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RETRIEVING FIBER ANGLE DISTRIBUTION FOR VARIABLE STIFFNESS LAMINATES FOR A BUCKLING LOAD PROBLEM

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Abstract. *A parametrization is presented for retrieving variable stiffness laminates from a lamination parameters distribution considering its buckling performance. The approach adopts the use of Lagrange design elements and interpolation functions in order to define fiber angle distribution along a given domain. A study case considered an optimized variable stiffness plate obtained from a lamination parameters based solution with its critical buckling load maximized. The parametrization approach is applied for post-processing the optimum design and retrieve a lay-up configuration. During post-processing, the proposed approach randomly changes its optimization starting point and assess the laminate critical buckling loads in order to penalize designs that diverge from the optimum design buckling performance. An optimization framework is implemented based on a sequential quadratic programming for obtaining a fiber angle distribution which best match with a given set of optimum lamination parameters. This method, besides reducing drastically the number of design variables, also guarantees that a smooth and continuous fiber angle distribution is achieved at the cost of minimum loss in buckling performance without limiting the design space.*

Keywords: *variable stiffness design, retrieving fiber angles, lamination parameters*

1. INTRODUCTION

Fiber-reinforced and resin-matrix composite materials are becoming important in weight-sensitive applications, especially in aerospace structures (Jones, 1998). There are several examples regarding the application of composite materials, such as the Boeing 787 Dreamliner and the Airbus A350, which are composed of composite materials in approximately 50% of its primary structure. A mainly reason for the application of composite is due to their superior strength and stiffness to weight ratio (Khani *et al.*, 2011). In addition, composite materials present directionally dependent elastic properties, which allows a unique tailoring advantage when compared to general monolithic materials (Daniel and Ishai, 1994). As a consequence, the design process of composite structures is composed by a system with larger number of degrees of freedom, which may lead to more efficient structural designs (Kaw, 2005) .

A constant stiffness composite structure is defined when a laminate is composed with constant ply angles with the same material and thickness for each ply. The design of such conventional composites is defined by the arrangement of ply stacking sequence through the thickness. In contrast, laminates in which the stiffness can spatially vary in order to increase structural performance are called variable stiffness composites (Wu *et al.*, 2012). These laminates are defined by an increased design space in which the mechanical properties are allowed to be locally tailored (Gürdal and Olmedo, 1992).

In general, variable stiffness designs present superior properties when compared to ones with constant stiffness. These capabilities have been demonstrated for buckling capacity (Hyer and Lee, 1991; Abdalla *et al.*, 2008; Ijsselmuiden *et al.*, 2008), elastic behavior (Gürdal and Olmedo, 1993), stiffness (Setoodeh *et al.*, 2006), first-ply-failure (Lopes *et al.*, 2008), maximum fundamental frequency (Blom *et al.*, 2008), post-buckling progressive damage (Lopes *et al.*, 2007) and flutter analysis (Guimaraes *et al.*, 2017). The buckling problem of variable stiffness laminate is specifically interesting since the structural response relies not only in the out-of-plane behavior, but also in the in-plane load redistribution (Olmedo and Gurdal, 1993).

On the other hand, as the design space increases, defining properly the design variables becomes a complex task (Ijsselmuiden *et al.*, 2010). In this context, lamination parameters have been introduced for optimization of composite structures, since the variable stiffness problem can be better explored within a larger set of possible laminate configurations (Hammer *et al.*, 1997a). In addition, the use of lamination parameters avoids non-convexity related to periodic functions without limiting the design space (Hammer *et al.*, 1997a). Although, once the optimal laminate stiffness distribution is obtained, it is required post processing to retrieve layer fiber angle distribution and stacking sequence.

Several methods have been developed in order to obtain a laminate stacking sequence based on lamination parameter distribution. Hammer *et al.* (1997b), Lipton (1994) and Autio (2001) demonstrated that there is minimum number of plies, each on different orientation, needed to find any given combination of lamination parameters, provided that the ply thickness is continuously varying. Ijsselmuiden *et al.* (2008) used a genetic algorithm to obtain a stacking sequence based on a candidate discrete ply fiber angle. Narita (2003) and Narita and Hodgkinson (2005) used layer-wise optimization as a sequential one-dimensional search to find the best angle distribution per layer. These approaches limit the set of feasible lamination parameters and constrains the relations between in-plane and flexural lamination parameters within this feasible set.

