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LAGRANGIAN VORTICES WITH CORRECTED CORE-SPREADING METHOD AND LARGE EDDY SIMULATION (LES)

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Abstract. *This paper contributes with a new methodology blending the corrected core-spreading vortex method and large eddy simulation (LES). The new scheme is characterized by a deterministic and efficient grid-free method. A splitting algorithm is necessary to control the consistency error by maintaining small vortex core sizes, which are used to discretize the vorticity field; that approach incorporates viscosity into vortex simulations. On the other hand, large eddy simulation (LES) is a mathematical model for turbulence used in computational fluid dynamics. In the past, our research group adapted a second-order velocity structure function model to include turbulence modeling in a two-dimensional Lagrangian vortex method; however, the viscosity had been incorporated using the random walk method. The manuscript was structured aiming to discuss important aspects of the new methodology including a test case of the study of aircraft wake development in the vicinity of ground plane, a situation commonly found during landing or takeoff operations at airports around the world.*

Keywords: *Lagrangian vortex method, vorticity dynamics, corrected core-spreading method, turbulence modeling, aircraft wake dissipation*

1. INTRODUCTION

In fluid mechanics, most of problems of interest for engineering practical applications depend on fluid-structure interaction (FSI), e.g., automotive industry, aeronautics and transportation industry. Undoubtedly, the development of numerical techniques more accurate is crucial to construct more precisely the instantaneous vorticity field, which is intrinsic to all those problems.

In the last two decades, the Lagrangian description has been developed and applied by our research group aiming to solve specific problems of FSI analysis (e.g., Alcântara Pereira et al., 2004; Alcântara Pereira et al., 2020; Bimbato et al., 2011; Oliveira et al., 2020; Moraes and Alcântara Pereira, 2021; Moraes et al., 2022). In two-dimensions, the vorticity field is discretized and represented by a cluster of particles (specifically, the so called Lamb discrete vortices), such that each individual particle is tracked during a typical numerical simulation to solve the vorticity transport equation (Batchelor, 2000). And, as the vorticity is a scalar in two-dimensions, the vortex blobs must to satisfy the convective vorticity transport. In Lagrangian manner, the attention is only directed to the fluid domain regions of high activities, i.e., the regions containing vorticity. Eulerian schemes, on contrary, need to consider the entire fluid domain even for the sub-regions with less importance flow activities. In Lagrangian meshless schemes, it is also avoided to impose the far away boundary conditions, because they are automatically satisfied by tracking all the Lamb discrete vortices. Furthermore, the velocity field is instantaneously computed only where the vortex blobs are placed into the fluid domain. In general, the velocity field computation needs of three contributions, i.e., incident flow (irrotational flow), solid boundaries and cluster of vortex blobs (the Biot-Savart law or vortex-vortex interaction).

The convective vorticity transport incorporates advection and diffusion (Chorin, 1973). The vorticity advection is solved by treating each vortex blob as a fluid particle; put in other words, it is only necessary to integrate the motion equation of each vortex particle because its vorticity variation is zero (Helmholtz, 1867). On the other hand, the vorticity diffusion incorporates viscous effects (the decay of vorticity) for the vortex methods, which can be computed by using statistical solution or deterministic solution (Takeda et al., 1997).

The key ideas above presented constitute the essence of the Lagrangian vortex method. Additionally, Alcântara Pereira et al. (2003) presented an important contribution for the methodology when it was incorporated two-dimensional turbulence computation using large eddy simulation (LES) through the second-order velocity structure function model. Of particular interest for the present paper, the mentioned turbulence modeling is here originally blended with the corrected core-spreading vortex method; the later simulates vorticity diffusion in a deterministic approach (Rossi, 1996).

Our research group usually simulates viscous effects in the Lagrangian vortex method through the random vortex method, as introduced by Chorin (1973). That approach essentially is related with fractional step method, in which the viscous effects are taking into account in the mean, simulating diffusion by a random walk, i.e., a Brownian-like motion of the vortex blobs. Alcântara Pereira et al. (2003) originally included two-dimensional turbulence computation into the random walk method using LES, as above commented. However, the literature reports that the corrected core-spreading vortex method presents the most potential for the applications of high-Reynolds number flows.

