frac{r_w}{\delta r_w} \right) T_W \]

The discretized algebraic equation is given by Eq. (6).

\[ A_p T_p = A_E T_E + A_W T_W \]  

For the volumes that belong to the boundary conditions, it is given by:

\[ A_p T_p = A_E T_E + A_W T_W \] (Specified Temperature)  
(7a)

\[ A_p T_p = A_E T_E + A_{W,2} T_{W,2} \] (Convection at the internal surface)  
(7b)

\[ A_p T_p = A_E T_{W,2} + A_W T_W \] (Convection at the external surface)  
(8)

In which the following relation must be satisfied: \( A_p = A_E + A_W \)

Finally, Table 1 presents the coefficients, \( A_E \) and \( A_W \), of Eqs. (6), (7a), (7b), and (8) for each node.
Table 1. Temperature coefficients for each volume (Authors, 2023)

<table>
<thead>
<tr>
<th>Volume Type</th>
<th>( A_E )</th>
<th>( A_W )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st volume (Specified Temperature)</td>
<td>( \frac{r_e}{\delta r_e} )</td>
<td>( \frac{r_w}{\delta r_w} )</td>
</tr>
<tr>
<td>1st volume (Convection)</td>
<td>( \frac{r_e}{\delta r_e} )</td>
<td>( \frac{r_w}{\delta r_w} )</td>
</tr>
<tr>
<td>Internal volumes</td>
<td>( \frac{r_e h_{\text{int}}}{k(1 + PE)} )</td>
<td>( \frac{r_w}{\delta r_w} )</td>
</tr>
<tr>
<td>Last volume</td>
<td>( \frac{r_e h_{\text{ext}}}{k(1 + PD)} )</td>
<td>( \frac{r_w}{\delta r_w} )</td>
</tr>
</tbody>
</table>

Where:

\[ PE = \frac{h_{\text{int}} \delta r_w}{k} \] (9)

\[ PD = \frac{h_{\text{ext}} \delta r_e}{k} \] (10)

4. NUMERICAL TEST DATA

The data used for the numerical experiment are presented in Table 2. It includes the geometric and thermal parameters of the case studied.

Table 2 Parameters used in the calculation (Authors, 2023)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal radius</td>
<td>( R_{\text{int}} )</td>
<td>0,020 m</td>
</tr>
<tr>
<td>Radius ratio</td>
<td>( \beta )</td>
<td>0,125 – 2,75</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>( k )</td>
<td>0,038; 0,043; 0,078; 0,1 W/m.K</td>
</tr>
<tr>
<td>Internal heat transfer coefficient</td>
<td>( h_{\text{int}} )</td>
<td>60 W/m².K</td>
</tr>
<tr>
<td>External heat transfer coefficient</td>
<td>( h_{\text{ext}} )</td>
<td>2; 16; 25 W/m².K</td>
</tr>
<tr>
<td>Internal working fluid temperature</td>
<td>( T_{\text{int}} )</td>
<td>50 °C</td>
</tr>
<tr>
<td>External working fluid temperature</td>
<td>( T_{\text{ext}} )</td>
<td>27 °C</td>
</tr>
</tbody>
</table>

5. RESULTS AND DISCUSSION

5.1 Critical radius ratio (\( \beta_{cr} \))

Let the critical insulation radius, Eq. (11), be a theoretical reference that indicates the value of the external radius \( (R_{\text{ext}}) \) of an insulating surface that will provide the highest heat exchange in a tube, determined by the ratio between the thermal conductivity coefficient \( (k) \) of the insulating material and the heat transfer coefficient by convection from the external environment \( (h_{\text{ext}}) \) (Çengel, 2009).

\[ R_{cr} = \frac{k}{h_{\text{ext}}} \] (11)

The term \( \beta \), referred to as the "radius ratio," represents the geometric coefficient calculated as the ratio between the insulation thickness and the internal radius of the tube, as given by Eq. (12).

\[ \beta = \frac{e}{R_{\text{int}}} = \frac{R_{\text{ext}} - R_{\text{int}}}{R_{\text{int}}} \] (12)

Considering the condition where \( R_{cr} = R_{\text{ext}} \) and applying it to Eq. (12), the coefficient \( \beta_{cr} \) ("critical radius ratio") is defined, indicating the value of the \( \beta \) coefficient for which there is maximum heat exchange in an insulated cylindrical
tube. Therefore, the heat transfer rate from the cylinder increases when $\beta < \beta_{cr}$, is maximum when $\beta = \beta_{cr}$, and decreases when $\beta > \beta_{cr}$. Another application for the use of $\beta_{cr}$ in thermal insulation cases is that, being a geometric coefficient, only conditions with $\beta_{cr} \geq 0$ deserve attention in the analyses, as it indicates that the calculated $R_{cr}$ is greater than the radius of the tube to be insulated.

This study will analyze the heat flow and the effects of insulation thickness based on the value of $\beta$. The advantage of using this coefficient lies in evaluating the heat exchange behavior considering the geometric conditions of the analyzed tube and the value of $\beta_{cr}$ for the tube's operating conditions. It also allows for the comparison of the insulation effectiveness of different geometric conditions.

5.2 Mesh independence

To analyze the influence of the mesh on the problem solution, four meshes were tested, as shown in Fig. 3. It was observed that the finest mesh (60 volumes) did not yield significantly different results compared to the coarsest mesh (10 volumes), considering the processing time. Therefore, the mesh with 40 volumes was chosen for further analysis.