In this paper, a parametrization model based on Lagrange interpolation functions is proposed for retrieving fiber angle distribution of a variable stiffness laminate. The proposed method is applied to an optimum lamination parameters distribution obtained for a buckling load problem in which both in-plane and out-of-plane lamination parameters are within the feasible region (Quadros, 2017). The parametrization approach reduces drastically the number of design variables, once the local angles are only defined at a given set of control nodes independently of the mesh size of finite element analysis.

2. METHODS

2.1 Optimum Lamination Parameters Distribution

The lamination parameters were initially introduced by Hahn and Tsai (1980) and represent a non-dimensional through the thickness integration of the layer angles. For structural optimization problems, using lamination parameters reduces significantly the number of design variables in comparison to using ply orientation angles and thickness – which may result in non-convex optimization problem related to periodic functions and discrete number of plies or constrained design space (Hammer *et al.*, 1997a). Based on Classical Laminate Theory (CLT), four lamination parameters V_{1A}, V_{3A}, V_{1D} and V_{3D} – as per Eq. (1) and Eq. (2) – are required to locally define laminate stiffness for balanced and symmetric laminates (Hahn and Tsai, 1980).

$$(V_{1A}, V_{3A}) = \int_{-1/2}^{1/2} (cos2\theta(\bar{z}), sin2\theta(\bar{z}), cos4\theta(\bar{z}), sin4\theta(\bar{z}))d\bar{z} \tag{1}$$

$$(V_{1D}, V_{3D}) = 12\int_{-1/2}^{1/2} (cos2\theta(\bar{z}), sin2\theta(\bar{z}), cos4\theta(\bar{z}), sin4\theta(\bar{z}))\bar{z}^2 d\bar{z} \tag{2}$$

From Eq. (1) and Eq. (2), $\bar{z} = z/t$ is the normalized through the thickness coordinate of the layers, t is the total laminate thickness and θ is the fiber angle at $\bar{z}$ (Hahn and Tsai, 1980).

The laminate stiffness is fully and linearly described as function of the material properties and lamination parameters, which may be beneficial for design optimization since it simplifies the obtainment of stiffness gradients when considering lamination parameters as design variables in structural optimization problems (IJsselmuiden, 2011). In addition, analogously to stiffness properties, a lay-up is represented by a unique set of lamination parameters, but one set of lamination parameters may be able to represent many lay-up configurations (Grenestedt, 1994).

In this work, an optimum lamination parameters distribution obtained for a maximization of critical buckling load from a previous work is considered as a reference for the retrieving of a laminate configuration (Quadros, 2017).

2.2 Parametrization Model for Retrieving Fiber Angle Distribution

The proposed parameterization consists in the use of Lagrange rectangular design element to define continuous fiber angle distribution over the plate domain though interpolation functions $\psi_i(\xi, \eta)$ – in which ξ and η are natural coordinates. This requires the definition of control nodes along the Lagrange design element – as illustrated in Fig. (1) for the case of a second order polynomial interpolation order ($p = 2$).

From Fig. (1), the local angles θ_k are locally defined, for each k laminate layer, at the control nodes of the Lagrange rectangular design element. Once the local angles θ_k are the design variables for the retrieving problem, the number of design variables increases according to the polynomial order of the interpolation functions and the number of layers considered. For this paper, a 9-control nodes Lagrange design element is considered.

