COB-2023-2259 - STUDY OF APPROXIMATION MANEUVERS WITH CLOSED-LOOP CONTROL FOR A SPACECRAFT OVER ASTEROIDS

Luan Henrique Glasser
Evandro Marconi Rocco
Instituto Nacional de Pesquisas Espaciais, Avenida dos Astronautas, 1758 - Jardim da Granja, São José dos Campos - SP, 12227-010
luan.glasser@gmail.com, evandro.rocco@inpe.br

Marcelo Lisbôa Mota
Instituto Federal de Educação, Ciência e Tecnologia de São Paulo, Avenida Thereza Ana Cecon Breda, 1896 - Vila São Pedro, Hortolândia - SP, 13183-250
prof.mlmota@ifsp.edu.br

Abstract. In the last forty years, asteroid exploration came into focus of nations as United States, China, Japan and also to European Union. An evidence of that is the number of asteroid missions that happened during this period, among which the Hayabusa and the Osiris-REX can be found. In this time, the complexity of the missions have evolved from distant flybys, to sample-return, to asteroid deflection missions, so that, consequently, have also evolved sensors, actuators, overall equipment, instruments and, by extension, the spacecraft. Asteroid exploration is one of the space science, engineering and technology frontiers. Mastering this knowledge and ability is a key for the past, in order to understand Solar System formation and, maybe, the origin of life, but also for the future, making space more accessible for humankind. The challenge imposed to a spacecraft for it to move in an interplanetary flight, approach an asteroid and land is central in this discussion. The objective of this work was to study procedures for approaching a spacecraft to the asteroids (8567) 1996 HW1 and (1580) Betulia, in order to compute maneuvers with less velocity increment usage, to find landing conducive conditions and to simulate the complete maneuvers of approximation and landing. The Spacecraft Trajectory Simulator is a simulation environment dedicated to study spacecraft dynamics and trajectories, capable of performing guidance and control, and it was the main tool used to realize this work. As research results, this work has produced two maps, one for each asteroid, of landing conducive conditions. The maps could be used in mission analysis, to select landing conducive conditions and as input for mission design. This work has also demonstrated the usage procedure of those maps, successfully computed the consumption of velocity increments for different maneuvers and simulated complete maneuvers of approximation and landing.

Keywords: spacecraft dynamics, PID controller, orbital maneuvers, asteroids, landing

1. INTRODUCTION

In he last 40 years, humankind has seen intense and growing activities regarding asteroid exploration, as Clark et al. (2018) and, among others, Veverka et al. (1994), Mitchell (2000), Hashimoto et al. (2010), Beshore et al. (2015), Taylor et al. (2017), Mingtao and Kaiduo (2022), and Daly et al. (2023) made clear. From basic flybys to sample return missions, from sample return missions to asteroid deflection, the theme has been growing in importance, as did the expertise regarding it, and the technological capabilities of the spacecrafts. Deep space exploration is important to make space more accessible, and also to allow better understanding of Solar System formation and, maybe, even the origin of life. Mastering this knowledge and ability is vital for future deep space exploration and planetary defense.

This research is an attempt to contribute to this scientific endeavor and moves along with the global increasing interest in deep space exploration. The objective of this work was to study maneuvers for approaching a spacecraft to the asteroids (8567) 1996 HW1 and (1580) Betulia, in order understand the velocity increment usage, to find landing conducive conditions (LCC) and to simulate the complete maneuvers of approximation and landing. The core tool used in this work was Spacecraft Trajectory Simulator (STRS), developed by Rocco (2008). The STRS simulator is one of the simulation tools of the Modeling and Simulation Laboratory of Dynamics and Control in Closed-Loop of the Orbit and Attitude of Space Vehicles (Lab MSDC Orbit & Attitude), at the Space Mechanics and Control Division (DIMEC) of the Instituto Nacional de Pesquisas Espaciais (INPE, National Institute of Spatial Research). The asteroids gravitational potential were modeled by Mota (2017), with the method of gravitational potential expansion in convergent series.