Historically, Leonard (1980) introduced the core spreading method as purely Lagrangian scheme to incorporate viscous effects (exactly solving the diffusion equation) by changing the core size of the vortex blobs. However, Greengard (1987) presented mathematical objections concerning the effectively of this method, until a solution based on vortex blob splitting was proposed about a decade later by Rossi (1996). In fact, there was a consistency error of core spreading provoked by the advection without deformation of larger vortex blobs as they spread. And the problem was solved when Rossi (1996) proposed the vortex splitting idea, which proved to be convergent and effective. In this paper, the numerical solution of the vorticity diffusion by using the vortex splitting idea is successfully compared with two analytical solutions, one related with the diffusion of a vortex sheet and other with radial distribution of the vorticity. Finally, turbulence computation using LES supported by the second-order velocity structure function model is blended with the vorticity diffusion in a deterministic approach (Rossi, 1996). As example, the corrected core-spreading vortex method with LES is then applied for the problem aircraft wake vortices in ground effect without crosswind (Carvalho et al., 2022).

2. MATHEMATICAL FORMULATION AND SOME NUMERICAL METHOD DETAILS

In Fig. 1, a pair of vortical structures with circulation strength $\Gamma^* = \pm W^* / (\rho b^* U^*)$ (where W^* is the aircraft weight, ρ is the fluid density, b^* is the wingspan and U^* is the aircraft approaching velocity) illustrates the initial conditions for the computation of the decay of a vortex pair in neutrally stable turbulent atmosphere. The dynamic interactions of aircraft wake vortices incorporating viscosity are numerically simulated blending the corrected core-spreading vortex method with large eddy simulation (LES).

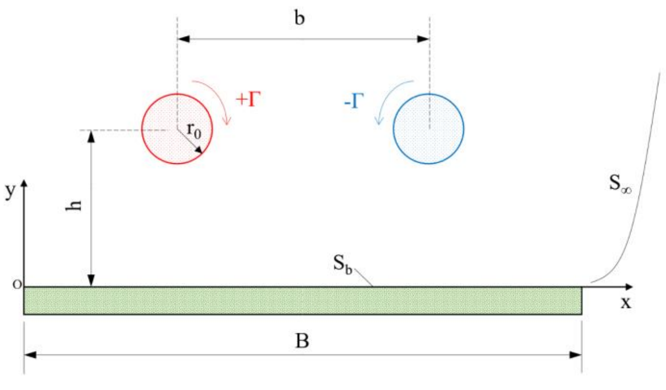


Figure 1. Problem geometry and definitions.

Still in Fig. 1, the origin of inertial frame is located at $x^* = 0$ and $y^* = 0$ and the centre of the vortical structures starts at $(B^*/2 \pm b^*/2;\ h^*)$. The fluid domain is defined by Ω and it is formed by solid contour $S = S_b \cup S_\infty$, where S_b defines the ground plane contour and S_∞ defines the fluid contour far from the ground plane surface. The boundary conditions of the problem must be imposed on the contour S. The flow generated by pair of vortical structures will induce vorticity generation on the ground plane surface of length B^*; that flow is assumed to be unsteady, incompressible and two-dimensional, and the fluid rheological behaviour is assumed to be Newtonian. There is no crosswind contribution for the computation of the decay of the vortex pair. So, the problem is non-dimensionalized by chosen b^* as representative length, Γ^*/b^* as representative velocity and b^{*2}/Γ^* as time scale. The symbol * denotes dimensional quantities.

The problem is governed by the continuity equation (the mass conservation) and the Navier-Stokes equation (the Newton´s second law of motion). Starting with the definition of vorticity, $\bar{\boldsymbol{\omega}} = \nabla \times \bar{\mathbf{u}}$, supported by the continuity equation, it can be taken the curl over the Navier-Stokes to obtain the well-known the vorticity transport equation. Here, it assumes the following particular form (Alcântara Pereira et al., 2003):

$$\frac{\partial \bar{\omega}}{\partial t} + \left(\bar{\mathbf{u}} \cdot \nabla\right)\bar{\omega} = \left(\frac{1}{Re} + \nu_t\right)\nabla^2 \bar{\omega} \ , \tag{1}$$

where $\bar{\mathbf{u}}$ is the velocity vector of the filtered field; the global Reynolds number and the local eddy viscosity coefficient (instantaneously computed for each vortex blob) are also defined as, respectively:

$$Re = \frac{\Gamma^*}{\nu} \ , \text{ and} \tag{2}$$

$$\nu_t\left(\mathbf{x}, \Delta, t\right) = 0{,}105 C_k^{-3/2} \Delta \sqrt{\bar{F}_2\left(\mathbf{x}, \Delta, t\right)} \, . \tag{3}$$

