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NUMERICAL ANALYSIS OF POWER LAW FLUID FLOW IN A CHANNEL PARTIALLY FILLED BY A POROUS MEDIUM USING THE LATTICE BOLTZMANN METHOD

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Abstract. *In this work the flow of power law fluid in a channel partially filled by a porous medium is investigated. The porous region, which is located in the bottom half of the channel, is modeled by a set of solid and disconnected square obstacles as a heterogeneous porous medium. Mass and momentum conservation equations are solved using the lattice Boltzmann method. The non-Newtonian fluid behavior is taken into account by varying the simulation relaxation factor locally with the fluid apparent viscosity. Results show the influence of porosity and power law index on the flow friction factor in the free region. It is observed that decreasing the power law index or increasing the porosity of the porous medium causes the reduction of the friction factor in comparison to the flow between parallel plates.*

Keywords: *fluid-porous interface, power law fluid, heterogeneous porous medium, lattice Boltzmann method*

1. INTRODUCTION

The flow of non-Newtonian fluids occurring adjacently to a porous medium has technological and biological applications such as the tangential filtration of suspensions (Hanspal et al., 2006), the flow of biological fluids (e.g., blood and chime) at the interior of the human body (Rao and Mishra, 2004) and the flow of drilling fluid through the annular space between the drilling string and the wellbore wall during the drilling process of oil wells in petroleum industry (Martins-Costa et al., 2013).

In the literature, one may find studies on modeling the flow of Newtonian and non-Newtonian fluids, e.g. power law fluid (Bird et al., 1987) next to the interface between a free (purely fluid) and a porous region based on different approaches to represent the fluid-porous interface. When the porous medium is treated as a heterogeneous medium (i.e., when the spatial resolution at which the problem is solved allows distinguishing the solid phase from the fluid one) the fluid-porous interface influence is given by the interaction between the fluid and the solid matrix of the porous medium. Otherwise, when it is not possible to distinguish the solid phase from the fluid one, the porous medium is viewed as a homogeneous medium. In this case, the influence of the fluid-porous interface may be taken into account by considering the spatial variation of the porous medium properties' along the interfacial region or modeled by a proper boundary condition that couples the equations that describe the flow in the fluid region and through the porous medium.

Chen et al (2009) used the lattice Boltzmann method to study the flow of power law fluid in a channel partially filled by a porous medium. The porous domain was modelled considering the homogeneous approach without any boundary conditions to describe the fluid-porous interface. Their results show that, for a constant pressure gradient, the flow velocity at the fluid-porous interface increases as the power law index or the porosity increase.

Using the theory of mixtures, Martins-Costa et al (2013) analyzed the flow in a channel partially filled by a porous medium considering boundary conditions that take into account the fluid velocity and its derivate to model the fluid-porous interface. The results presented by Martins-Costa et al (2013) show that the greater the power law index the greater is the fluid velocity next to the fluid-porous interface.

Based on the fluid-porous interface model proposed by Ochoa-Tapia and Whitaker (1995) for Newtonian fluids, Silva et al. (2016) proposed a mathematical model for power law fluids. They studied the flow between parallel plates in which both plates are covered by a porous medium. They found that, for a constant mass flow rate through the channel, when the power law index is increased the fluid flows preferentially through the porous medium so that the mass flow rate through the central region of the channel decreases.

In order to complement the results obtained by the aforementioned studies, the goal of this work is to investigate the flow of power law fluid in a channel partially filled by a porous medium represented by a set of solid and disconnected obstacles (heterogeneous approach) using the lattice Boltzmann method (LBM). Specifically, it is analyzed both the porosity ant the power law index (fluid model parameter) effects over the friction factor of the flow through the fluid region of the channel, $C_{f,p}$.

2. MATHEMATICAL MODEL

Figure 1 shows both geometry and boundary conditions of the flow of power law fluid in a channel partially filled by a porous medium. The flow occurs in a two-dimensional channel of length L and height h_{fr} + h_{pr}. The fluid-porous interface is considered to be at $x_2 = 0$, dividing the (upper) free region and the (lower) porous one. Regarding the porous region, it is composed by a set of solid obstacles of size S equally spaced by a distance s. The porous medium is characterized by its porosity, ϕ, and density of obstacles. The porosity is defined as the ratio between the volume occupied by the fluid phase and the total volume of the porous medium, while the density of obstacles represents the number of obstacles per unit of area of the porous medium.