5.3 Results and Analysis

The simulations for the analysis of heat flow were conducted in two scenarios for the internal conditions of the tube. "Case 1" refers to the scenario where internal convective effects are not considered in the heat flow analysis, while "Case 2" considers the effect of convection in the internal region of the tube under study. For each case, simulations were performed by varying $\beta$, $k$, and $h_{ext}$ according to the data provided in Table 2. The results presented in this section were obtained using a computational code developed in the Matlab R2022a platform.

Table 3 presents the critical radii (mm) for the values of $k$ and $h_{ext}$ shown in Table 1. Table 4 provides the corresponding values of $\beta_{cr}$ for each calculated $R_{cr}$ in Table 3.

<table>
<thead>
<tr>
<th>$h$ (W/m².K)</th>
<th>$k$ (W/m.K)</th>
<th>$R_{cr}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.038</td>
<td>0.043</td>
<td>0.078</td>
</tr>
<tr>
<td>2</td>
<td>19.00</td>
<td>21.50</td>
</tr>
<tr>
<td>16</td>
<td>2.38</td>
<td>2.69</td>
</tr>
<tr>
<td>25</td>
<td>1.52</td>
<td>1.72</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$h$ (W/m².K)</th>
<th>$k$ (W/m.K)</th>
<th>$R_{cr}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.038</td>
<td>0.043</td>
<td>0.078</td>
</tr>
<tr>
<td>2</td>
<td>-0.05</td>
<td>0.08</td>
</tr>
<tr>
<td>16</td>
<td>-0.88</td>
<td>-0.87</td>
</tr>
<tr>
<td>25</td>
<td>-0.92</td>
<td>-0.91</td>
</tr>
</tbody>
</table>

It can be observed from Table 4 that there are only 3 scenarios in which attention should be given to the effect of undesired increased heat transfer for pipe insulation, which are the cases where $\beta_{cr} \geq 0$.

The temperature distribution on the external surface of the tube for various values of $\beta$ with respect to the internal radius is shown in Figure 4. It can be observed that the temperature profile is no longer linear, as typically seen in a flat
Figures 5a and 5b show the behavior of heat flux for cases 1 and 2, respectively, with respect to the variation of $\beta$ for all values of $k$ presented in Table 2 and $h_{ext} = 2 \text{ W/m}^2 \cdot \text{K}$. In cases where $\beta_{cr} \geq 0$, an increase in heat flux out of the tube is observed when $\beta < \beta_{cr}$, a peak value for $\beta = \beta_{cr}$, and a decrease in heat flux for $\beta > \beta_{cr}$. Only one curve in these two cases presented in Figure 5 exhibits a decreasing heat flux behavior for all values of $\beta$, which corresponds to the value of $k = 0.038 \text{ W/m} \cdot \text{K}$, the only one that has $\beta_{cr} < 0$. Figure 6 provides a better comparison for the most significant cases of $\beta_{cr} > 0$.

![Figure 4. Influence of $\beta$ on the temperature profile (Authors, 2023).](image)

![Figure 5. Influence of $\beta$ on the heat flux for different convective coefficients and thermal conductivities for $h_{ext} = 2 \text{ W/m}^2 \cdot \text{K}$: (a) case 1; (b) case 2 (Authors, 2023).](image)
Figures 7a and 7b show the behavior of heat flux for cases 1 and 2, respectively, with respect to the variation of $\beta$ for all values of $k$ presented in Table 2 and $h_{ext} = 16 \text{ W/m}^2 \cdot \text{K}$. They exhibit an effective insulation behavior for all tested values of $k$, consistent with what is expected since all the $\beta_{crit}$ values for these cases were negative. This means that, for the simulated condition, there is no risk of reverse heat transfer effects caused by the existence of a critical insulation radius. It is notable the difference in heat flux values between the simulated conditions for low values of $\beta$, and there is a tendency for this difference to decrease as the value of $\beta$ increases.

A comparison between the conditions presented in Figures 5a, 5b, 7a, and 7b indicates the conservative nature of a heat flux analysis in a tube when the internal convective conditions are disregarded.

![Figure 6. Comparison of the influence of $\beta$ on the heat flux for different convective coefficients and thermal conductivities for $h_{ext} = 2 \text{ W/m}^2 \cdot \text{K}$: (a) case 1; (b) case 2 (Authors, 2023).](image)

![Figure 7. Influence of $\beta$ on the heat flux for different convective coefficients and thermal conductivities for $h_{ext} = 16 \text{ W/(m}^2 \cdot \text{K})$: (a) case 1; (b) case 2 (Authors, 2023).](image)
In this work, the mathematical and numerical modeling of steady-state heat diffusion in cylindrical thermal insulators was performed using the finite volume method and implemented in the Matlab software. The results highlighted the importance of the radius ratio ($\beta$) in thermal insulation efficiency. The analysis of heat flow and the effects of insulation thickness were based on the value of $\beta$ and the critical radius ratio ($\beta_{cr}$).

It was found that only three scenarios had $\beta_{cr} \geq 0$, indicating situations where attention should be given to the effect of undesired increased heat transfer in insulated tubes. The temperature distribution on the external surface of the tube exhibited a nonlinear profile as the value of $\beta$ increased. The behavior of heat flux in relation to the variation of $\beta$, thermal conductivity, and external heat transfer coefficient was also evaluated.

Through this study, the importance of the critical insulation thickness and its application in the optimization process of insulation material thickness or the determination of compatibility between a coating and a material was understood. The proposed methodology can serve as a basis for future research involving the analysis of ideal insulation thickness in different geometries and operating conditions.

6. REFERENCES


7. RESPONSIBILITY NOTICE

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