In Eq. (3), C_k = 1.4 is the Kolmogorov constant and $\bar{F}_2$ is the local second-order velocity structure function of the filtered field (Lesieur and Métais, 1996). The later utilizes the concept of velocity differences instead derivatives and it must be computed for each vortex blob with core radius σ_0 during each time step, in the following manner (Alcântara Pereira et al., 2003):

$$\bar{F}_{2i} = \frac{1}{NVC}\sum_{j=1}^{NVC}\left\|\bar{\mathbf{u}}_i\left(\mathbf{x}_i, t\right) - \bar{\mathbf{u}}_j\left(\mathbf{x}_i + \mathbf{r}_{ij}, t\right)\right\|^2 \left(\frac{\sigma_{0i}}{r_{ij}}\right)^{2/3} , \tag{4}$$

where NVC is the number of vortex blobs necessary to compute the eddy viscosity coefficient for the ith vortex blob. So, all the average velocity differences related with the ith vortex blob are computed among a center of an annulus and points placed between its internal and external radius, as originally proposed by Alcântara Pereira et al. (2003).

The boundary conditions of the problem at S_b (see Fig. 1) are the impermeability condition and the no-slip condition, respectively:

$$u_n - v_n = 0 \text{, and} \tag{5}$$

$$u_\tau - v_\tau = 0 \, , \tag{6}$$

where u_n e u_τ are the normal and tangent components of the fluid velocity, and v_n and v_τ are normal and tangential components of the ground plane velocity (in the present problem v_n and v_τ are equal to zero).

The boundary condition of the problem at S_∞ (see in Fig. 1) is automatically satisfied and it is represented in the following manner (note there is no crosswind interference):

$$\left|\mathbf{u}\right| \to 0 \, . \tag{7}$$

The impermeability condition is ensured through the image method (Katz and Plotkin, 1991). On the other hand, Lamb discrete vortices must be generated on the ground plane surface during each time step to ensure both the no-slip condition and the global circulation conservation.

3. THE LAGRANGIAN VORTEX METHOD WITH TURBULENCE MODELING

Chorin (1973) proposes the “Viscous Splitting Algorithm”, where the vorticity transport equation, Eq. (1), is splitted in two problems. The first problem governs the vorticity advection, such that:

$$\frac{D\bar{\omega}}{Dt} = \frac{\partial \bar{\omega}}{\partial t} + \left(\bar{\mathbf{u}} \cdot \nabla\right)\bar{\omega} = 0 \, . \tag{8}$$

Note that Lagrangian manner is assumed in Eq. (8) and, consequently, the equation for vortex blobs trajectory is immediately obtained:

$$\frac{d\mathbf{r}_i}{dt} = \overline{\mathbf{u}}_i \, . \tag{9}$$

Equation (9) is solved during each time step Δt by using the second-order Adams-Bashforth scheme (Ferziger et al., 2020), such that:

$$\mathbf{r}_i(t+\Delta t) = \mathbf{r}_i(t) + \left[1.5\overline{\mathbf{u}}_i(t) - 0.5\overline{\mathbf{u}}_i(t-\Delta t)\right]\Delta t \, , \tag{10}$$

where $\mathbf{r}_i$ is the position vector of the ith vortex blob and $\overline{\mathbf{u}}_i$ is velocity vector induced at the same vortex blob due to contributions of the all vortex blobs and its images.