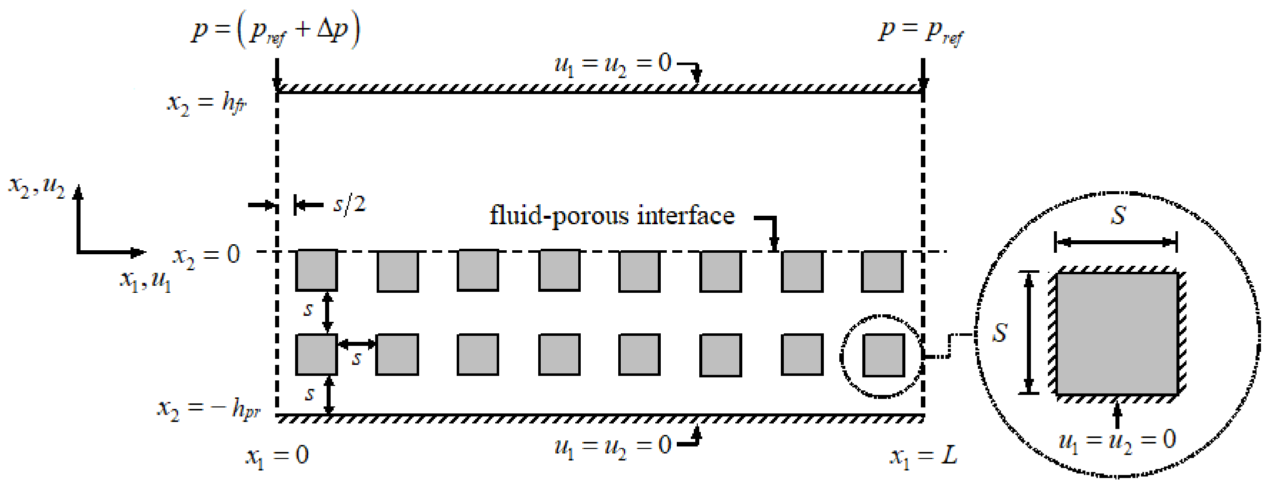

Figure 1. Problem geometry and boundary conditions

The flow occurs due to prescribed pressure conditions imposed at the inlet and outlet of the channel, p_{ref} + Δp and p_{ref}, respectively. No slip conditions are considered over channel walls and the surface of the obstacles.

Mathematically, the flow is described by the mass and momentum conservation equations, Eqs. (1) and (2), respectively, considering the following assumptions: i) steady state, ii) incompressible fluid, iii) two-dimensional flow and iv) negligible gravitational effects:

$$\frac{\partial u_\alpha}{\partial x_\alpha} = 0 \tag{1}$$

$$\rho u_\beta \frac{\partial u_\alpha}{\partial x_\beta} = -\frac{\partial p}{\partial x_\alpha} + \frac{\partial \tau_{\alpha\beta}}{\partial x_\beta} \tag{2}$$

where ρ is the fluid density, u_α is the velocity vector, p is pressure and $\tau_{\alpha\beta}$ is the stress tensor.

Considering the power law fluid model (Bird et al., 1987), $\tau_{\alpha\beta}$ is defined according to Eq. (3):

$$\tau_{\alpha\beta} = \eta(\dot{\gamma})\dot{\gamma}_{\alpha\beta} = \eta_c |\dot{\gamma}|^{n-1} \dot{\gamma}_{\alpha\beta} \tag{3}$$

where $\eta(\dot{\gamma})$ is the apparent viscosity of the fluid, η_c is the consistency index and n is the power law index.. $\dot{\gamma}$ is the magnitude of the rate of strain tensor, $\dot{\gamma}_{\alpha\beta}$, which is calculated from Eq. (4):

$$\dot{\gamma} = \sqrt{\frac{1}{2}\dot{\gamma}_{\alpha\beta}\dot{\gamma}_{\alpha\beta}} \tag{4}$$

