

24th COBEM - 2017



24th ABCM International Congress of Mechanical Engineering  
December 3-8, 2017, Curitiba, PR, Brazil

## COBEM-2017-1981

# DYNAMIC MODELING OF PASSIVE HYDRAULIC ENGINE MOUNTS

Flávio Yukio Watanabe

Guilherme Oliveira Miguel

Federal University of São Carlos, Department of Mechanical Engineering  
Rod. Washington Luís (SP-310), km 235 - São Carlos - SP - Brazil - 13565-905  
fywatanabe@ufscar.br, gmiguel@ufscar.br

**Abstract.** Hydraulic engine mounts are used in the automotive industry and are designed to support the weight of the engine and to isolate engine and chassis from the high frequency vibration caused by engine disturbance forces and low frequency vibration caused by road irregularities. This paper concerns linear and nonlinear modeling of three types of passive hydraulic engine mounts with different flow passage mechanisms. The linear model of a floating-decoupler and inertia track hydraulic mount is presented and compared with a simple nonlinear decoupler model. The dynamic models and behavior of these supporting and damping systems are compared in time and frequency responses. Numerical models were implemented in Matlab/Simulink utilizing lumped parameters. The results show the complexity involving the modeling of such systems and some hysteresis behavior presented in nonlinear systems.

**Keywords:** hydraulic engine mounts, vehicle dynamics, linear modeling, nonlinear modeling

## 1. INTRODUCTION

The first vehicles in the XIX century had rigid connections between the engine and chassis. This led to vibration, noise and fatigue to the system components. To mitigate those problems, some vibration attenuators like the elastomeric engine mounts were created. The necessity of quieter, safer and more comfortable vehicles conflicts with the trend of lighter and more powerful engines, which tend to worsen the vibration problems (Marzbani et al., 2014).

There are two main excitation sources in a vehicle: one, which originates in the engine, with high frequency and low amplitude, and other one generated by the track, with low frequency and high amplitude (Yu et al., 2001). To reduce the vibration of the engine, it is then necessary low stiffness and damping components, so that the force transmissibility can be reduced. The vibration originated from the track requires high stiffness and damping components, so that the relative displacement and fatigue of the engine are minimized.

This dual behavior indicates that the ideal engine mount must have frequency and amplitude dependent stiffness and damping. In this context the hydraulic engine mounts, invented in the 1980's, can act as a rigid or flexible component, depending on the type of excitation. Despite those advantages, they are still expensive and difficult to model, since this dual behavior creates nonlinearities. Figure 1 shows the cross section of a typical hydraulic engine mount.

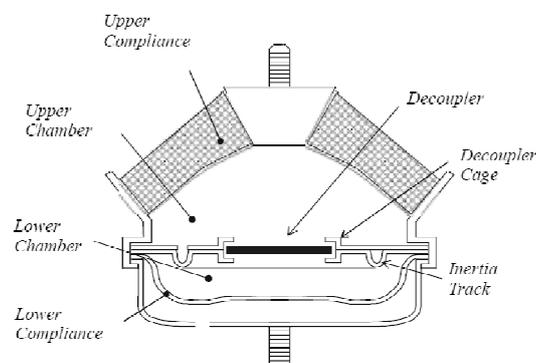


Figure 1. Cross section of a typical hydraulic engine mount

In a simplified way, one can separate a hydraulic mount in three structures: a superior chamber, a lower chamber

and a device which allows flow between the two chambers. Those parts are wrapped by a metallic structure with rubber membranes and a viscous fluid with well controlled viscous characteristics fills both chambers (Geisberger, 2000).

If the hydraulic mount is subject to some kind of excitation, there is a relative displacement in the vertical axis of the absorber parts. This oscillation causes variations in the volume of the upper and lower chambers, pushing the fluid from one chamber to the other. The flow regulator device between the chambers, which may be a simple orifice, a long channel or inertia track, a decoupler, or combinations of these devices acting as a resistance to the flow, generating damping. The achieved type of damping depends on the flow passage mechanism: some are more suitable for low frequencies damping, such as the decoupler, while others are more suitable for high frequencies, such as the inertia tracks. Such devices behaviors are nonlinear and are hard to model.