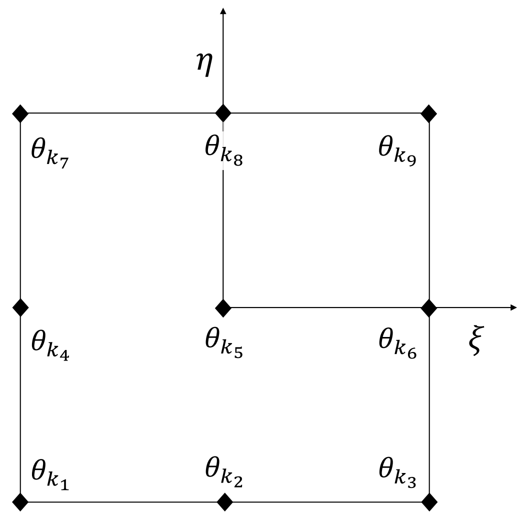


Figure 1: 9-control nodes Lagrange design element

Once defined at control nodes, it is possible to recovery the continuous distribution of the fiber orientation angles $\theta_{k_i}(\xi, \eta)$ by using Lagrange interpolation function $\psi(\xi, \eta)$ according to Eq. (3) .

$$\theta_k(\xi, \eta) = \sum_{i=1}^{n} \theta_{k_i} \cdot \psi_i(\xi, \eta) \tag{3}$$

From Eq. (3), the Lagrange interpolation function $\psi(\xi, \eta)$ is defined by the product of the interpolation functions along ξ and η at a given control node i (Reddy, 2006):

$$\psi_i(\xi, \eta) = f_i(\xi) \cdot g_i(\eta) \tag{4}$$

In which $f_i(\xi)$ and $g_i(\eta)$ are defined as function of the nodal coordinates ξ_k and η_k as per Eq. (5) and Eq. (6) for $n = (p+1)^2$ control nodes (Reddy, 2006).

$$f_i(\xi) = \prod_{j=1, j\neq i}^{n} \left(\frac{\xi - \xi_i}{\xi_j - \xi_i} \right) \tag{5}$$

$$g_i(\eta) = \prod_{i=1, j\neq i}^{n} \left(\frac{\eta - \eta_i}{\eta_j - \eta_i} \right) \tag{6}$$

The advantage of using the Lagrange interpolation functions is that one can represent a continuous distribution by only initially defining this function at the design element control nodes. The values defined at these nodes represent the coefficient of the recovery functions and also the value of the continuous function at its respective coordinate (Wu *et al.*, 2012).

From an optimum lamination parameters distribution, an optimization routine to retrieve the fiber angle distribution is implemented. The objective function is defined by averaging the relative error, according to Eq. (7), for a given number n_p of points evaluated. In Eq. (7), $V_{iA,D}$ represents the optimum lamination parameters and $\tilde{V}_{iA,D}$ the lamination parameters obtained when assigning the design variables θ_k to Eq. (1) and Eq. (2) for a laminate with k layers.

$$RE = \frac{1}{n_p} \sum_{i=1}^{n_p} \sum_{j=1}^{4} \left| \frac{\tilde{V}^i_{jA,B,D} - V^i_{jA,B,D}}{V^i_{jA,B,D}} \right| \tag{7}$$

The lay-up configuration is obtained based on the minimization of the relative error. In the optimization framework, the objective function is proposed in order to find a given set of angles in which the respective lamination parameters are close enough to the optimum solution, such as $\tilde{V}_{jiA,B,D}(\xi,\eta) \rightarrow V_{jiA,B,D}(\xi,\eta)$. Equation (8) reveals the objective function which computes the relative error of the candidate solution by evaluating n_p points over the reference element domain (ξ,η).

$$\begin{aligned} \min_{\theta_{k_i}} \quad & RE \\ \text{s.t.:} \quad & g_p(\theta_{k_i}) \leq 0, \ p = 1, ..., n_p \\ & h_q(\theta_{k_i}) = 0, \ q = 1, ..., n_p \end{aligned} \tag{8}$$

In this work, the inequality constraints in Eq. (8) defines the limits for the design variables, such as $-90° \leq \theta_{k_i} \leq 90°$, while the equality constraints guarantees a balanced and symmetric laminate. The functions g_p and h_q are defined over the n_p points over the parametrized domain.