The next sections will present the fundamental theory needed for this work, followed by the methodology used. Then, the results will be presented, firstly regarding the asteroid (8567) 1996 HW1, then for the asteroid (1580) Betulia. Conclusions and references can be found further.
2. FUNDAMENTAL THEORY

2.1 Dynamics from the gravitational potential

Two coordinate systems were used to compute and represent the trajectories: Body-Center Inertial (BCI) and Body-
Center Body-Fixed (BCBF). Both of the coordinate systems are centered in the asteroids mass centers, but BCI has an
arbitrary inertial orientation, while BCBF has its axes fixed in the asteroid body frame, in the direction of the principal
axes. Always in the beginning of any simulation, BCI and BCBF were aligned. Always, during the simulation, BCI and
BCBF z-axes were aligned.

According to Mota (2017), the dynamic model of the system is

\[ \ddot{r} = \nabla U, \]  

where \( \ddot{r} \) is the acceleration of the vehicle and \( U \) is the gravitational potential.

Mota (2017) developed a method for modeling the gravitational potential in a convergent series expansion, which
resulted in

\[ U = \sum_{i=1}^{N} U_i + \epsilon = G \frac{m}{V} \sum_{i=1}^{N} \int \int \int \int Q \frac{P_i(\tilde{u}) \frac{R'_i r'_i}{r'^2_1 + 1}}{d} \, dV + \epsilon. \]  

where \( U_i \) is the i-th term of the potential and \( \epsilon \) is the truncation error. In this model, \( U \) can be expanded until order \( N \). Also, \( G \) is the universal gravity constant, \( m \) is the asteroid mass, \( V \) is the volume, \( Q \) is any of the tetrahedral elements that compose the asteroid form discretization, \( P_i(\tilde{u}) \) are Legendre polynomials. The asteroids were considered homogeneous. The module of the distance of a mass element inside the asteroid, with respect to the asteroid mass center, is \( R'_i \), and \( r'_i \) is the module of the position vector of a particle with respect to the asteroid mass center. This was the model used in this research.

2.2 Solution of the Lambert’s problem

Battin (1999) explains the problem to connect two position vectors \( r_1 \) and \( r_2 \), angled from each other by \( \Delta \theta \), with an orbit, is called Lambert’s problem. The first person recorded to solve it was Carl Friedrich Gauss, in 1801, and his solution was used to predict Ceres position in the same year. The Lambert’s problem solution returns two velocity increments, \( \Delta v_1 \) and \( \Delta v_2 \), which must be applied in the beginning and in the end of a maneuver. Figure 1 describes the problem.

![Figure 1. Schematics to the Lambert’s problem.](image)

The procedure to solve this problem is as it follows. The goal is to find \( v_1 \) and \( v_2 \):

\[ v_1 = \frac{r_2 - f(z)r_1}{g(z)}, \quad v_2 = \frac{g(z)r_2 - r_1}{g(z)} \]  

where \( f(z) \) and \( g(z) \) are universal variables, according to Battin (1999) and da Silva Fernandes and de Paula Santos Zanardi (2018). To determine these function values, you should calculate:

\[ A' = \text{sign}(\pi - \Delta \theta) \sqrt{|r_1||r_2|(1 + \cos \Delta \theta)}, \quad \Delta \theta = \arccos \left( \frac{r_1 \cdot r_2}{|r_1||r_2|} \right). \]
Then, evaluate the movement direction. If $\text{sign}(\pi - \Delta \theta) = -1 \rightarrow \Delta \theta > \pi$ and the trajectory to be performed is the long one; otherwise, it is the short one.

