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# MATHEMATICAL MODELING OF SENSORS ELEMENTS USING GRAPHITE

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***Abstract.*** *This work shows the mathematical modelling of graphite piezoresistive sensor elements type 2B in paper substrate. The parameters analyzed were the sensitivity factor, variation of the electrical resistance with the applied mechanical tension (longitudinal and transversal) and piezoresistive coefficients. The results of the computational simulations were compared with the experimental data obtained through an experimental arrangement using the clamped beam method. The obtained experimental factor gauge is of the order 12 indicating that the type of graphite used is a good material for developing sensor elements.*

***Keywords:*** *Mathematical modeling, sensor elements, piezoresistors.*

## 1. INTRODUCTION

The graphite has been investigated and used in a wide range of applications due to its excellent mechanical, electrical and thermal properties. This paper shows the mathematical modelling and characterization of piezoresistive sensor elements using graphite films deposited on A4 paper type polymeric substrate, a process known as GOP - Graphite on Paper shown in Figure 1.

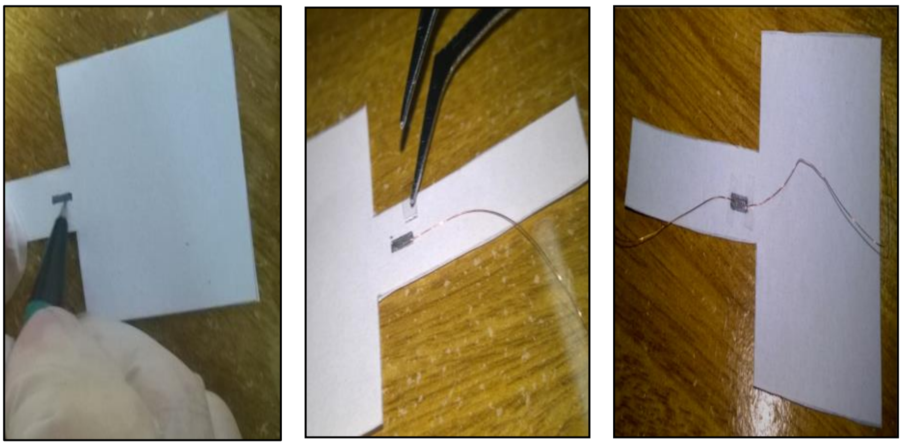

Figure 1: Graphite on Paper

The effect of piezoresistivity is the change in the electrical resistance of a material when a certain mechanical stress is applied to it (Moi, *et al*., 2013). Due to the cross effects between the physical properties of the material, the temperature also influences the electrical and mechanical properties of the sensor element (Hammes, *et al*., 2015).

This work proposes the use of carbon in the allotropic form of graphite, deposited in flexible polymer (Hammes, *et al*., 2015), for the construction of piezoresistive sensors, aiming at the modeling and computational simulation of the same. The understanding of the material properties is important for the future use of this material as a mechanical strain sensor device.

Paper has been recognized as a low cost substrate for piezoresistive strain sensors and is being used in different arrangements of MEMS devices. However, its properties are influenced by the type of fibers, weight, roughness, porosity and impurities in the paper.

Other physical and chemical characteristics are related to the types of cellulose fibers found besides the influence of humidity and temperature.

## 2. EXPERIMENTAL PROCEDURE

The sensor elements were fabricated using pencil graphite 2B on the flexible polymer according to figure 2.