For the solution of Eq. (8), a sequential quadratic programming (SQP) algorithm is implemented. Due to the non-convex nature of the problem and in order to guarantee a robust approximation, the solution obtained from a given starting point is disturbed based on a random function and a new analysis is defined based on previous solution. In addition, a numerical analysis using the software Abaqus is performed by a Finite Element modeling in which the mesh size is defined by n_p. Finally, the lay-up configuration is obtained when both the RE is minimized and the obtained laminate presents similar buckling performance to the optimum lamination parameters based solution.

As per the optimum lamination parameters distribution, the retrieved laminate buckling performance is normalized with respect to the critical buckling load of a quasi-isotropic laminate with the same geometry and loading conditions. Laminates such as $[0, \pm 60]_s$, $[0, 36, \pm 72, -36]_s$ and $[0, 30, 60, 90, -60, -30]_s$ are considered as quasi-isotropic reference for the analysis.

2.3 Study Case

The study case mimics the loading conditions adopted for obtaining the optimum lamination parameters solution from Quadros (2017). Thus, a buckling optimization problem of a simply supported plate subjected to axial compression is considered as per Fig. (2). The plate has length $a = b = 100mm$, ply thickness $l_t = 0.19mm$ and IM6/SC1081 carbon epoxy material properties were adopted. A laminate with $k = 6, 10$ and 12 layers is considered for retrieving the lamination parameters.

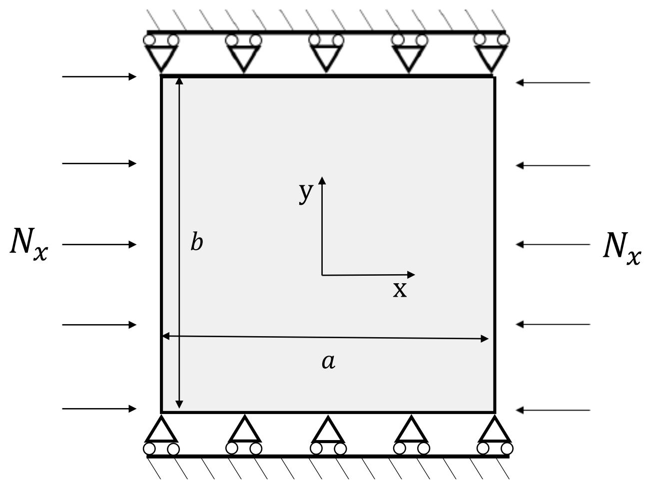
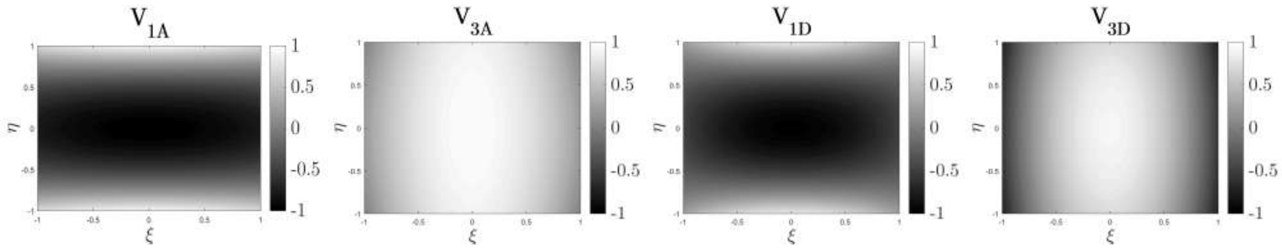


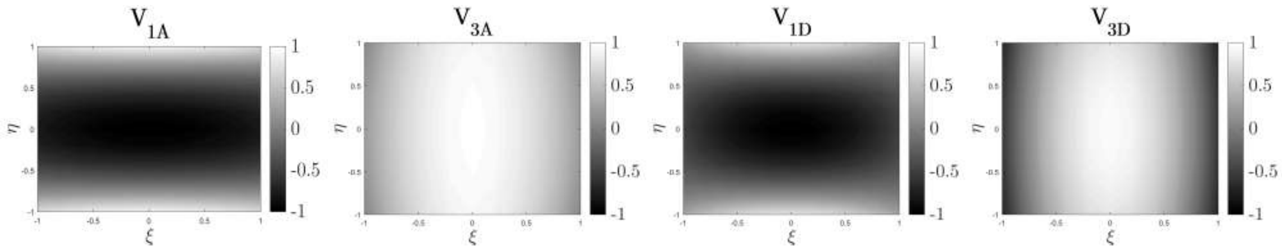
Figure 2: Panel subjected to axial compression

