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## THEORETICAL-EXPERIMENTAL COMPARISON OF THE CLASSICAL LAMINATE THEORY FOR COMPOSITE MATERIALS OF FIBERGLASS AND POLYESTER MATRIX

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**Abstract.** Synthetic fiber composite materials have good mechanical properties and it presents a good alternative material in view of metallic materials. This work makes an experimental and theoretical comparison, through the classical laminate theory (CLT), of fiberglass composite material and polymer matrix. For the experimental analysis, tensile tests of the fiberglass laminate composite were developed according to ASTM D3039. For the determination of stresses and strains, the elastic properties of the composite were determined by the rule of the mixtures, and then these values were inserted into the CLT matrices for the definition of the representative strain/strain curve. The results of the experimental and theoretical longitudinal modules diverged by 35%, but for the ultimate stress the error was 25%. The difference between the theoretical and the experimental results occurs according to the considerations of the microstructural analysis. Experimentally the fiber and matrix do not present a perfect adhesion, there are voids in the matrix and the alignment of the fibers are not parallel to the force application, contributing to the difference between the results presented in the work.

**Keywords:** tensile test, classical laminate theory, polyester matrix, elastic properties.

### 1. INTRODUCTION

The increasing use of composite materials demonstrates the ability to perform various functions in different engineering fields. For such applications, it is necessary to submit the material to several standardized tests, in order to quantify the properties of the material and meeting the required quality.

In this work, it was performed the tensile stress test of the laminated composite  $[0]_3$  for fiberglass and epoxy matrix according to ASTM D3039, comparing with the experimental results with the classical laminate theory (CLT) materials.

### 2. THEORETICAL REFERENCE

Nesta seção serão discutidos, os modos de determinação das propriedades elásticas do compósito, através do cálculo da regra das misturas, e as tensões e deformações do material para os resultados analíticos da teoria clássica dos laminados.

In this section, we discuss the ways of determining the elastic properties of the composite by calculating the rule of mixtures (ROM) as well as the stress and strains of the material for the analytical results of the classical laminate theory when subjected to uniaxial stress.

## 2.1 Classical laminate theory (CLT)

The components of the stiffness matrix show at Eq. (1),  $[\bar{Q}_{ij}]_k$ , from a laminate of k plies, are stress transformation equations with sine and cosine components calculated from the principal coordinates (coordinates parallel to the applied force). (Gibson, 2012)

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (1)$$

Equation (2) correlates all strains-displacements relations with the inverse matrix ABD for the theory of laminated plates and the input parameters: forces and moments per unit length of the laminate. (Almeida and Fujiyama, 2017). In this case, there were not no moments and forces in y-direction, only forces at x-direction representing the application of the axial force in the tensile test of the composite material.

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \\ k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix}^{-1} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} \quad (2)$$

## 2.2 Rule of mixtures (ROM)

ROM is one of the various micromechanical models for predicting the effective moduli of continuous fiber-reinforced laminae. Equation (3) show the determination of the longitudinal modulus for a unidirectional composite. Where  $E_f$  and  $E_m$  represents the effective modulus of fiber and matrix, respectively, and  $V_F$  is the fiber volume fraction.

$$E_1 = E_f V_F + E_m (1 - V_F) \quad (3)$$

Equation (3) is validated among some assumptions: the bond between fibers and matrix is perfect; the elastic moduli, diameters, and space between fibers are uniform; the fibers are continuous and parallel; the fibers and matrix follow Hooke's law (linearly elastic); the fibers possess uniform strength and the composite is free of voids. (Kaw, 2006)

## 3. MATERIALS

In this section, the values of each constituent of the composite are available in order to determine the effective modulus by calculating the rule of mixtures and the stress-strain diagram of the composite for the analytical values of classical laminate theory (CLT). Subsequently, the composite specimens are prepared according to ASTM D3039.

### 3.1 Input data for laminate theory

Table 1 shows the properties of each constituent, for the determination of the longitudinal modulus of the laminate used at the rule of mixtures, then at CLT equations.