## 2. MATHEMATICAL MODELING PROCEDURE

The mathematical models of hydraulic engine mounts are generated using the conservation of momentum and fluid continuity equations. The cross section e dynamic model of a floating-decoupler and inertia track hydraulic engine mount model are illustrated in Fig. 2 and an approach from Jazar and Golnaraghi (2002) and Geisberger (2000) is utilized.

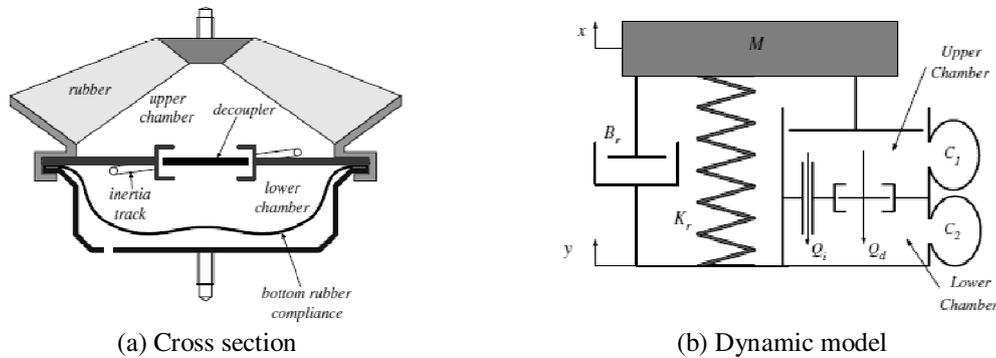


Figure 2. A floating-decoupler and inertia track hydraulic engine mount

The obtained system of equations for this type of hydraulic engine mount is showed in matrix form in Eq. (1), where,  $M$  is the effective fluid mass,  $B$  is equivalent viscous damping coefficient,  $C$  is the chamber compliance,  $K = (C_1 + C_2)/C_1C_2$ ,  $A$  is cross-sectional area,  $A_p$  is equivalent piston area, and  $i$  and  $d$  subscripts are related to inertia track or decoupler, respectively.

$$\begin{bmatrix} M_d & 0 \\ 0 & M_i \end{bmatrix} \begin{Bmatrix} \ddot{x}_d \\ \ddot{x}_i \end{Bmatrix} + \begin{bmatrix} B_d & 0 \\ 0 & B_i \end{bmatrix} \begin{Bmatrix} \dot{x}_d \\ \dot{x}_i \end{Bmatrix} + \begin{bmatrix} A_d^2 & A_d A_i \\ A_d A_i & A_i^2 \end{bmatrix} \begin{Bmatrix} x_d \\ x_i \end{Bmatrix} = \frac{A_p}{C_1} \begin{Bmatrix} A_d \\ A_i \end{Bmatrix} x \quad (1)$$

The time and frequency analysis may be conducted with the transmitted force  $F_T$  of the engine mount, which can be shown to be different for high frequency and low amplitude, Eq. (2), and for low frequency and high amplitude excitations, Eq. (3).

$$F_T = K_r x + B_r \dot{x} + (A_p - A_d)(P_1 - P_2) + A_p P_2 + A_d R_d Q_d \quad (2)$$

$$F_T = K_r x + B_r \dot{x} + A_p (P_1 - P_2) + A_p P_2 \quad (3)$$

### 2.1 NONLINEARITIES

Many nonlinearities affect the behavior of hydraulic mounts. He and Singh (2007) studied the asymmetric step responses and the stiffening of chambers under increased mean loads, which is caused by a discontinuous behavior of the top and bottom compliances. The compliances are modeled as functions of pressure and are intended to describe the influence of a vacuum region, a linear region and a stiffening region.

As noted by Geisberger (2000), the stiffness and damping of the chambers are usually modeled using a Viogt model for rubber, which assumes that they are invariable to the frequency and amplitude. However, the behavior of these variables depends of the steady-state amplitude, frequency of excitation and preload magnitude. Because of this, the nonlinear behavior of the stiffness and damping can be modeled by a function of the frequency of oscillation, amplitude

and preload. Another source of nonlinearities is the inertia track. The linear inertia track model considers constant resistance and inertia. Because the flow through the inertia track is oscillatory, the Reynolds number changes at each point of the oscillation. This means that the flow can be turbulent, passing through a laminar state and zero, which means that the friction factor and stiffness change with time.