From Fig. (2), it is also assumed that the edges at $x = a/2$ and $x = -a/2$ are forced to remain straight, there is no overall transverse deformation and there is no out-of-plane displacement along the z axis.

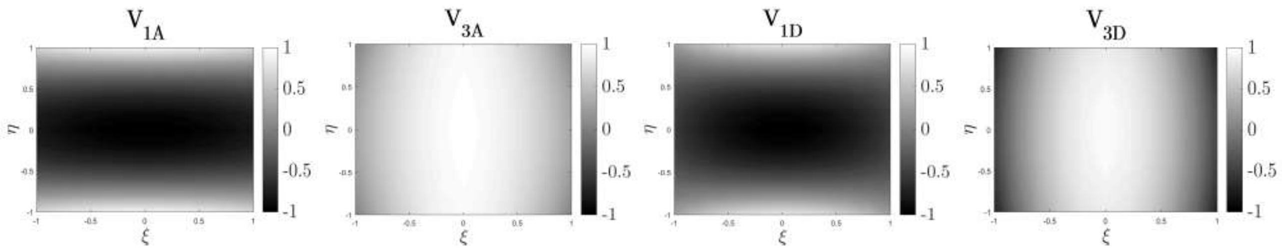
The optimum lamination parameters $V^i_{jA,B,D}$ previously obtained by Quadros (2017) are mapped for a 9-control nodes Lagrange design element. The optimum distribution were obtained based on the feasible regions defined by Grenestedt and Gudmundson (1993) and, therefore, corresponds to at least one possible laminate configuration. Figure (3) illustrates the optimum lamination distribution for a laminate with $k = 6, k = 10$ and $k = 12$ layers.



(a) $k = 6$



(b) $k = 10$

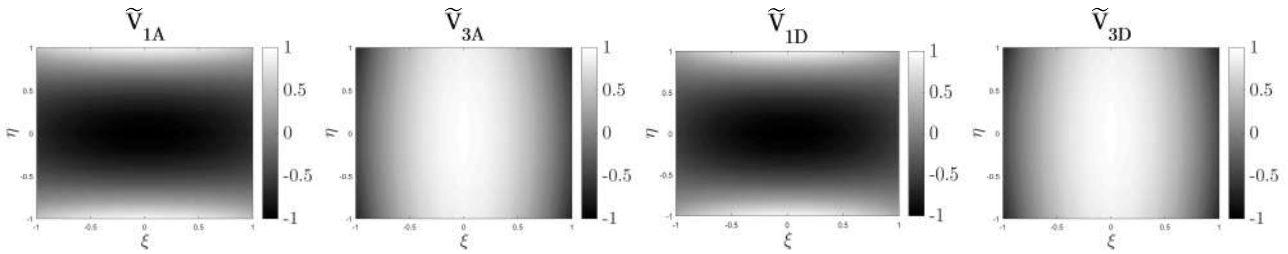


(c) $k = 12$

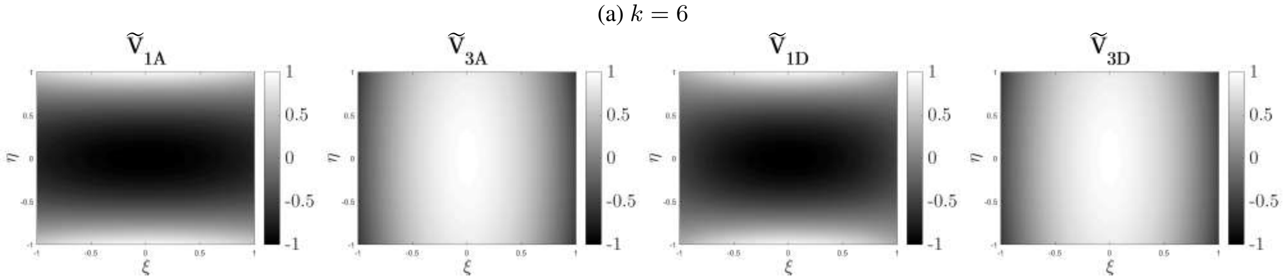
Figure 3: Optimum Lamination Parameters distribution