Then, solve:

$$F(z) = x^3(z)S(z) + A'\sqrt{y(z)} - t_m\sqrt{\mu} = 0$$  \hspace{1cm} (5)

where $t_m$ is the maneuver time and:

$$S(z) = \frac{\sqrt{z} - \sin \sqrt{z}}{\sqrt{z^2}}, \quad C(z) = \frac{1 - \cos \sqrt{z}}{z}, \quad y(z) = |r_1| + |r_2| - A' \frac{1 - zS(z)}{\sqrt{C(z)}}, \quad x(z) = \frac{\sqrt{y(z)}}{C(z)}.$$ \hspace{1cm} (6)

Rocco (2015) developed an ingenious approach to solve this problem. Considering that $z \in \mathbb{R}_+ : 0 \leq z \leq (2\pi)^2$, the function $F(z)$ can be numerically solved by scanning the interval in which $z$ is allowed to vary, until a solution converges. Then, $f(z)$ and $g(z)$ can be calculated:

$$f(z) = 1 - \frac{y(z)}{|r_1|}, \quad g(z) = A' \sqrt{\frac{y(z)}{\mu}}, \quad \dot{g}(z) = 1 - \frac{y(z)}{|r_2|}.$$ \hspace{1cm} (7)

And, finally, the maneuver velocity increments can be obtained by:

$$\Delta v_1 = v_1 - v_0, \quad \Delta v_2 = v_f - v_2,$$ \hspace{1cm} (8)

where $v_0$ and $v_f$ the velocities of the initial and final orbits. This method was used to calculate all the maneuvers presented in this work.

### 2.3 Controlling the spacecraft trajectory and perturbations

According to Ogata (1997), to control a system is to impose the system a desired behavior. A process drives this: (1) sensors observe/estimate (maybe with some extra computation) the better they can the system state; (2) the observed/estimated state is compared to a reference state, that may have been generated by the guidance system or given, and an error is calculated; (3) the error is fed to the control system, which will generate a control signal from the error, according to some control law; (4) the control signal is fed to the actuators, that change the system dynamics, which also change under influence of perturbations; (5) another observation is made by the sensors and the process goes on, in a closed-loop flow.

There is a huge amount of methods to achieve this, but a simple and effective one is the use of a PID controller, which stands for Proportional, Integral and Derivative controller:

$$s_c = K_p e_r + K_i \int_0^t e_r \, dt + K_d \frac{de_r}{dt},$$ \hspace{1cm} (9)

where $s_c$ is the control signal, $e_r$ is the error, $K_p$ is the proportional gain, $K_i$ is the integral gain, and $K_d$ is the derivative gain. The proportional term amplifies the error signal, the integral term integrates the error signal, and the derivative term derives the error signal. The gains used in this research were: $K_p = 0.050$, $K_i = 0.001$, and $K_d = 0.025$.

### 3. METHODOLOGY

#### 3.1 Orbit definitions

The core of this research was simulation. In order to plan the maneuvers and simulate them, it was needed a useful definition of orbital regions around the asteroid, from which and to which the spacecraft could move. To make the proper definition, it was adopted the terminology: high, middle, low and terminal orbits, which are defined by Fig. 2, based on the sphere of influence radius (SIR), as calculated by da Silva Fernandes and de Paula Santos Zanardi (2018), and the radius of the circumscribing sphere $R_c$.

#### 3.2 Spacecraft Trajectory Simulator

The STRS was developed on MATLAB/Simulink by Rocco (2008), Rocco (2010), Rocco (2013b) and Rocco (2013a). Several researches have been made with the simulator, such as Gonçalves et al. (2013), Gomes dos Santos et al. (2014),
Gonçalves et al. (2017), Rocco and Gonçalves (2017), Mahler and Rocco (2017), Venditti and Rocco (2017), Mota and Rocco (2019), Macêdo and Rocco (2019), and Rocco (2019). The objective of the simulator is to provide means for simulating spacecraft trajectories. Several features and capabilities are included in STRS. It gathers orbital mechanics theory, different gravitational models for a diverse scope of spatial bodies, control systems theory, guidance theory, and many other disciplines, providing a rich simulation environment and allowing an even more rich scope of studies. Figure 3 shows the STRS architecture. STRS was the main tool used in this research.