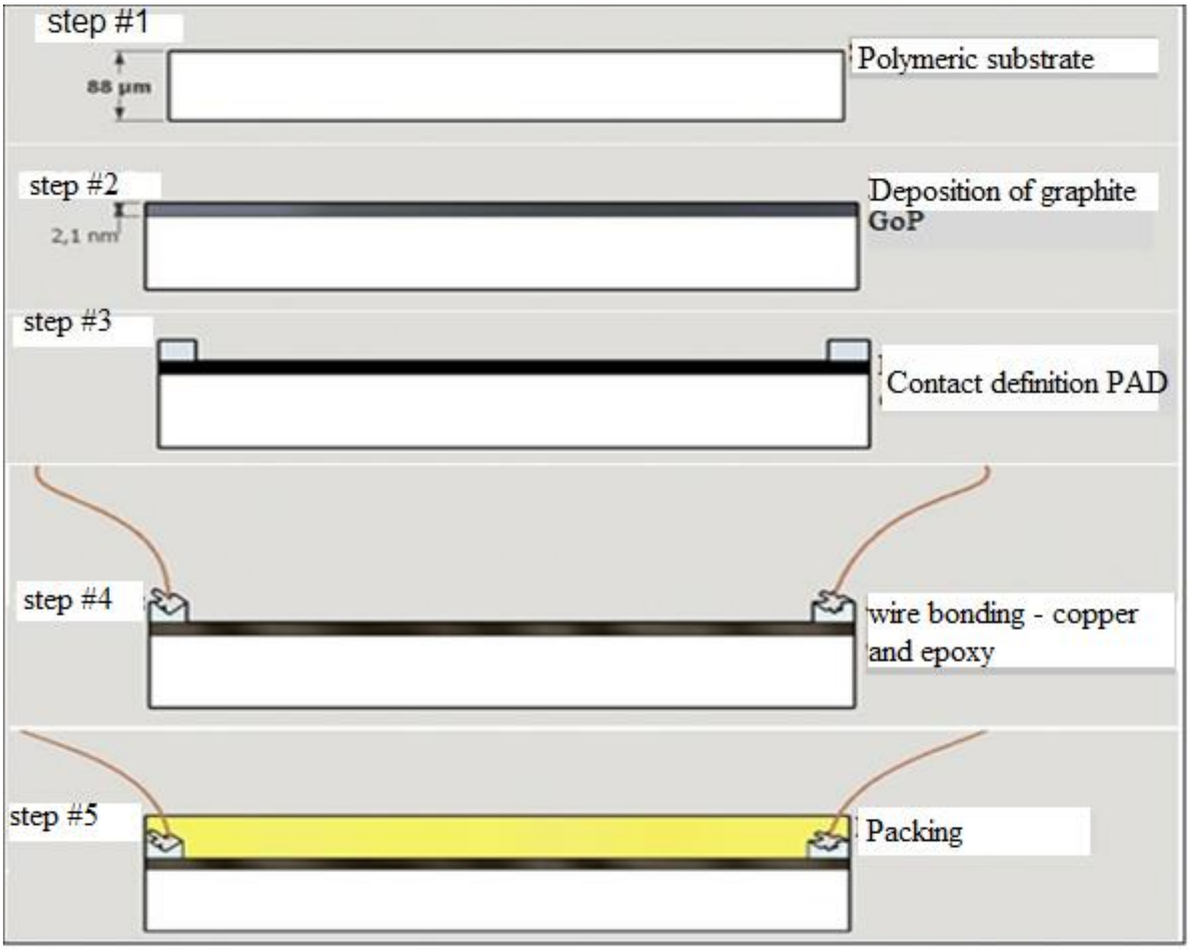


Figure 2: manufacturing steps of the sensor elements

The electrical contacts were made with copper wires (Ren, *et al*., 2012). The structure for testing the sensor element was based on the method of the cantilever (Ren, *et al*., 2012) as illustrated in Figure 3.

