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THE INFLUENCE OF THE BEARING ANGULAR STIFFNESS IN ROTORDYNAMICS

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Abstract. *The rotordynamic analysis plays an important role in a high-speed machine mechanical design. Unwanted shaft lateral vibrations due to resonance can be easily avoided in general mechanical systems, but even in slow speed hydro-generators, the rotating system must be able to withstand an over speed of about three times the rated speed. For this reason, an accuracy rotordynamics model to predict its dynamic behaviour is very important for reliable and uninterrupted operation of these machines. In some cases, the bearings can combines radial and axial constrains. This kind of assembly adds stiffness in the slope degree-of-freedom on the bearing. The majority of bearing models for rotordynamic calculation presents stiffness in the displacement degrees-of-freedom only, a two by two order matrix, disregarding the slope stiffness. To handle the angular stiffness, the bearing model must be expanded to at least a fourth order matrix, to accommodate also the angular stiffness coefficients. In this work, a methodology to identify the angular stiffness coefficient, besides to show its influence in the unbalance response, is presented. The methodology is based in an inverse problem, using the unbalance response obtained experimentally and its numerical equivalent in an optimization environment. The resultant parameters and its equivalent obtained through analytical model found in literature will be compared. For identification propose, an experimental set up will be implemented.*
Keywords: *rotordynamic analysis, bearing model, angular stiffness, optimization.*

1. INTRODUCTION

Rotating machines are widely used in industry usually performing critical roles in production processes. There are a large number of types of rotating machines, for example turbines, compressors, electrical motors, energy generators, among others. The shaft lateral vibration due to resonance is one of main problems that must be avoided in rotating systems. Although this kind of vibration can be easily prevented, even in slow speed hydro-generators, the rotating system must be able to withstand an over speed about three times the rated speed. In order to ensure a safe and reliable operation of such equipment, it is fundamental to have a correct prediction of their dynamic behaviour throughout the design stages and an appropriate predictive monitoring and diagnosis when in operation.

These characteristics are closely related to the bearing dynamic coefficients (Miliavacca, 2015). The approach to obtain the stiffness and damping matrix of bearings in the most of models is limited to translation coefficients, that is, a second-order matrix. According to Samali et al. (1986), the dynamic coefficients due to momentum, although neglected in many analyzes, may have significant importance. In some cases, the bearings can combines radial and axial constrains, adding stiffness in the slope degree-of-freedom on the bearing (Håkansson, 2008). Furthermore, if the rotor shaft generates a significant moment on the bearing, the lack of angular stiffness coefficients can lead to errors in determining critical speeds (Lim e Singh, 1990).

The present paper proposes a methodology to determine the angular stiffness coefficients of the ball bearing through an inverse problem, using unbalance responses acquired by experimental test and its numerical equivalent in a nonlinear optimization environment. On this purpose, an experimental set up will be mounted in laboratory. Thus the bearing coefficients obtained by optimization can be compared with an analytical model present in literature (Zhang, 2013).

2. MOTIVATION

The effect of angular stiffness can be clearly noted when comparing two unbalance responses in a rotor with self-aligning ball bearings and deep groove ball bearings. In Fig. 1, the rotor with self-aligning ball bearings is represented by the blue curve and the rotor using deep groove ball bearings by black curve.