3.3 Trajectory and perturbation control

STRS is a simulation environment that has full support for guidance and control, allowing the control of the trajectory and the perturbations. Trajectory and perturbation control happens through the following process: the perturbation is calculated, affects the actual dynamics and its value is passed to the guidance system; the guidance system and the reference state propagator generates a reference state; the actual, perturbed state is also propagated, and both reference and actual states are fed to the control system; the control system calculates the error between states and generates the control signal; the propulsion system actuates with respect to the control signal; and the process goes on. Trajectory and perturbation control can be turned on and off at any time, by the users will or by programming.

3.4 Landing Conducive Conditions

In the beginning of this research, the STRS was explored to get better understanding and intuition about the system dynamics. The asteroid used in these investigations was (8567) 1996 HW1. This process led to the observation of an interesting phenomenon: there were some conditions on equatorial orbits from which the vehicle would land in less than 50,000 s, only by influence of the gravitational perturbation. These conditions were defined as landing conducive conditions (LCC). Once discovered a couple of LCC, the next logical step was to understand if there were more LCC around the asteroid. To do so, a set of 372 simulations was configured and executed for each asteroid. Each simulation baseline was as the following: BCI and BCBF coordinates aligned in the initial time; initial circular orbits; radial position and relative angular position to the asteroid were varied on each simulation, in order to get a map of points around the asteroid that were or were not LCC, which led to valuable results.
3.5 Approximation and landing maneuvers and its performance

The maneuvers studied in this research were made in two phases of approximation, an initial approximation from a high to a low orbit and a terminal approximation from a low to a terminal orbit, by a vehicle modeled as a point of mass. In maneuvers where landing happened there were also a hovering phase and a landing phase. Initial approximation begins in a high circular orbit, with a true anomaly of about zero degree and ends in a low circular orbit with a true anomaly of about 180 degrees. Terminal approximation begins in a low circular orbit, when the true anomaly is approximately 180 degrees, and ends in a circular terminal orbit, when the true anomaly is approximately 360 degrees. The velocity increments were calculated by solving the Lambert’s problem. Initial and terminal approximation and hovering were done always with perturbation control, while the final landing approach was always uncontrolled. Also, propulsion was not impulsive, but continuous and modular, done by 2 N thrusters. The performance of the approximation maneuvers was evaluated in terms of total velocity increments.

4. RESULTS AND DISCUSSION

4.1 Maneuvering around (8567) 1996 HW1

4.1.1 Total velocity increment usage

A set of 10 maneuvers was simulated. The initial conditions of the high and the final conditions of the terminal target orbits were fixed, while, at each maneuver, the target low orbits sizes were changed. The higher the number of the maneuver, the higher was the low orbit semimajor axis. The high initial orbit had a 32.250 m semimajor axis, initial true anomaly of 0 degree and other keplerian elements were zero. The final terminal target orbit had a semimajor axis of 3.000 m, final true anomaly of 360 degrees and other keplerian elements were zero. Low orbits had their semimajor axis varying from 3.900 m to 10.200 m, by 700 m increments, while having initial true anomalies of 180 degrees and other keplerian elements were zero. This information was used to solve the Lambert’s problem for each maneuver, that were simulated with the STRS. Figure 4 shows the results.

![Figure 4. Total velocity increments computed for the approximation maneuvers around (8567) 1996 HW1.](image)

Three kinds of information can be obtained from Fig. 4: the total velocity increment for the whole maneuvers, the total velocity increment for the initial approximation and the total velocity increment for the terminal approximation. Regarding the initial approximation, the lower the low target orbit is, the higher is the velocity increment usage. Such a behavior cannot be identified in the terminal approximation line, which has a bigger variation and does not show a regular behavior. The 3 maneuvers with less velocity increment consumption are 1, 5 and 10. The greater usage of velocity increments was on maneuvers 3 and 9.