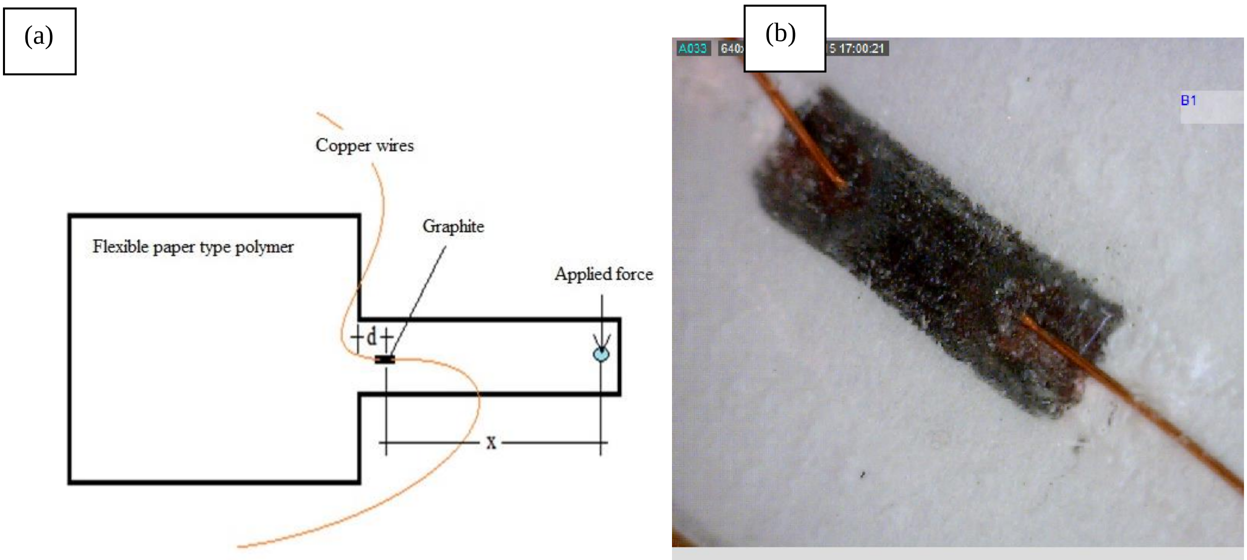


Figure 3. Experimental setup. (a) Design of the sensor element (Hammes, *et al*., 2015); (b) Photographs of the deposited sensor element.

In a clamped beam both the mechanical stress (T) and the contact force (F) depend on the modulus of elasticity (E) and the geometry of the material length (L), width (w) and thickness (t) of which is done (Hammes, *et al*., 2015).

In order to obtain the data, a $6^{1/2}$ digit multimeter Hp 34401A was used to read the variation of the resistance as a function of the applied mechanical stress, shown in Figure 4. In the application of the mechanical stresses small "weights" were used. With masses ranging from 0.095 g to 1.5 g coupled to the beam as shown in Figure 3 (b). The mathematical equations and models were implemented using MatLab$^{TM}$ software.

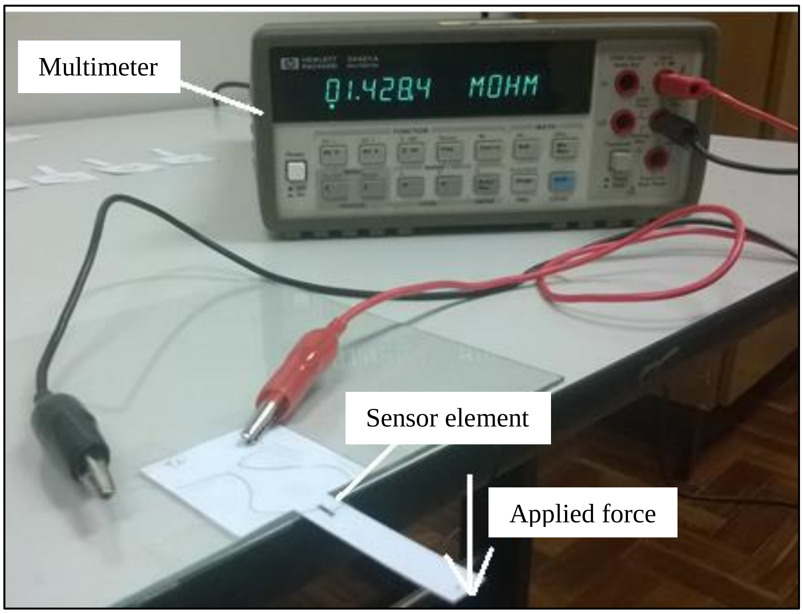


Figure 4: Digit multimeter Hp 34401A

When a force is applied on the crimped beam, the carbon atoms expand or compress sensibly, inducing a change in resistance electric. These changes may generate changes in the piezoresistance behavior of a sensing device.