Using the Buckingham Pi theorem (White, 1999), one may define the following non-dimensional parameters for the problem: Reynolds number, Re, power law index, n, porosity, ϕ, non-dimensional density of obstacles, DO, and height ratio, H, where Re, ϕ, DO and H are given by Eqs. (5) to (8):

$$\mathrm{Re} = \frac{\rho u_{ref}^{2-n} D_{h,fr}^{n}}{\eta_{c,\mathrm{mod}}} \tag{5}$$

$$\phi = 1 - \left(\frac{S}{S+s}\right)^2 \tag{6}$$

$$DO = \frac{1}{(S+s)^2} D_{h,fr}^2 \tag{7}$$

$$H = \frac{h_{fr}}{h_{pr}} \tag{8}$$

where u_{ref} is the average velocity across the free region when the porous medium is impermeable, $D_{h,fr} = 2h_{fr}$ is the hydraulic diameter of the free region and $\eta_{c,mod}$ is the modified consistency index of the power law fluid, defined as in Eq. (9):

$$\eta_{c,\mathrm{mod}} = \eta_c \left[\frac{1}{12}\left(8 + \frac{4}{n}\right)^n\right] \tag{9}$$

According to the Reynolds number definition given by Eq. (5), one may show that the friction factor of the flow through the free region, defined in Eq. (10), $C_{f,i}$, (Bird et al, 2002):

$$C_{f,i} = \frac{\left(-\frac{\Delta p}{L}\right)\frac{D_{h,fr}}{4}}{\frac{1}{2}\rho u_{ref}^2} \tag{10}$$

is given by Eq. (11) when the porous medium is impermeable:

$$C_{f,i} = \frac{64}{\mathrm{Re}} \tag{11}$$

3. NUMERICAL MODEL

The mathematical model described in Section 2 is solved using the lattice Boltzmann (LB) method, a numerical method based on kinetic theory to describe physical phenomena in fluids (Chen and Doolen, 1998). The fluid is interpreted as a collection of particles and is statistically represented by the velocity distribution functions $f(x_\alpha, c_\alpha, t)$ defined in such way that $f(x_\alpha, c_\alpha, t)dx_\alpha dc_\alpha$ corresponds to the probable number of particles with position between x_α and $x_\alpha + dx_\alpha$ with velocity within c_α and $c_\alpha + dc_\alpha$ at time t. In LB method, the physical space is discretized in a lattice structure in which the velocity distribution functions stream along predefined directions i with fixed velocities $c_{\alpha,i}$ and collide with each other at lattice sites. In this work, we use the so called D2Q9 (Qian et al., 1992) model, where 2 stands for the problem dimensionality and 9 for the number of possible velocities for particles to stream. The D2Q9 model is schematically shown in Fig. 2 and the values of $c_{\alpha,i}$ are presented in Table 1, where $c = \Delta x/\Delta t$, Δx is the horizontal or vertical distance between two neighbor lattice sites and Δt is the time interval in which particles stream from one lattice site to another.

The dynamics of the system is governed by the Boltzmann transport equation in its discrete form, where external forces are neglected and particle collisions are modeled by the BGK approximation (Bhatnagar et al., 1954), as in Eq. (12):

$$f_i(x_\alpha + c_\alpha \Lambda t, t + \Lambda t) = f_i(x_\alpha, t) + \frac{\Lambda t}{\lambda}\left[f_i^{eq}(x_\alpha, t) - f_i(x_\alpha, t)\right] \tag{12}$$

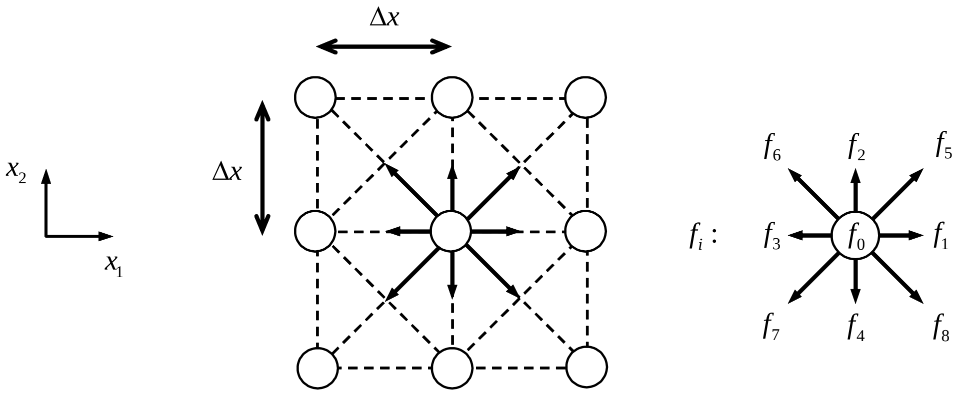

Figure 2. D2Q9 lattice structure

Table 1. Particle velocities for the D2Q9 model.