4.1.2 Landing conducive condition map

To map the LCC, a set of 372 simulations was made, respecting the definition of LCC presented previously. In the beginning of each simulation, BCI and BCBF coordinate systems were aligned, the initial orbit was circular, and it was varied the radial position vector magnitude and the initial angular position to the asteroid. Figure 5 shows the results of this simulations, making evident what initial condition was or was not an LCC.

This result could be immensely valuable for asteroid landing mission planning, because it could provide a reduction on velocity increment consumption, hence mass reduction, hence cost reduction for deep space missions. To better explain this idea, let the following be considered. A spacecraft is in a circular high orbit of an asteroid. It must go to a low, then to a terminal orbit, from where it could land. In order to do that, the vehicle moves from the high orbit through a transfer orbit. Two transfer orbits drive the vehicle to the low and then to the terminal orbit, where the vehicle uses its propulsion system to adjust the orbital velocity and keep itself in a circular terminal orbit. From now, how could the spacecraft land? The trivial case would be to reduce the orbital velocity to zero, so it would fall vertically under the effect of the gravitational field. Another way to land would be by using the LCC map, which states: if the vehicle is in a radial and
angular position that is an LCC, and if the orbit of the vehicle when it is in this position is circular, the vehicle will land in less than 50,000 s, because of the gravitational perturbations of the asteroid. But the vehicle was already in a circular orbit when it reached the terminal orbit, so it would not be needed another velocity increment in order for it to land. Hence, there would be the economy of the velocity increment that would be spent on the trivial case. Also, as an additional fact, none of the LCC points had produced final landing velocities, with respect to the asteroid surface, greater than 3.0 m/s.

A procedure for using the LCC map could be the following:

1. define the approximation maneuver in terms of high, low and terminal orbits conditions;
2. configure a simulation with the calculated maneuver, controlling the perturbations, so the vehicle can stay as long as possible hovering in a terminal orbit;
3. execute the simulation;
4. using the LCC map, identify in the simulation results what orbital position is an LCC and get the simulation time where the condition is achieved;
5. evaluate if in this time the orbit is circular, if not, calculate the necessary velocity increment to make the orbit circular;
6. configure another simulation such as the previous, but, now, when the vehicle reaches the selected LCC, turn off the perturbation control and apply the velocity increment to make the orbit circular;
7. evaluate the landing results.

This procedure was applied to all the approximation and landing maneuvers simulations presented in this work.

4.1.3 Approximation and landing maneuver

A complete simulation of approximation and landing maneuver was made and is presented on Fig. 6. The red line is the trajectory represented on BCI coordinates, and the blue line is the trajectory represented in BCBF coordinates. The central asteroid shall be ignored when analyzing the BCI trajectory, while it has to be considered in the analysis of the BCBF trajectory.

The BCI trajectory makes it clear the initial approximation, the terminal approximation, the hovering around the asteroid and the landing after reaching the LCC. This LCC is the one represented by a radial position magnitude of 2.900 m and angular position to the asteroid of 90 degrees, and it can be found by analyzing the BCBF trajectory. At the moment the LCC was reached, a velocity increment to circularize the orbit was applied and, as foreseen by the LCC map, the vehicle has landed in less than 50,000 s with a final velocity lesser than 3 m/s. This result demonstrates the usability of the procedure presented previously and of the LCC map.

4.2 Maneuvering around (1580) Betulia

4.2.1 Total velocity increment usage

A set of 10 simulations was made in order to allow the evaluation of the velocity increment usage around (1580) Betulia. The same maneuvering concept was used to bring the vehicle from an initial equatorial high circular orbit, with a semimajor axis of 90.855 m, to a target circular low orbit, and then from the low orbit to a target circular terminal orbit, with a semimajor axis of 4.400 m. The low orbits were scanned: the initial semimajor axis was 6.200 m and the final was
27th ABCM International Congress of Mechanical Engineering (COBEM 2023)
December 4-8, 2023, Florianópolis, SC, Brazil

Figure 6. Simulation of a complete approximation and landing maneuver on (8567) 1996 HW1. The plot shows trajectories on BCI coordinates (red line) and on BCBF coordinates (blue line).