The second problem governs the vorticity diffusion, in which (Chorin, 1973):

$$\frac{\partial \overline{\omega}}{\partial t} = \left(\frac{1}{\mathrm{Re}} + \nu_t\right)\nabla^2 \overline{\omega} \, . \tag{11}$$

Equation (11) is solved during each time step Δt by using the vortex blob splitting (Rossi, 1996). Here, the original solution is adapted to include turbulence modeling and it assumes the following form:

$$\sigma_{0i}(t+\Delta t) = \sqrt{\sigma_{0i}{}^2(t) + 4\frac{\Delta t}{\mathrm{Re}}\left(1 + \nu_t \,\mathrm{Re}\right)} \, . \tag{12}$$

In Eq. (12), σ_{0i} is the i^{th} vortex blob core. According to Rossi (1996), if the core radius σ_{0i} reaches a maximum permissible/controlled value, then it must be submitted to partition scheme. That approach generates four new vortex blobs (starting from original vortex blob), each with a quarter of the original circulation and a core of $\sigma = \alpha\sigma_{0i}$, which are placed at 90° each other at a distance r, i.e.:

$$r = 2\sigma_{0i}\sqrt{1-\alpha^2} \, , \tag{13}$$

from the original vortex blob; and the parameter α [0, 1] governs the partition scheme. It's important to note that the maximum permissible value for the vortex blob core is not a determining factor for the convergence of the method, unlike α.

In this paper, the analytical solutions given by Eqs. (14)-(15) are compared with correspondence numerical solutions. As a first comparison, it is considered the viscous diffusion of a finite length of the vortex sheet, say, between $0 < x < \ell$. As presented by Batchelor (2000), the exact solution for the vorticity distribution may be expressed through (here in the non-dimensionalized form):

$$\omega(y,t) = \frac{\gamma(x)/2}{\sqrt{\pi t/\mathrm{Re}}}\exp\left(-\frac{y^2}{4t}\mathrm{Re}\right), \tag{14}$$

where γ(x) = 2U is the sheet strength, U = 1 is the inlet velocity, and 4t /Re is the vorticity penetration depth during the time t.

The second exact solution is evaluated for diffusion of a point vortex in two-dimensional flow (Batchelor, 2000), in which the analytical vorticity distribution in space and time may be expressed through (here in the non-dimensionalized form too):

$$\omega(r,t) = \frac{\mathrm{Re}}{4\pi t}\exp\left(-\frac{r^2}{4t}\mathrm{Re}\right). \tag{15}$$

Firstly, the numerical solution for the problem governed by Eq. (14) is computed by generating M = 30 vortex blobs on y = 0 (Fig. 2b), that will spread and then create 4 new discrete vortices each, in a single time step of Δt = 0.28. And the numerical solution for the vorticity distribution is calculated as follows (Leonard, 1980):

$$\omega_{jk}(y,t) = \sum_{k=1}^{Z} \frac{\Delta\Gamma_k}{\pi\sigma_k^2} \exp\left(-\frac{r_{jk}^2}{\sigma_k^2}\right). \tag{16}$$

where Z = 120 is the total number of vortex blobs after the diffusion by corrected core-spreading vortex method; each vortex blob has strength $\Delta\Gamma_k = \gamma(x)\,\ell / M$, being that $\ell = 0.3$. Other adopted parameters were Re = 7,650 and $\alpha = 0.8$ (α is a spatial refinement factor). So, the vortex blobs grew from $\sigma_0 = 0.01$ to $\sigma = 0.0125$, in which each one generates four new vortex blobs of core radius $\sigma_0 = 0.01$. The most important results are presented in Figs. 2a and 2b, which let conclude that the numerical vorticity distribution agrees very well with its analytical solution for y > 0 at x = ℓ / 2 (see Fig. 2a).

Secondly, the numerical solution for the problem governed by Eq. (15) is computed by initially placing one point vortex (x; y) = (0.0; 0.0), which will spread during seven time steps of magnitude Δt = 0.22 resulting Z = 16,384 vortex blobs at t = 1.54, see Fig. 3b. In the present numerical simulation, the parameters adopted were Re = 7,650, $\alpha = 0.96$. Here, the vortex blobs grew from $\sigma_0 = 0.001$ to $\sigma = 0.01$.