i	$c_{\alpha,i}$
0	(0,0)
1,2,3,4	(c, 0), (0, c), (-c, 0), (0, -c)
5,6,7,8	(c, c), (-c, c), (-c, -c), (c,-c)

where λ is the relaxation factor of f_i towards its equilibrium value, f_i^{eq}, which, according to the model used in this work (He and Luo, 1997), is defined as in Eq. (13):

$$f_i^{eq} = w_i\left\{\rho + \rho_0\left[\frac{c_{\alpha,i}u_\alpha}{c_s^2} + \frac{c_{\alpha,i}u_\alpha c_{\alpha,i}u_\alpha}{2c_s^4} + \frac{u_\alpha u_\alpha}{2c_s^2}\right]\right\} \tag{13}$$

where ρ_0 and ρ are the initial and instantaneous fluid density, respectively. The weighting factor, w_i, and the speed of sound, c_s, are defined according to the lattice structure. For the D2Q9 model, $w_0 = 4/9$, $w_{1,2,3,4} = 1/9$, $w_{5,6,7,8} = 1/36$ and $c_s = c/\sqrt{3}$.

The fluid density and flow velocity are calculated at each lattice site in terms of the velocity distribution function according to Eqs. (14) and (15):

$$\rho(x_\alpha, t) = \sum_{i=0}^{8} f_i(x_\alpha, t) \tag{14}$$

$$u_\alpha(x_\alpha, t) = \frac{1}{\rho_0}\sum_{i=0}^{8} c_i f_i(x_\alpha, t) \tag{15}$$

Using the Chapman-Enskog analysis (Chapman and Cowling, 1970) it is possible to show that the LB model described above represents the mathematical model for the flow of power law fluid provided that Eqs. (16) and (17) are satisfied:

$$p = \rho c_s^2 \tag{16}$$

$$\lambda = \frac{\eta_c |\dot{\gamma}|^{n-1}}{\rho_0 c_s^2} + \frac{\Delta t}{2} \tag{17}$$

In order to simulate the problem stated in Section 2, periodic pressure boundary conditions (Liao and Jen, 2008) are considered at the inlet and outlet of the channel, while halfway bounce-back conditions (Guo and Shu, 2013) are used to model channel walls and obstacle surfaces.

4. NUMERICAL TESTS

In order to verify the LB model used in this work, two test cases are studied. Firstly, we analyze the flow of power law fluid between parallel plates. Figure 3 shows the numerical and analytical (Bird et al., 2002) velocity profiles for n = 0.25, 1.00 and 4.00 and Re = 1.00. The maximum relative error between numerical and analytical results is 1.25%.

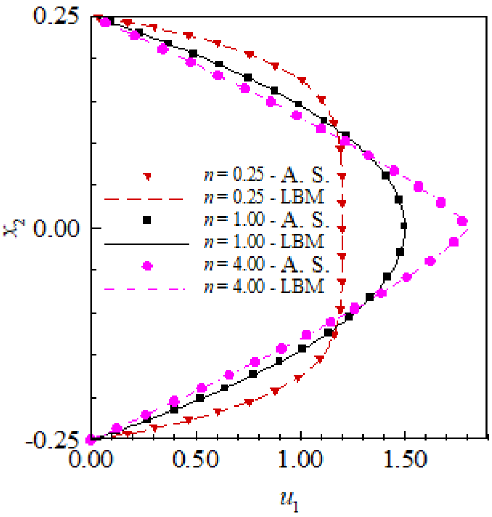

Figure 3. Numerical and analytical velocity profiles for the flow of power law fluid (n = 0.25, 1.00 and 4.00) between parallel plates for Re = 1.00.

The second problem analyzed as a test case is the flow of power law fluid through a porous channel, which, in the context of this work, consists of considering the flow through the channel shown in Fig. 1 when h_{fr} = 0.0. According to Fadili et al. (2002), the mass flow rate, $\dot{m}$, and the applied pressure gradient, $-\Delta p/L$, present the following relationship:

$$\dot{m} \propto \left(-\frac{\Delta p}{L}\right)^{\frac{1}{n}} \tag{18}$$

Figure 4 shows a plot, in logarithmic scale, of mass flow rate as a function of pressure gradient for n = 0.25, 1.00 and 4.00. In accordance to Eq. (18), these plots are straight lines with angular coefficient equal to 1/n.