## 3. MATHEMATICAL MODELING

The resistance R of a piezoresistor, shown in Figure 5, can be calculated according to (Gniazdowski, *et al*., 2000)

$$R = R_{\theta_{amb}} + \rho_0 \pi_L \int_{X_i}^{X_f} T_L(x) dx + \rho_0 \pi_T \int_{X_i}^{X_f} T_T(x) dx \tag{1}$$

Where $\pi_L$ and $\pi_T$ are the longitudinal and transverse piezoresistive coefficients respectively, $T_L(x)$ and $T_T(x)$ are the voltages applied along the piezoresistor. $R_{\theta_{amb}}$ is the initial resistance of the material at room temperature given by Eq. (2) and $\rho_0$ is the resistivity and $t$ is the thickness of the deposited material.

$$R_{\theta_{amb}} = \rho_0 \frac{L}{w.t} \tag{2}$$

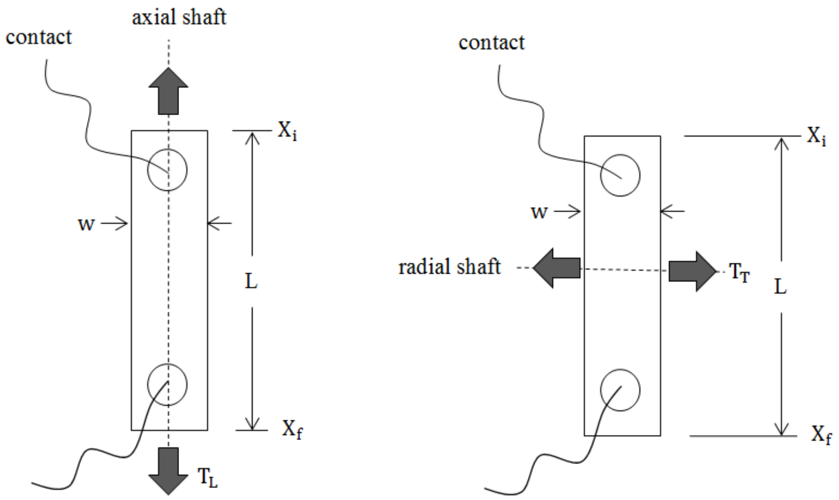


Figure 5: Physical model of a piezoresistor

When two different piezoresistors are positioned on a test structure, we consider the existence of a system of two linear equations with unknown piezoresistive coefficients given by Eqs. (3) and (4),

$$T'_L = \int_{X_i}^{X_f} T_L(x) dx \tag{3}$$

$$T'_T = \int_{X_i}^{X_f} T_T(x) dx \tag{4}$$

Then $\Delta R = R - R_{\theta_{amb}}$, thus:

$$\frac{\Delta R}{R_{\theta_{amb}}} = T'_L \pi_L + T'_T \pi_T \tag{5}$$

If two different resistances are considered, we have a system of two linear equations with two unknown piezoresistive coefficients:

$$\begin{bmatrix} \frac{\Delta R_1}{R_{\theta_{amb}}} \\ \frac{\Delta R_2}{R_{\theta_{amb}}} \end{bmatrix} = \begin{bmatrix} \pi_1 T_L & \pi_1 T_T \\ \pi_2 T_L & \pi_2 T_T \end{bmatrix} \begin{bmatrix} \pi_L \\ \pi_T \end{bmatrix} \tag{6}$$

In matrix notation, this system has the generic form given by:

$$\frac{\Delta R}{R} = T\Pi \tag{7}$$

## 4. RESULTS AND DISCUSSION

From the mathematical models described it was possible to represent graphically important factors for the fabrication of sensor elements. These graphs represent a comparison between the (ideal) situation and the (real) experimental situation, and provide important parameters for the design of piezoresistive sensor elements.

As can be seen in Figure 6, there is an exponential behaviour of the mechanical stress as a function of the GF indicating that the measure of longitudinal mechanical stress increases the GF follows this tendency to a certain limit in that there will be no more sensitivity factor of the material in detriment of the rupture of the graphite film.