The decoupler switching action is another well-studied nonlinearity source. When the decoupler plate doesn't touch the cage, the decoupler is open and the fluid passes what is considered a simple orifice. As with the inertia track, the Reynolds number doesn't remain constant. When the decoupler plate touches the cage, all the flow through the decoupler is considered blocked, which again changes the resistance parameters.

Equation (1) is obtained from a simplified linear model. Many nonlinear models have been proposed to better describe the time and frequency response of hydraulic mounts. One of the most studied nonlinearities is relative to the closing decoupler, which can be a difficult task to accurately describe. Jazar and Golnaraghi (2002) proposed a simple yet effective nonlinear decoupler model, which was used in this study. The model consists of a function which changes the fluid resistance to flow and which depends of the decoupler velocity, displacement, a nonlinear coefficient  $E$  and the decoupler gap size  $\Delta$ . The model can be seen in Eq. (4).

$$\begin{bmatrix} M_d & 0 \\ 0 & M_i \end{bmatrix} \begin{Bmatrix} \ddot{x}_d \\ \ddot{x}_i \end{Bmatrix} + \begin{bmatrix} B_d & 0 \\ 0 & B_i \end{bmatrix} \begin{Bmatrix} \dot{x}_d \\ \dot{x}_i \end{Bmatrix} + K \begin{bmatrix} A_d^2 & A_d A_i \\ A_d A_i & A_i^2 \end{bmatrix} \begin{Bmatrix} x_d \\ x_i \end{Bmatrix} + E \frac{x_d^2}{\Delta^2} \dot{x}_d \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = \frac{A_p}{C_1} \begin{Bmatrix} A_d \\ A_i \end{Bmatrix} x \quad (4)$$

For the direct decoupler and inertia track hydraulic engine mount of Fig. 3, an approach from Jazar and Golnaraghi (2002) is utilized. The conservation of momentum and fluid continuity equations are again applied and system of equations in the matrix form is represented in Eq. (5), where,  $K = 1/C_1 + 1/C_2 + 1/C_3$ .

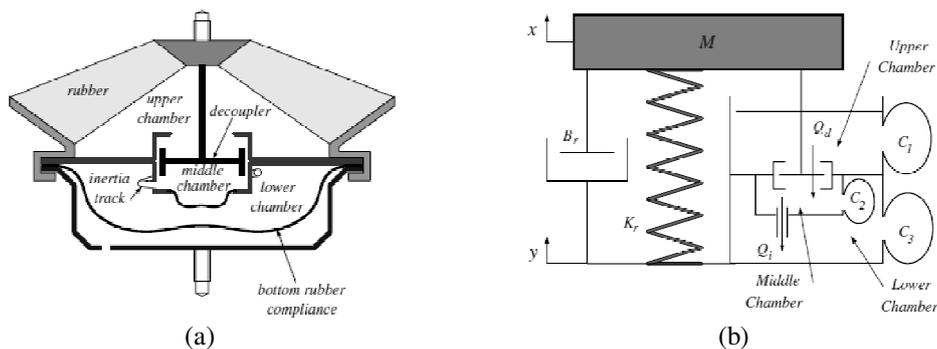


Figure 3. Direct decoupler and inertia track hydraulic engine mount

$$\begin{bmatrix} M_d & 0 \\ 0 & M_i \end{bmatrix} \begin{Bmatrix} \ddot{x}_d \\ \ddot{x}_i \end{Bmatrix} + \begin{bmatrix} B_d & 0 \\ 0 & B_i \end{bmatrix} \begin{Bmatrix} \dot{x}_d \\ \dot{x}_i \end{Bmatrix} + \begin{bmatrix} A_d^2 K & -\frac{A_d A_i}{C_2} \\ -\frac{A_d A_i}{C_2} & \frac{A_i^2}{C_2} \end{bmatrix} \begin{Bmatrix} x_d \\ x_i \end{Bmatrix} = \begin{Bmatrix} A_d \left( \frac{A_p}{C_1} - A_b K \right) \\ \frac{A_i A_b}{C_2} \end{Bmatrix} x \quad (5)$$

The hydraulic engine mount of Fig. 4 has a bell system to improve flow motion (Marzbani et al., 2014), and the Geisberger (2000) approach is conducted. The application of conservation of momentum and fluid continuity equations conducts to the system of Eq. (6-7) that represents the model behavior.