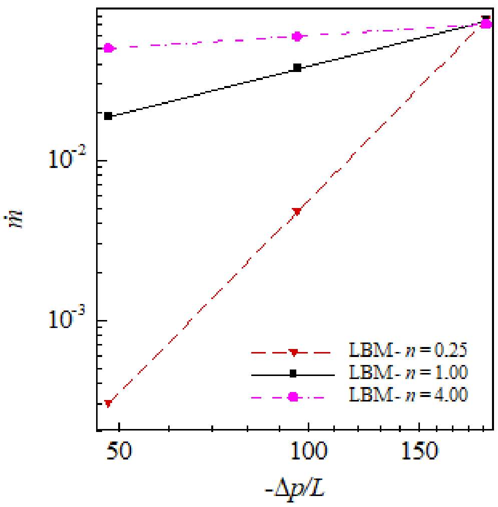

Figure 4. Numerical results for mass flow rate as a function of the pressure gradient for different values of power law index (n = 0.25, 1.00 and 4.00) for the flow through a porous channel.

The analysis shown in this section reveal that the LB model used in this work is suitable to model the flow of power law fluids in geometries with different degrees of complexity. In both cases (flow between parallel plates and through a heterogeneous porous medium), numerical results shown good agreement with results found in literature.

5. RESULTS AND DISCUSSION

The analysis of the flow through a channel partially filled with a porous medium considers the influence of porosity and power law index over the friction factor ratio, $C_{f,p}/C_{f,i}$, where $C_{f,i}$ and $C_{f,p}$ are calculated from Eq. (10). The values of porosity vary between 0.51 and 0.96, while the power law index range from 0.25 to 4.00. In all cases, the other non-dimensional parameters are kept constant (DO = 16, H = 1.00, Re = 1.00). All results were obtained for a lattice structure with 320 lattice sites ($\Delta x = 3.13 \times 10^{-3}$ m) and $\Delta x/\Delta t$ ratio equal to 32768 m/s. These values were obtained following the grid test procedure described in Meira (2016) so that LB compressible errors could be neglected and numerical stability was ensured (Succi, 2001).

Figure 5 shows the friction factor ratio, $C_{f,p}/C_{f,i}$, versus porosity, ϕ, curves for different values of power law index.

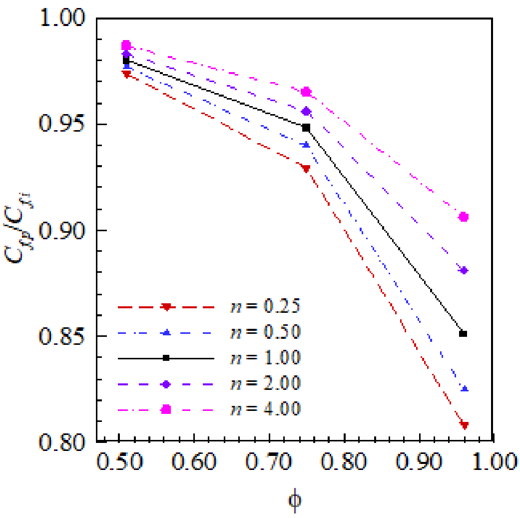

Figure 5. $C_{f,p}/C_{f,i}$ as a function of porosity for different values of power law index (n = 0.25, 0.50, 1.00, 2.00 and 4.00).

Analyzing Fig. 5, one may verify that $C_{f,p}/C_{f,i}$ tends asymptotically to unity as porosity decreases, whereas for greater values of porosity $C_{f,p}/C_{f,i}$ is reduced. This behavior reflects the influence of porosity over the permeability of the fluid-porous interface. As porosity is decreased, the solid fraction of the fluid-porous interface increases so that the resistance imposed by the solid matrix to the fluid flowing next to the porous region also increases. For the limiting case where ϕ = 0.00, the porous region becomes a solid block and the fluid-porous interface turns into a solid impermeable wall. In this case, $C_{f,p}/C_{f,i}$ = 1.00 regardless of the value of n.