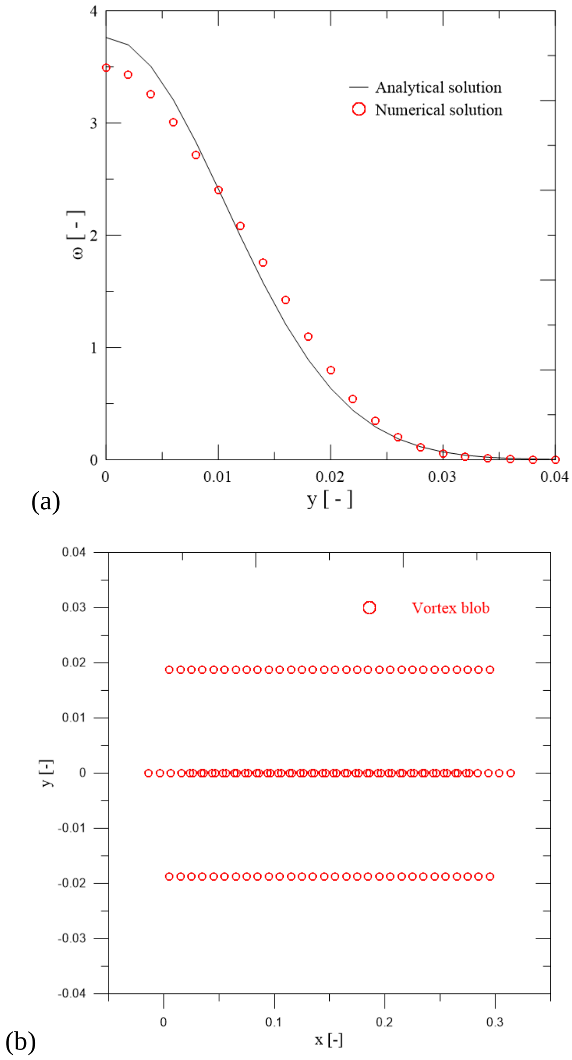


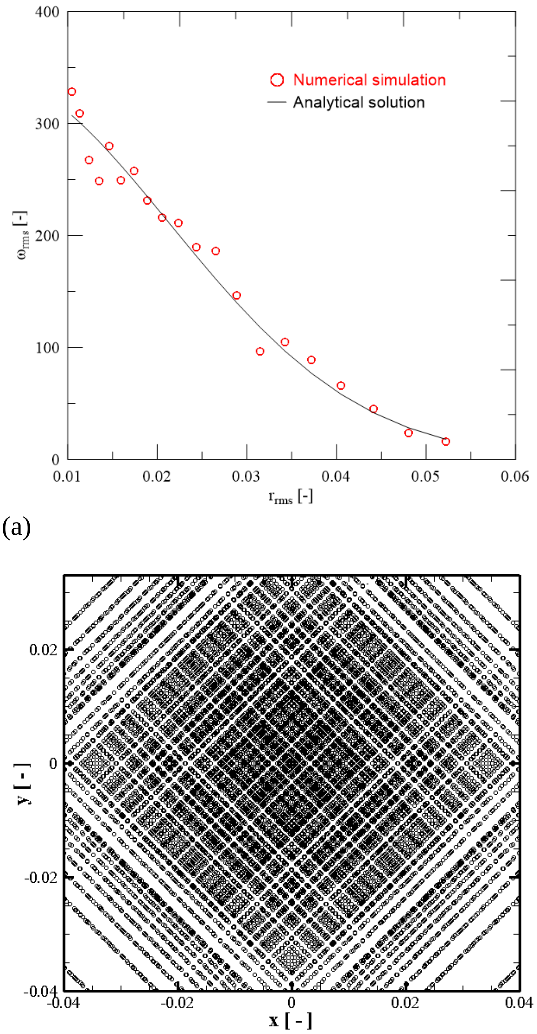
Figure 2. Diffusion of a vortex sheet simulated by the corrected core-spreading method generating 120 vortex blobs during one time step of magnitude Δt = 0.22: (a) vorticity distribution and (b) vortex blobs distribution with α = 0.8.

The comparison of corrected core-spreading vortex method with exact solution presented in Fig. 3a was obtained from Fig. 3b; in the latter was defined a series of annular bins separated by spaced radii r_i in accordance with a distribution in crescent geometric progression, or GP, with the first radius being $r_1 = 0.01$ and the last $r_{20} = 0.05$.

Suppose that n_i vortex blobs are captured by a i^{th} annular area lying between r_i and r_{i+1} in Fig. 3b. Thus, the vorticity at r.m.s. radius $\sqrt{\frac{1}{2}\left(r_i^2 + r_{i+1}^2\right)}$ of strip i may be computed in the following manner:

$$\omega_i = \frac{n_i}{Z\pi\left(r_{i+1}^2 - r_i^2\right)} \,. \tag{17}$$

The test case illustrated in Fig. 3a adopted twenty annular strips for sampling the scattered vortex blobs, where a very satisfactory prediction of vortex distribution was obtained.