3. RESULTS AND DISCUSSION

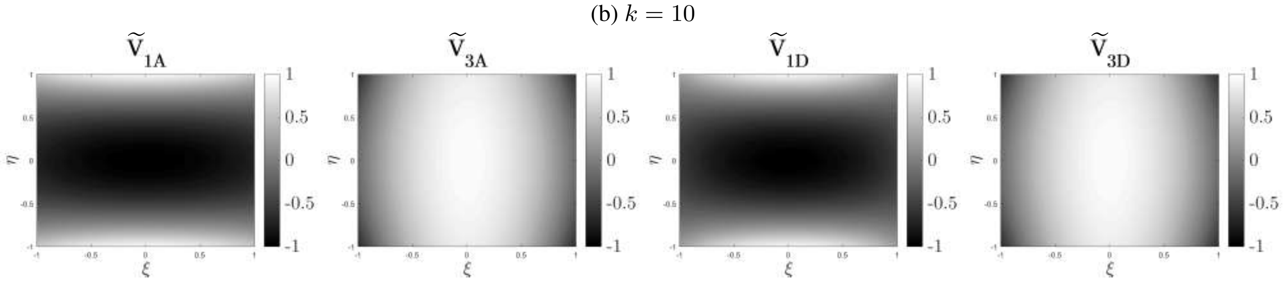
From the solution of the solution of Eq. (8), the approximated lamination parameters $\tilde{V}_{iA,D}$ distribution are shown on Fig. (4) for $k = 6, 10, 12$. It is clear, when comparing Fig. (3) to Fig. (4), that the distribution obtained mimics the optimum lamination parameters distribution for all cases evaluated. As the number of layers increases, it is noteworthy that the number of possible laminate configurations also increases since it is directly proportional to the design space for post-processing. In addition, with respect to the critical buckling loads, the obtained approximate laminate represented about 1% of performance loss with respect to the optimum lamination parameters distribution behavior.



(a) $k = 6$



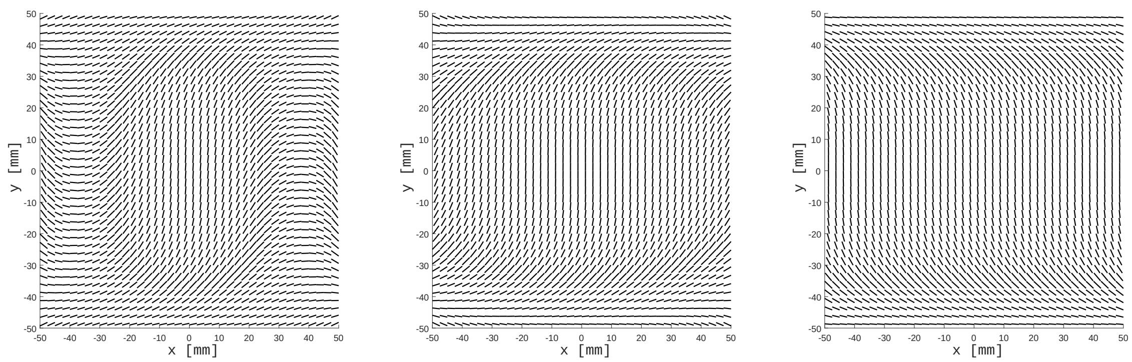
(b) $k = 10$



(c) $k = 12$

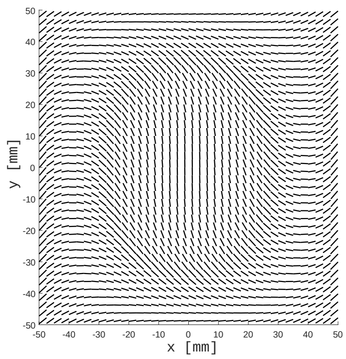
Figure 4: Approximated lamination parameters distribution