Figure 6 shows the velocity profiles for n = 0.25 (a), 1.00 (b) and 4.00 (c) for ϕ = 0.51, 0.75 and 0.96. It is important to mention that each of these profiles is obtained through the arithmetic mean of all the velocity profiles taken along the entire channel. Analyzing Fig. 6, it is possible to verify that the velocity of the flow next to the fluid-porous interface increases with porosity, indicating that $C_{f,p}/C_{f,i}$ is reduced. It is possible to notice that the velocity increase at the fluid-porous interface is more pronounced for ϕ = 0.96, ranging from 1.90×10^{-2} (n = 0.25) to 1.42×10^{-2} (n = 4.00). Regarding the influence of the power law index, the flow velocity at fluid-porous interface increases when n is decreased. For n = 0.25 the influence of porosity is more remarkable, ranging from 0.03×10^{-2} (ϕ = 0.51) to 1.90×10^{-2} (ϕ = 0.96).

Considering the influence of the power law index over $C_{f,p}/C_{f,i} \times \phi$, the results presented in Fig. 5 show that $C_{f,p}/C_{f,i}$ decreases as n is reduced. This behavior is a result of the interaction between the fluid and the fluid-porous interface. The lower the value of n, the lower is the fluid apparent viscosity next to the fluid-porous interface. Figure 7 shows the apparent viscosity field for different values of power law index (n = 0.25, 1.00 and 4.00) and ϕ = 0.75, confirming that for lower values of n the apparent viscosity of the fluid decreases next to the obstacle at the fluid-porous interface. As a result, the fluid flows more easily through the free region when n is decreased. For smaller values of porosity, the interaction between the fluid and fluid-porous interface becomes less significant as well as the effect of the power law index. Recalling that when ϕ approaches 0.00 the value of $C_{f,p}/C_{f,i}$ tends to unity (regardless of the value of n) it is possible to justify the change in the slope of $C_{f,p}/C_{f,i} \times \phi$ as the value of n varies. For grater values of ϕ the influence of n

is more pronounced and $C_{f,p}/C_{f,i}$ varies significantly with n. On the other hand, for ever smaller values of ϕ the effect of n over $C_{f,p}/C_{f,i}$ decreases and $C_{f,p}/C_{f,i}$ tends to a constant value independent of ϕ.