(a)

(b)

Figure 3. Prediction of the diffusion of a point vortex by the corrected core-spreading method: (a) radial vorticity distribution and (b) vortex blobs distribution with $\alpha = 0.96$.

Finally, the developed algorithm to simulate trajectory of aircraft wake vortices is implemented using an in-house code via parallel computing (OpenMP) in Fortran through 9 steps, such that: (i) construction of the ground plane geometry including the number NP of "shedding points" above the pivotal points (one pivotal point is located at center of each flat panel of length B / NP); (ii) creation of the pair of vortical structures shedding from wing tips; (iii) generation of NP new vortex blobs on ground plane; (iv) velocity field computation at vortex blobs locations into the fluid domain (see Fig. 1); (v) advection of the vortex cloud using Eq. (10); (vi) vorticity diffusion using Eqs. (12)-(13); (vii) reflection of vortex blobs that accidently migrate into the imaginary plan, i.e. y < 0) as sketched in Fig. 1; (viii) velocity field computation at each pivotal point to impose the no-slip condition and, consequently, to obtain the next vortex blobs generation on ground plane; and (ix) advance by the time Δt.

4. RESULTS AND CONCLUSIONS

In order to perform the simulations for the test case of aircraft wake development in the vicinity of ground plane, it is necessary to adopt some parameters, as presented in Table 1, which were obtained after some initial tests.

The value $\Delta t = 0.05$ is chosen based on the second-order Adams-Bashforth scheme, (Eq.10), resulting into an advection error with a magnitude of 0.0011%.

For the corrected core-spreading method, the selected parameters are the spatial refinement factor (α), which determines an appropriate diffusion scheme. A higher value is desirable, but it also results in an increase in computational cost.

The minimum cutoff strength for vortex blobs partitioning is selected to allow calculations with only significant vortices intensities, thus avoiding an excessive number of vortex blobs at the end of simulation that would exceed the physical memory of the machine.

Table 1. Dimensionless numerical parameters adopted in the present simulation.

Parameter	Value
Runway length (B)	8.0
Release height of the primary vortical structures (h)	2.0
Initial centroid distance of the primary vortical structures (b)	1.0
Circulation strength of each primary vortical structure ($\lvert\Gamma\rvert$)	1.0
Initial radius of each primary vortical structure (r)	0.1
Time increment (Δt)	0.05
Number of time steps	1,000
Number of flat panels on ground plane (NP)	40
Number of vortex blobs in each primary vortical structure (N)	50
Initial core radius of each vortex blob (σ_0)	0.001
Vortex blob core radius before partition (coremax)	0.00125
Vortex blob core radius after partition (coremin)	0.001
Spatial refinement factor (α)	0.8
Minimum strength necessary to break one vortex blob ($\lvert\Gamma_{cut}\rvert$)	0.0004
Reynolds number (Re)	7,650

Figure 4 presents a comparison between the trajectory of the centroid of the right primary vortical structure and the experimental result given by Liu & Srnsky (1990). The trajectory fits the experimental data well until x = 5.5. This can be explained by the conditions that the experimental tests was made: (a) the experiment was conducted in a closed water tank, and the dissipation of the wake at increased time instants interacted with the walls of the tank, differing from the trajectory of a non-disturbed dissipation, and (b) the experimental measurement of the center of the vortical structure is difficult to be made, this leads to inevitable errors on the real trajectory.