The results shown Fig. (4) can be represented by a discrete fiber angle distribution by recovering the continuous fiber angle distribution $\theta_k(\xi, \eta)$ from the nodal design variables θ_{k_i}. Figure (5), Figure (6) and Figure (7) illustrates the fiber angle distribution obtained for a laminate with $k = 6, 10, 12$ layers, respectively.

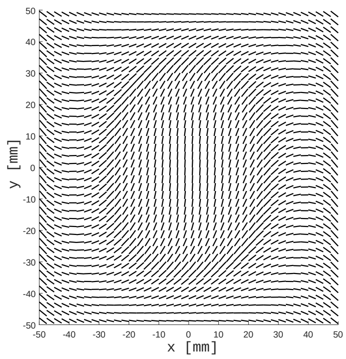


(a) Layers 1 and 6 (b) Layers 2 and 5. (c) Layers 3 and 4

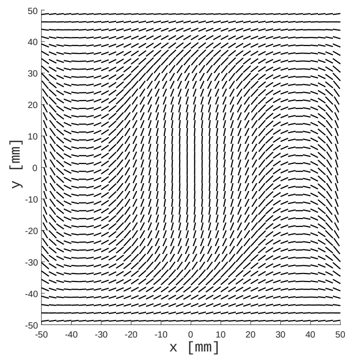
Figure 5: Lay-up configuration obtained considering $k = 6$



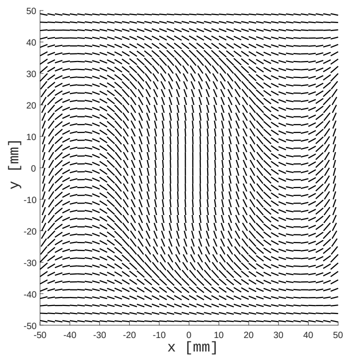
(a) Layers 1 and 10



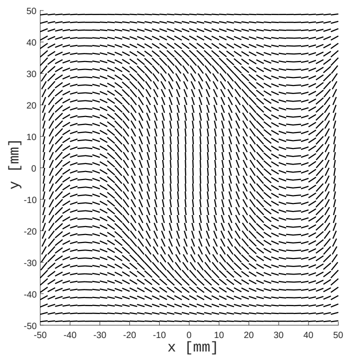
(b) Layers 2 and 9.