Table 1. Literature results

	Effective modulus	Author
Commercial fiberglass	70 GPa	(Gibson, 2012)
Terephthalic polyester matrix	0.964 GPa	(Almeida and Fujiyama, 2017)

Laminate has three plies of fiberglass and polyester matrix, with fiber mass fraction of 27.44% or 15.6% of fiber volume fraction, so the final properties of composite material is depicted at Tab. 2. (Sousa e Fujiyama, 2017)

Table 2. Results from ROM

Fiber volume fraction ( $V_F$ )	15.6%
Effective composite modulus	11.7GPa
Major Poisson Ratio	0.315

### 3.2 ASTM D3039

According to ASTM D3039 (ASTM D 3039, 2014), the dimensions for unidirectional laminate is given by Fig. 1a and the specimen of fiberglass/polyester laminate  $[0]_3$  is shown in Fig. 2b. The final thickness of each specimen was about 1.25mm.



Figure 1. Specimen for laminates. (a) dimensions in mm; (b) final specimen.

### 4. METHODOLOGY: COMPUTATIONAL AND EXPERIMENTAL PROCEDURE

Figure 2 shows a flow chart for the comparison of the experimental results and the CLT. For the theoretical results, it was necessary to determine the elastic properties of the composite by calculating the rule of mixtures for the effective modulus. Then, some force values per unit length ( $N_x$ ) were calculated for the determination of stresses and strains during the CLT. After the input of the variables and forces, the stiffness matrix of the laminate (matrix ABD) and strain values were calculated. In order to calculate the stresses, it was necessary to multiply the constants of the stiffness lamina matrix by the strain.

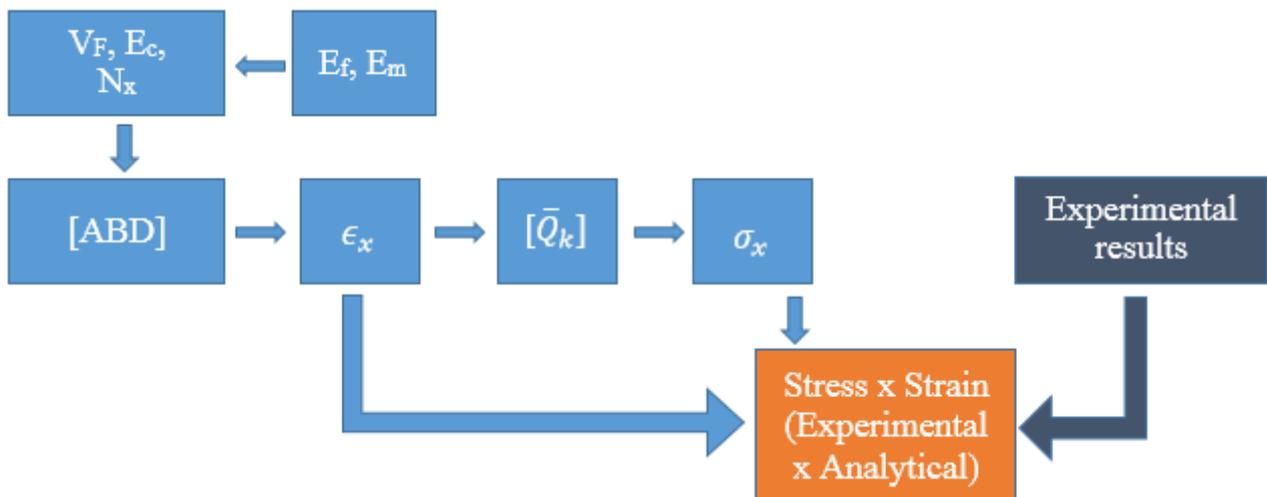


Figure 2. Work flow chart.

## 5. RESULTS AND DISCUSSION

Table 3 shows the average results obtained experimentally from the tensile stress test, and Fig. 3 presents the characteristic curve of the laminate, i.e. the curve of the specimen that most closely matches to the average experimental results, compared with the curve from laminate theory.