Figure 7. Total velocity increments computed for the approximation maneuvers around (1580) Betulia.

24.200 m, the increments of semimajor axis were of 2.000 m. The higher the number of the maneuver, the higher was the semimajor axis of the low orbit. Figure 7 shows the results.

Figure 7 shows a similar result if compared to what was shown for (8567) 1996 HW1. The initial approximation is approximately linear, in a sense that the smaller the semimajor axis of the low orbit, the greater the velocity increment usage. For the terminal approximation it does not happen, the behavior seen in the plot varies. For the terminal approximation it can be said that the greater the trajectory from low to terminal orbit, the greater the velocity increment usage. Similarly to (8567) 1996 HW1, the maneuver that spent less velocity increments were 1, 5 and 10, while the greater velocity increments usage values happened on maneuvers 3 and 9.

4.2.2 Landing conducive condition map

The asteroid (1580) Betulia is more regular in its mass distribution than the (8567) 1996 HW1, which implies that its gravitational field is more regular. Hence, proportionally, its gravitational perturbation is smaller. So one could ask if the perturbations would be enough so LCC could be observed. Now we will explore the results associated with this question, showing indeed that there are LCC for (1580) Betulia. In order to investigate the existence of LCC on (1580) Betulia, the same procedure applied to (8567) 1996 HW1 was used. Figure 8 shows the results.

What can be seen is that (1580) Betulia also have LCC, as the map shows. Interestingly, the LCC occupies approximately the same regions in both asteroids. Since (1580) Betulia is more regular than (8567) 1996 HW1, it would be licit to assume that LCC maps could be found for other asteroids, specially if they have intermediate regularity (between the asteroids studied here). It also would make sense to suppose that asteroids more irregular than (8567) 1886 HW1, being more disturbed, would have LCC maps. This observations provide a direction for future researches regarding this theme.

4.2.3 Approximation and landing maneuver

Figure 9 shows the simulation for approximation and landing on (1580) Betulia, zoomed for better observation. The red trajectory is the movement represented on BCI coordinates, for which the plotted asteroid has to be ignored, and the blue trajectory is the BCBF one, for which the asteroid has to be considered. It can be seen in the BCBF trajectory that the LCC was reached around a radial position of approximately 4.900 m and an angular position relative to the asteroid...
This subsection showed an application of the LCC map to an approximation and landing maneuver on (1580) Betulia. This results indicate the procedure made for (8567) 1996 HW1 have some degree of generality and can be reproduced and used for maneuvers around other asteroids. This results corroborates the value of the LCC map for planning space missions.

5. CONCLUSIONS

In this study, approximation and landing maneuvers were examined for the asteroids (8567) 1996 HW1 and (1580) Betulia. The research successfully improved our understanding of velocity increment usage. Additionally, the study revealed the existence of LCC maps for both asteroids and showed their use through approximation and landing maneuver simulations. A procedure was proposed for utilizing the LCC maps, which was also demonstrated through simulation. The usage of the LCC maps suggested that this tool could lead to advantages in asteroid missions by reducing the required total velocity increment, what ultimately would lead to a mass reduction. Furthermore, based on the LCC map results, it can be inferred that asteroids with intermediate regularity in their gravitational fields, lying between (8567) 1996 HW1 and (1580) Betulia, much likely possess LCC maps as well. This insight shows the direction of future research.

6. ACKNOWLEDGEMENTS

We thank CAPES for deeply supporting this research financially. Also, we thank INPE for providing an amazing structure so we could produce this work. And we thank for the laboratory colleagues, with whom good talks enlightened our understanding and helped with some tricky algorithm problems.
7. REFERENCES


Rocco, E., 2015. “Gravitational disturbances generated by the sun, phobos and deimos in orbital maneuvers around...


8. RESPONSIBILITY NOTICE

The authors are solely responsible for the printed material included in this paper.