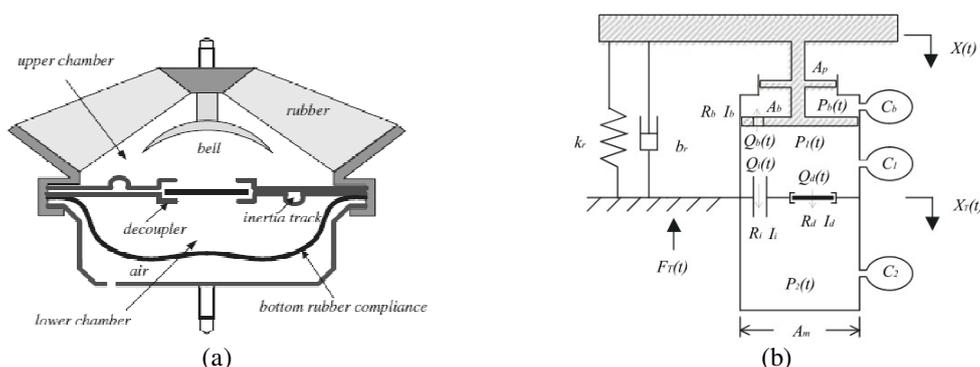


Figure 4. Floating decoupler, inertia track and bell system hydraulic engine mount

$$k_1 \left[ \left( 1 - \frac{A_b}{A_m} \right) x - \frac{A_b}{A_m} x_b - \frac{A_d}{A_m} x_d \right] + k_b \left[ \left( 1 - \frac{A_b}{A_m} \right) x - \frac{A_d}{A_m} x_b - \frac{A_b}{A_m} x_d \right] = \frac{A_m}{A_b} [m_b \ddot{x}_b + b_b (\dot{x}_b - \dot{x})] \quad (6)$$

$$k_1 \left[ \left( 1 - \frac{A_b}{A_m} \right) x - \frac{A_b}{A_m} x_b - \frac{A_d}{A_m} x_d \right] - \frac{k_2 A_d}{A_m} x_d = \frac{A_m}{A_d} (m_b \ddot{x}_b + b_b \dot{x}_d) \quad (7)$$

### 3. RESULTS AND DISCUSSION

Equations (1) and (4) were implemented in Matlab/Simulink using block diagrams. The values used for both simulations can be seen in Tab. 1 and diagram blocks for the linear and nonlinear models are presented in Fig. 5 and 6, respectively.

Table 1. Values used in the Simulink models

Symbol	Linear floating decoupler	Nonlinear floating decoupler	Units
$A_i$	5.72e-5	5.72e-5	m <sup>2</sup>
$A_d$	2.3e-3	2.3e-3	m <sup>2</sup>
$A_p$	5.027e-3	5.027e-3	m <sup>2</sup>
$B_i$	2.9	2.9	N s/m
$B_d$	4.83e-3	4.83e-3	N s/m
$B_r$	2000	2000	N s/m
$C_1$	4.6e-10	4.6e-10	m <sup>3</sup> /N
$C_2$	4.6e-10	4.6e-10	m <sup>3</sup> /N
$K$	2.196e9	2.196e9	N/m <sup>5</sup>
$K_r$	266e3	266e3	N/m
$M_i$	0.37e-2	0.37e-2	kg
$M_d$	2.645e-2	2.645e-2	kg
$E$	-	2.9095	-
$\Delta$	-	1.0e-3	m