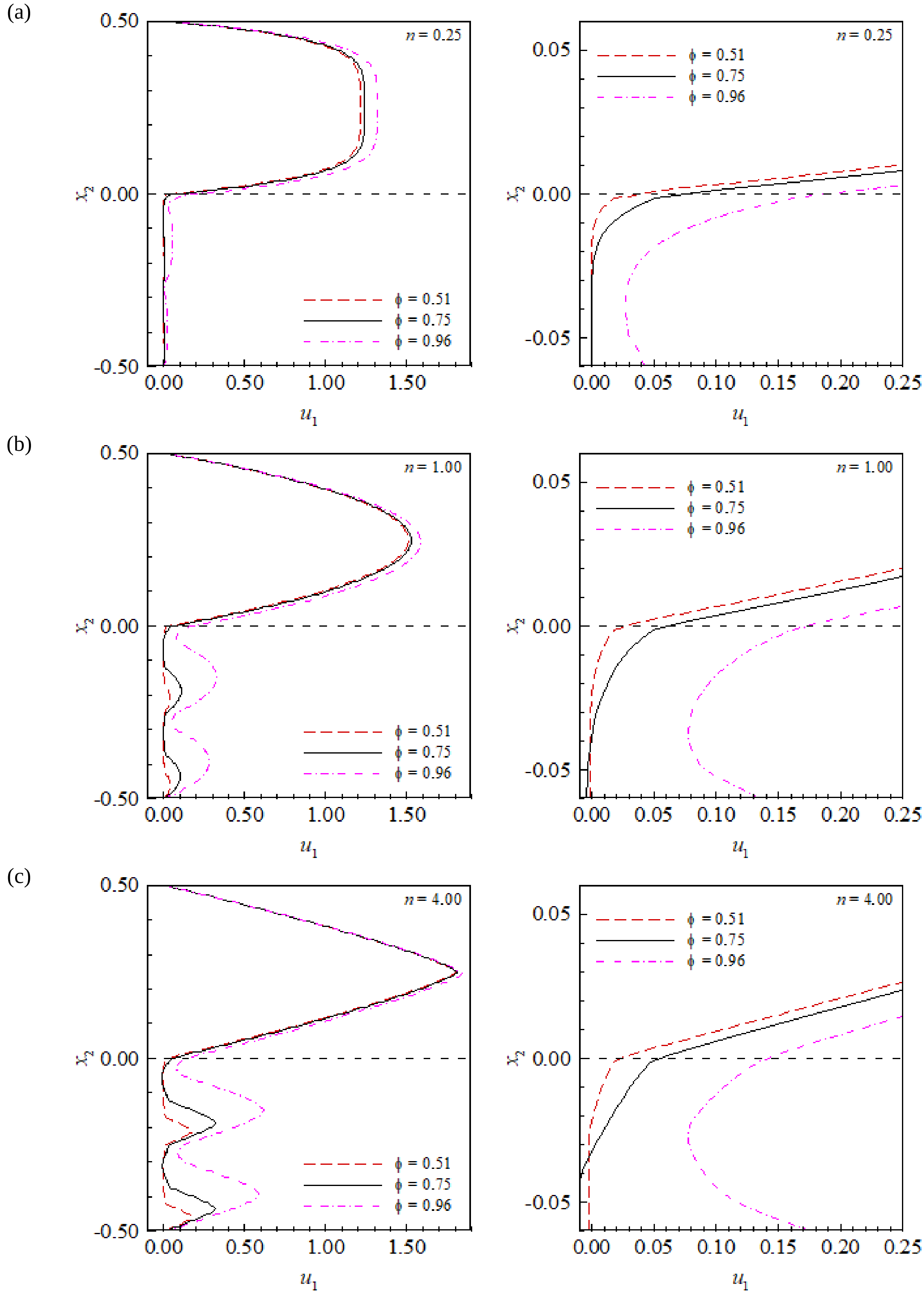

Figure 6. Velocity profiles for the flow of power law fluid in a channel partially filled by a porous medium for n = 0.25 (a), 1.00 (b) and 4.00 (c) and ϕ = 0.51, 0.75 and 0.96.

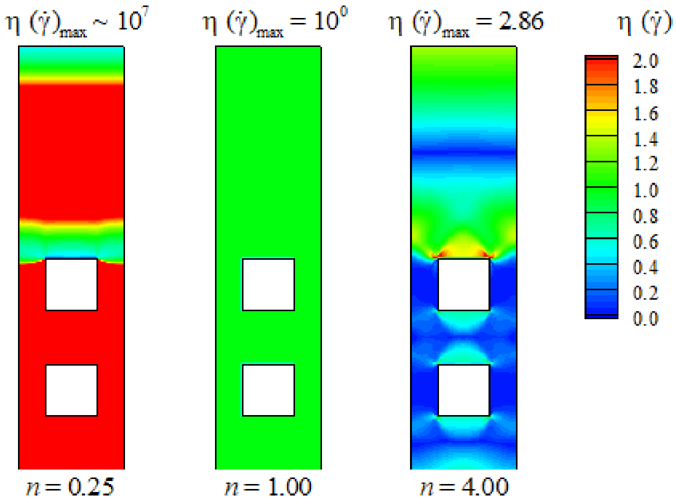

Figure 7. Fluid apparent viscosity field for the power law fluid flow in a channel partially filled by a porous medium for different values of power law index.

6. CONCLUSIONS

In the present work, the flow of power law fluid through a channel partially filled by a porous medium was numerically investigated using the lattice Boltzmann method. The porous domain is represented by a set of square obstacles located at the bottom half of the channel. Results show that, when all other non-dimensional parameters are kept constant, the friction factor ratio $C_{f,p}/C_{f,i}$ decreases when porosity is increased or power law index is reduced, where $C_{f,p}$ is the friction factor through the free region when porous medium is permeable and $C_{f,i}$ when the porous medium is impermeable. This behavior is a result of the interaction of the fluid and the solid matrix of the porous medium at the fluid-porous interface. As the porous medium becomes less permeable, the fluid-porous interface approaches to a solid wall and $C_{f,p}/C_{f,i}$ tends to unity. Regarding the influence of the power law index, as n decreases the apparent viscosity of the fluid flowing next to the fluid-porous interface decreases. Thus, the fluid flows more easily across the free region and $C_{f,p}/C_{f,i}$ decreases.

7. ACKNOWLEDGEMENTS

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