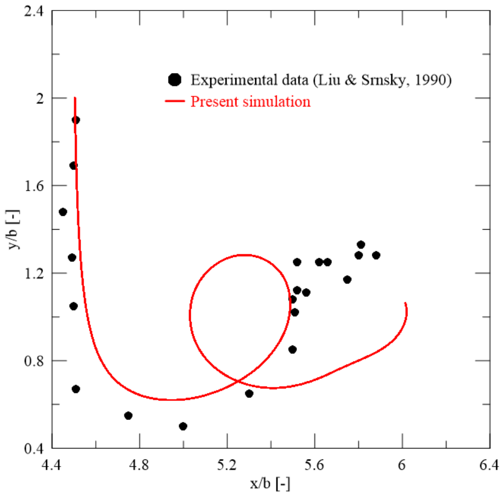
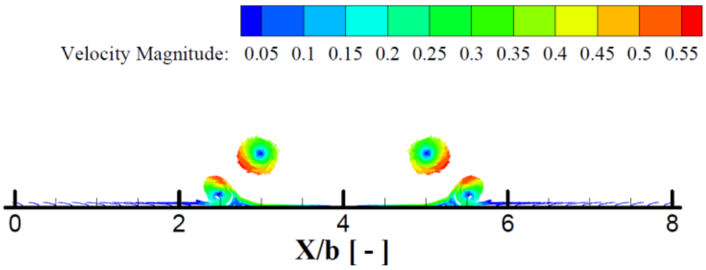
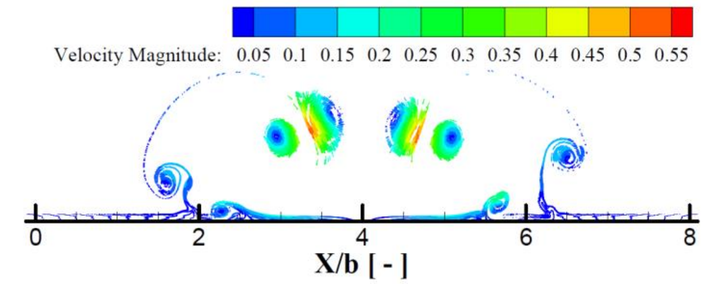


Figure 4. Trajectory of the centroid of the aircraft wake vortices at Re = 7,650.

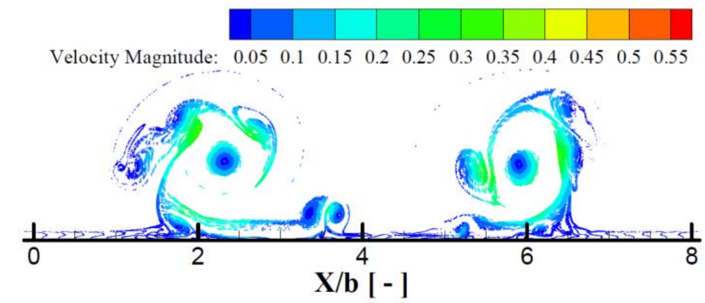
Figure 5 presents the position of the vortex blobs at different time steps. At t = 15.0, for instance, it is possible to verify a symmetry in the position and the formation of the counter rotating secondary vortical structures on the ground. On the other hand, at t = 30, it be can see the tertiary and quaternary vortical structures generated from the shedding of the hydrodynamic boundary layer formed on the ground plane with a degree of asymmetry because of turbulence effects. At t = 45, the primary vortical structures slows down and the subsequent structures keep developing, dissipating the vorticity. The discretized vorticity field is represented by Z = 49,138 vortex blobs because of the corrected core-spreading vortex method with $\alpha = 0.8$. The final CPU time was about 122 min when using an Intel® Core™ i7-2600 CPU @ 3.4 GHz, with parallel computation.



(a) t=15



(b) t=30



(c) t=45

Figure 5. Aircraft wake vortices positions in different time steps simulated by the corrected core-spreading method at Re = 7,650.

Closing, the present corrected core-spreading vortex method has been successfully implemented, since two analytical solutions for diffusion problems were compared with numerical results here obtained. Discrete vortex modeling of boundary layers considering the viscous diffusion of an infinite vortex sheet (Fig. 2) and the motion of a diffusing vortex of initial strength Γ centered at origin of the (x; y) plane (Fig. 3) were chosen as test cases. Even for the Reynolds number of Re = 7,650, two dimensional flow assumption with turbulence modeling is assumed at the present numerical simulation, which is expected to capture the main features of aircraft wake vortices in ground effect with no crosswind in a qualitatively sense. Despite the differences presented in Fig. 4, our results are promising, which encourages performing additional tests in order to explore the phenomena in more details.

Finally, in order to handle effects of crosswind and buoyancy forces on aircraft wake vortices near a roughened ground plane, a new method will be carried out (Carvalho et al, 2022 and Moraes et al., 2022).

5. ACKNOWLEDGEMENTS

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