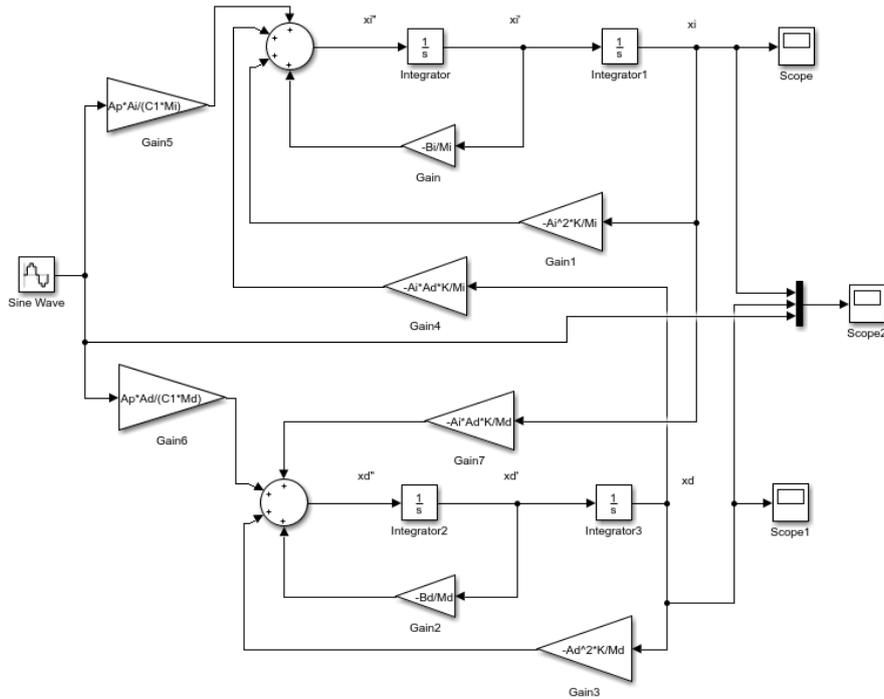


Figure 6. Block diagram for the linear decoupler model

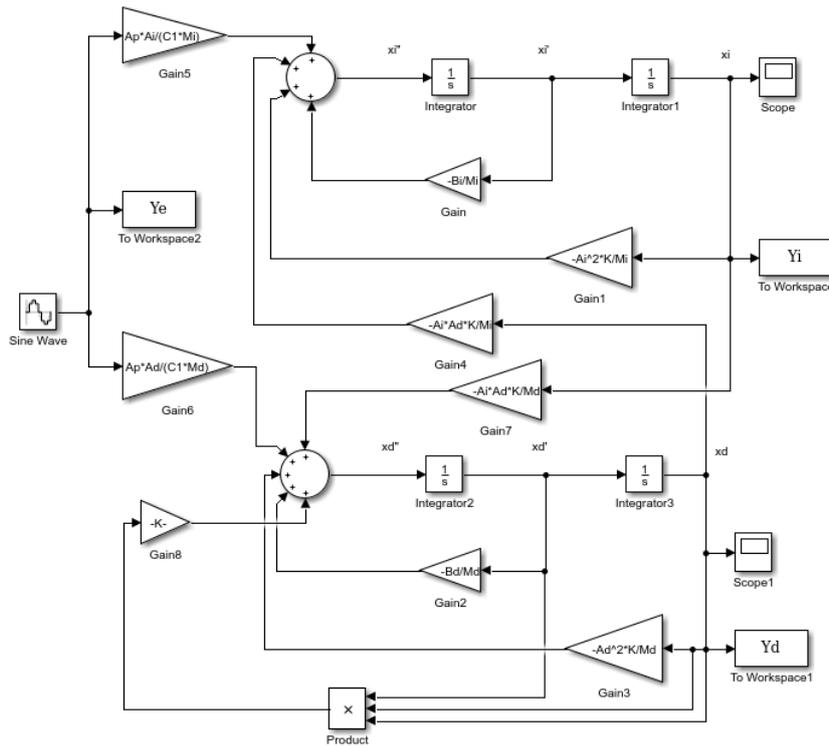


Figure 7. Block diagram for the nonlinear decoupler model

For a sinusoidal input of amplitude 0.1m and 0.01m and a frequency around the natural frequency (627 rad/s), the time response for both linear and nonlinear models can be seen in Fig. 8 and 9, respectively.