Table 3. Average results from tensile stress test of each specimen

Maximum load (kN)	5.94 ( $\pm 0.79$ )
Deformation (mm)	6.92 ( $\pm 0.60$ )
Ultimate tensile stress (MPa)	398.7 ( $\pm 57.53$ )
Strain (%)	4.95 ( $\pm 0.60$ )
Elastic modulus (GPa)	7.46 ( $\pm 0.23$ )

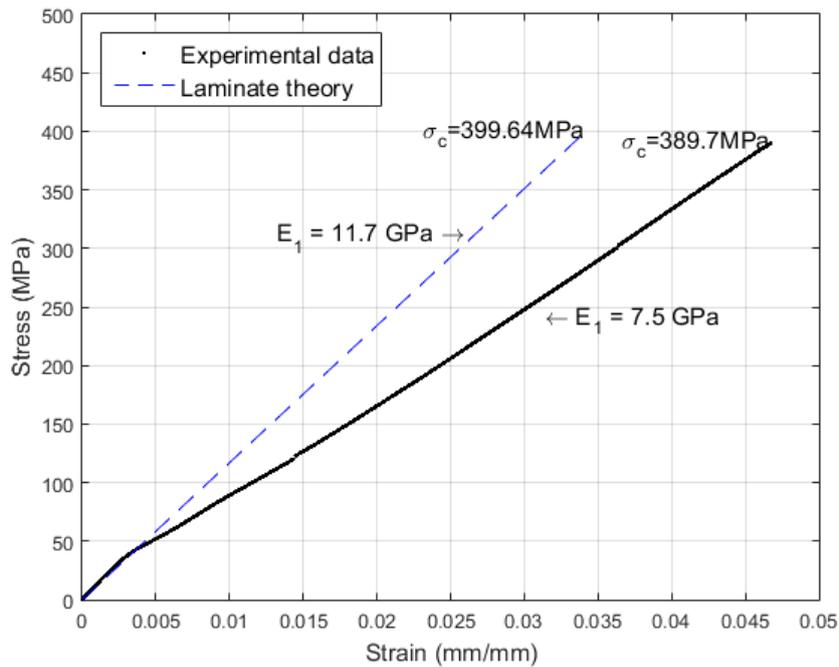


Figure 3. Characteristic curve of experimental and theoretical results.

From the graph of Figure 3, the difference between the ultimate tensile stress between the experimental values and the CLT diverged by 25%, but for the longitudinal modulus the error was in the range of 36%. For another work (Bourchak and W. Harasani, 2015), the error between the values of the longitudinal modulus and the tensile strength are in the order of 19% and 44%, respectively, for an angle-ply composite.

It is also worth mentioning that the moduli of elasticity are identical at the beginning of the tensile test of the fiberglass laminate, up to 50 MPa. In this range, both the fiber and the matrix in each of the three laminas of the laminate, the characteristic curve of the tensile stress test the curve of the CLT, present the same rate of strain, validating the rule of the mixtures for that region.

The results differed according to the elastic properties calculated by the ROM, since the model for the determination of the longitudinal modulus presents considerations as matrix and fiberglass are isotropic and homogenous; the constituents follow Hook's Law; the adhesion between fiber and matrix are perfect; fibers, matrix and composite presents equal strains (Gibson, 2012). This last consideration, experimentally, would be valid for ultimate strain of the single fiber. The adhesion between the fiber and the matrix is not perfect at the experimental test and the matrix presents porosities, which contributes to the reduction of the mechanical properties of the composite in relation to the theoretical model.

## 6. CONCLUSION

The work made a comparison of the CLT with the experimental data of the tensile stress test of fiberglass/polyester composite. The experimental results show the longitudinal modulus of specimens with a value of 7.5GPa and rupture stress 390MPa. The results presented errors consistent with the literature (Bourchak and W. Harasani, 2015) and based

on the theoretical considerations of ROM. The experimental analysis was consistent with CLT only at the beginning of the tensile stress test, proving the considerations applied at the rule of mixtures.

## **7. ACKNOWLEDGMENTS**

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