(c) Layers 3 and 8



(d) Layers 4 and 7



(e) Layers 5 and 6

Figure 6: Lay-up configuration obtained considering $k = 10$

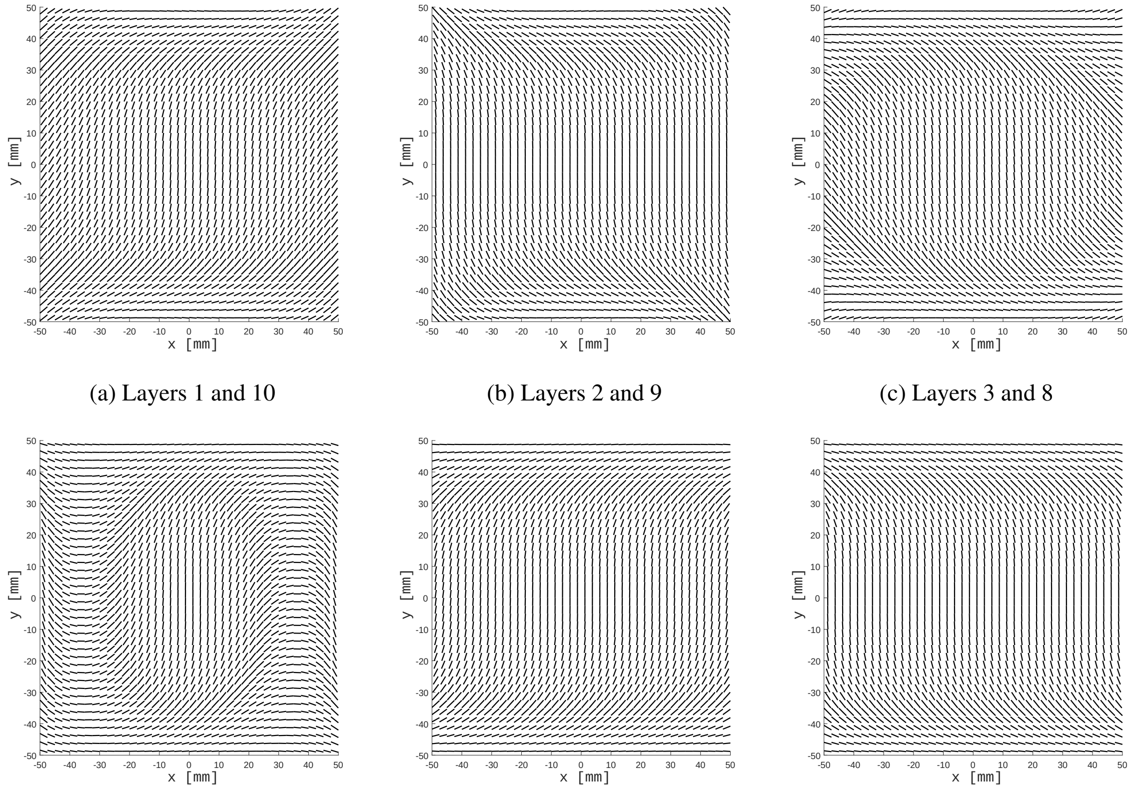
Therefore, when retrieving a laminate from an optimum lamination parameters distribution, the corresponding fiber orientation angles shall be oriented such that longitudinal, transverse and out-of-plane stiffness properties will be tailored in order to promote in-plane load redistribution and, as consequence, increase in critical buckling.

Regions in which the fiber orientation angles are small present larger axial stiffness than regions in which the fiber orientation angles are large (Olmedo and Gurdal, 1993). From Fig. (5), Fig. (6) and Fig. (7), it is possible to verify that most of the fiber orientation angles are small near the plate edges, which means that the obtained designs are axially reinforced in these regions. On the other hand, all the proposed designs are composed by layers in which the center of the plate is mainly composed by fiber angles oriented at 90°, which means that the center of the plate tends to be soft and does not carry much load.

From Quadros (2017), the increase in critical buckling loads achieved by the structural optimization using the lamination parameters are shown in Tab. 1 for the case of $k = 6$. The increase represents the ratio between the critical buckling load achieved by the optimum lamination parameters and the quasi-isotropic reference laminate. In previous work, structural optimization was evaluated for different mesh sizes for the numerical analysis and different modes were tracked. Similar results were found for $k = 10$ and $k = 12$.

Table 1: Increase in the normalized buckling load obtained for $k = 6$ (Quadros, 2017)

	$p = 2$		
Mesh Size	Mode 1	Mode 2	Mode 3
100	2.2233	2.3989	3.7521
400	2.2969	2.4010	3.8782
900	2.3001	2.4072	3.8494



(a) Layers 1 and 10 (b) Layers 2 and 9 (c) Layers 3 and 8

(d) Layers 4 and 7 (e) Layers 5 and 6 (f) Layers 5 and 6

Figure 7: Lay-up configuration obtained considering $k = 12$

4. CONCLUSION

A model for obtaining laminates based on Lagrange design elements was introduced in order o increase the efficiency for retrieving variable stiffness from an optimized lamination parameters distribution. This approach reduced drastically the number of design variables without compromising the performance – herewith about 1% of loss in terms of critical buckling loads – without limiting the set of feasible lamination parameters, design variables and constrains. Due to the easy polynomial interpolation order control over the Lagrange element, the method allows obtaining of a smooth and continuous fiber angle distribution for the design of variable stiffness laminates.

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