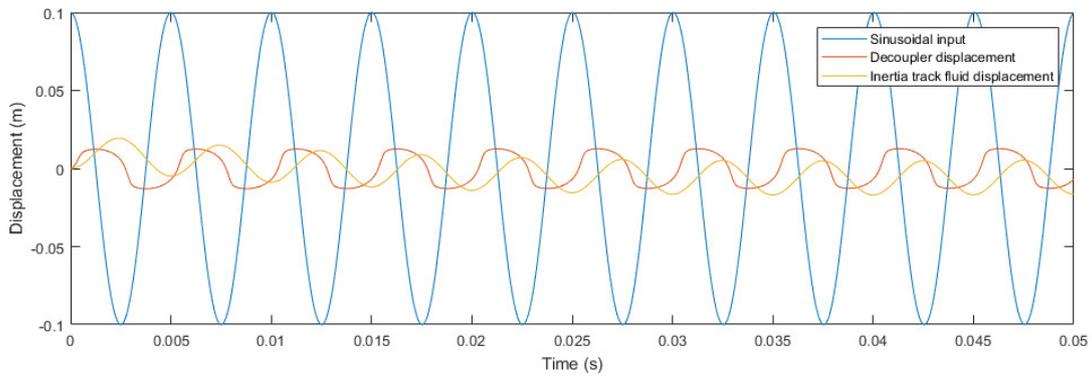


Figure 8. Time response for the linear model

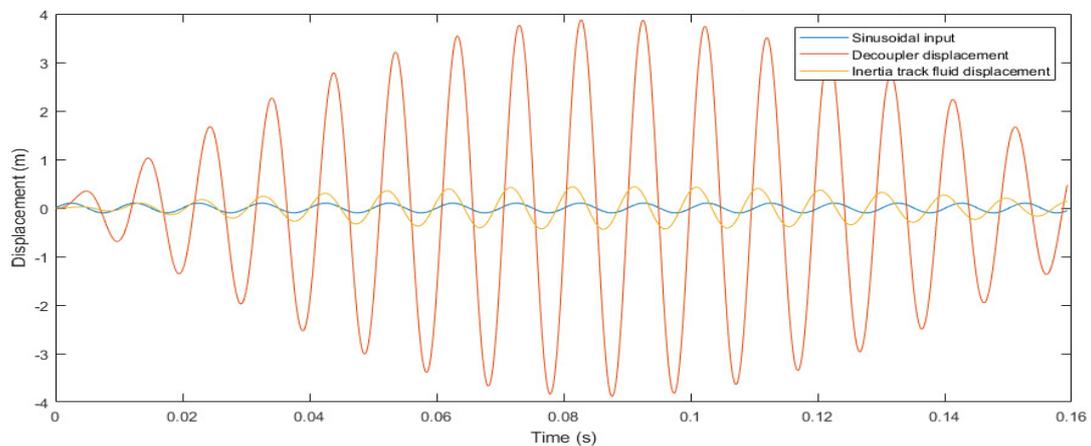


Figure 9. Time response for the nonlinear model

Figure 8 shows that the linear model couldn't represent the resonance phenomenon. The fluid displacement in the decoupler and inertia track were slightly shifted and smaller than the input. In this case, a similar fluid amount passes in both the inertia track and decoupler. In the nonlinear model presented in Fig. 9, the resonance phenomenon is better viewed. The decoupler displacement is much larger than the input, but it also isn't correct. Because the decoupler is situated in a cage, there is a physical limit to the fluid displacement, which is much smaller than the maximum showed in the graph. In order to better analyse the nonlinear behavior at resonance, the nonlinear model needs to be seen in a frequency scale.

Figure 10 shows the frequency response for the linear model. As one can see, in the linear model the flow is much bigger through the decoupler. There is no difference in phase between the inertia track and decoupler responses.

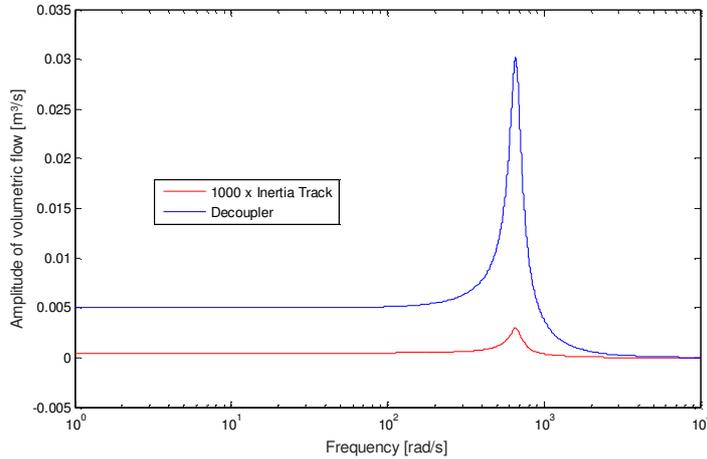


Figure 10. Frequency response for the linear model

Jazar (2002) proposes a multiple-scale perturbation method, using a small bookkeeping parameter  $\epsilon$  to show the order of magnitude of nonlinear, damping, forcing, and coupling terms. Supposing that the excitation  $y$  is a harmonic low-amplitude function near resonance, the following system can be described:

$$\ddot{y} + \epsilon u_i \dot{y} + \epsilon v y_i + v y_d = v y \quad (8)$$

$$\ddot{y}_d + \epsilon(u_d + y_d^2)\dot{y}_d + y_d + \epsilon y_i = v y \quad (9)$$

$$y = \epsilon w \cos(\omega t) \quad (10)$$

where  $w$  is approximately equal to 1. Solving the system for the steady-state solution, Jazar (2002) shows that the system is equal to

$$u_d^2 r^2 + (2\sigma r - v r - 0.25 q r^3)^2 = w^2 \quad (11)$$

The implicit Eq. (11) relates the amplitude of the response  $r$  with the detuning frequency  $\sigma$  and the amplitude of the excitation  $w$ . The frequency response curve at Fig. 11 is showed for some given values of  $v$ ,  $q$  and  $w$ . For a nonlinear system, the decoupler behavior at resonance is a backbone curve. This nonlinear behaviour represents hysteresis and instability.

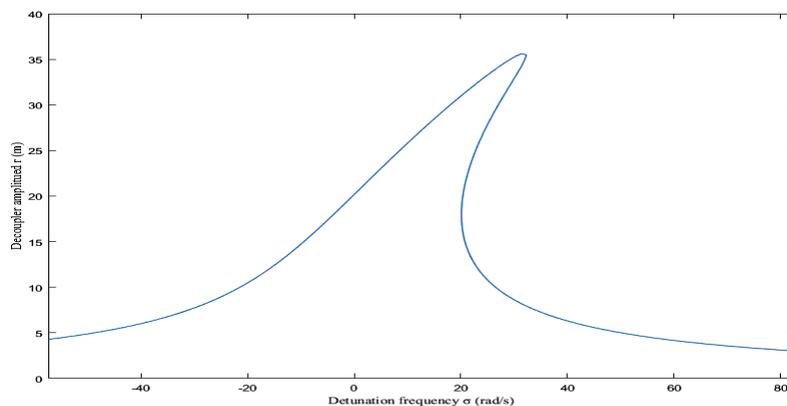


Figure 11. Nonlinear frequency response of the decoupler around resonance

#### **4. CONCLUSIONS**

This study revised some of the linear and nonlinear models presented in the literature, using block diagrams to analyze the behavior in time domain. The linear model presented a poor solution for a near resonance analysis, which was better described in the nonlinear model. The hydraulic engine mounts, although a viable and more efficient solution than traditional elastomeric mounts, are hard to model and analyze due to the various nonlinearities sources. A deeper analysis is necessary to compare the behavior of the direct decoupler and bell system hydraulic mounts.

#### **5. ACKNOWLEDGMENTS**

This work was supported by São Paulo Research Foundation - FAPESP.

#### **6. REFERENCES**

- Geisberger, A.A., 2000. *Hydraulic engine mount modeling, parameter identification and experimental validation*. Master's Thesis, University of Waterloo, Waterloo.
- He, S. and Singh, R., 2007. Discontinuous compliance nonlinearities in the hydraulic engine mount. *Journal of Sound and Vibration*, Vol. 307, p. 545-563.
- Jazar, G.N. and Golnaraghi, M.F., 2002. Nonlinear modeling, experimental verification, and theoretical analysis of a hydraulic engine mount. *Journal of Vibration and Control*, Vol. 8, p. 87-116.
- Marzbani, H., Jazar, R.N. and Fard, M., 2014. Hydraulic engine mounts: a survey, *Journal of Vibration and Control*, Vol. 20(10), p. 1439-1463.
- Yu, Y., Naganathan, N.G. and Dukkipati, R.V. 2001. A literature review of automotive vehicle engine mounting systems, *Mechanism and Machine Theory*, Vol. 36, p. 123-142.

#### **7. RESPONSIBILITY NOTICE**

The authors are the only responsible for the printed material included in this paper.