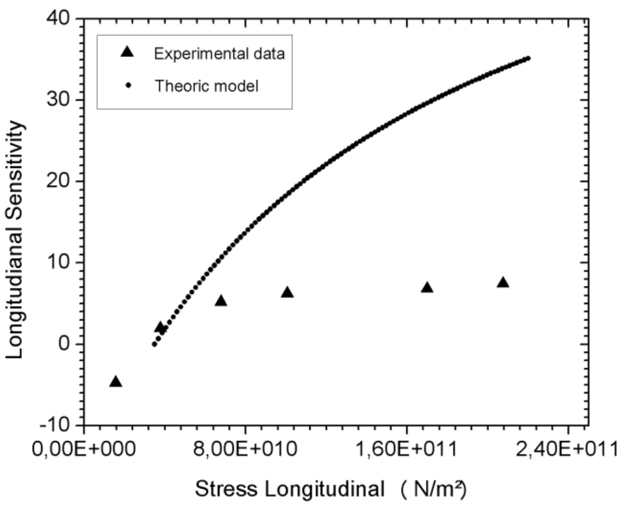


Figure 6: Comparison between the Theoretical and Experimental Model.

The results presented in Figure 7 are in agreement with those shown in Figure 6 indicating the piezoresistive coefficient values of the graphite.

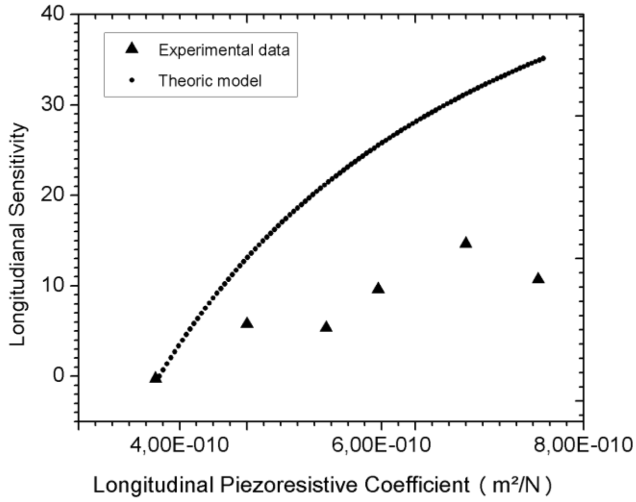


Figure 7: Piezoresistive Graphite Coefficient compared to simulated and experimental data.

The results shown in Figure 8 show a linear trend according to the experimental data. However small nonlinearities are observed which are due to the film structure itself and the experimental arrangement used.

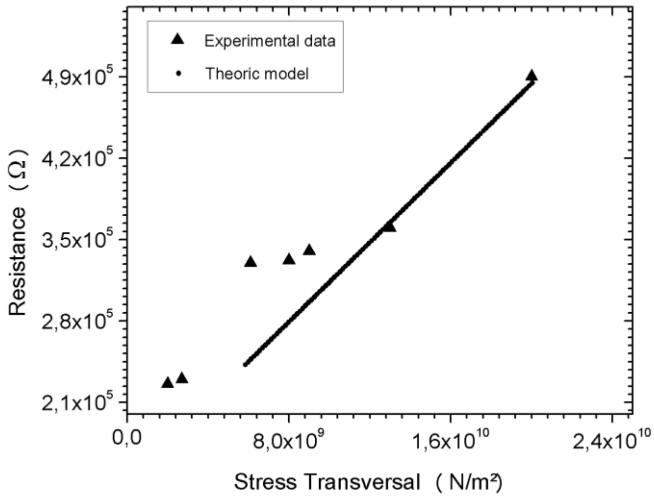


Figure 8: Comparative result between theoretical and experimental model.

## 5. CONCLUSIONS

The mathematical models implemented predicted the behavior of the sensor element, despite the fact that the experimental data presented some peaks and some discrepancies. The discrepancies are related to the deposition method and the type of substrate used. However, the results found are promising for the manufacture of sensor devices using paper-based printing.

## 6. ACKNOWLEDGEMENTS

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## 8. RESPONSIBILITY NOTICE

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