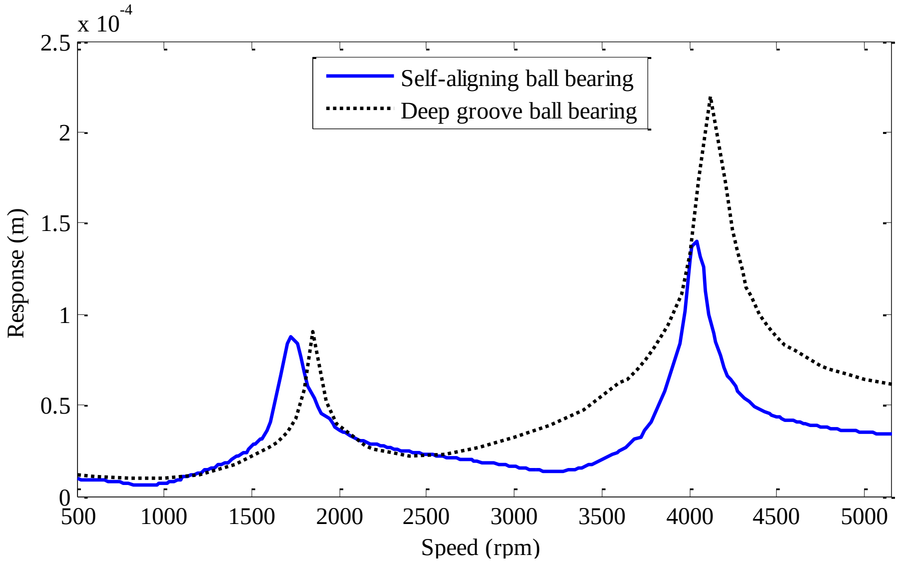

Figure 1 – Experimental unbalance responses.

For the rotor with self-aligning ball bearing, the first and the second critical speeds are 1722 rpm (28.7 Hz) and 4030 rpm (67.16 Hz). For the rotor with deep groove ball bearing, the first critical speed is 1850 rpm (30.83 Hz) and the second critical speed is 4125 rpm (68.75 Hz). This is directly linked with the effect of angular stiffness in the deep groove ball bearing. If the angular stiffness is not present in the model, it may to show an error of 128 rpm for first and 95 rpm for second critical speeds.

3. MATHEMATICAL MODEL

The model of a mechanical rotating system can be represented by

$$M\ddot{q}(t)+\left(C+G(\Omega)\right)\dot{q}(t)+\left(K_s+K_b\right)q(t)=f(t), \tag{1}$$

where M is the mass matrix, C the damping matrix, G the gyroscopic matrix, K_s the rotor stiffness matrix, K_b the bearing stiffness matrix, Ω is the rotational speed, q is the generalized coordinate and f represent the external forces applied in the system . The mechanical rotating system is a rotor that is basically composed of a shaft, one or more disks and several bearings. The model presented in Eq. (1) is used in numerical simulations and the matrices are defined by finite element method which can be found in Lalanne and Ferraris (2001).

3.1 Bearing Mathematical Model

The radial ball bearing stiffness was estimate by model presented in Krämer (1993), as follows,

$$k_r = 1.3\, z^{2/3} d^{1/3} F^{1/3} 10^6 \quad N/m\,, \tag{2}$$

where z is the number of balls of bearing, d is the ball diameter and F is the load applied radially in the bearing.

The bearing stiffness is adding in the finite element model of the rotor using the follow matrix,

$$K_b = \begin{bmatrix} \boxed{\begin{matrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{matrix}} & \begin{matrix} k_{x\theta} & k_{x\varphi} \\ k_{y\theta} & k_{y\varphi} \end{matrix} \\ \begin{matrix} k_{\theta x} & k_{\theta y} \\ k_{\varphi x} & k_{\varphi y} \end{matrix} & \begin{matrix} \boldsymbol{k_{\theta\theta}} & k_{\theta\varphi} \\ k_{\varphi\theta} & \boldsymbol{k_{\varphi\varphi}} \end{matrix} \end{bmatrix}, \tag{3}$$

that is a fourth order matrix, where the terms in k_{xx}, k_{xy}, k_{yx} and k_{yy} represents a second order matrix which consider only radial stiffness in the bearing.

In this study it is assumed that the radial stiffness is orthotropic, that is, the stiffness strictly horizontal or vertical are the same, $k_{xx} = k_{yy} = k_r$. Hence, the terms of cross stiffness are neglected. This occurs in this case, because when is applied a force in a specific direction, there is no significant reaction in any other direction. Considering the angular stiffness $k_{\varphi\varphi}$ and $k_{\theta\theta}$, the bearing stiffness matrix - Eq. (3) - can be rewritten as follows,

$$K_b = \begin{bmatrix} k_r & 0 & 0 & 0 \\ 0 & k_r & 0 & 0 \\ 0 & 0 & \boldsymbol{k_{\theta\theta}} & 0 \\ 0 & 0 & 0 & \boldsymbol{k_{\varphi\varphi}} \end{bmatrix}. \tag{4}$$

The coefficients that represent the angular stiffness will be used as parameter to be estimated in an optimization environment. The mathematical model for angular stiffness will be presented in next section.

The addition of bearing stiffness is exemplified as shown in Fig. 2. The nodes of the element matrix are given by K_{11}, K_{12}, K_{21} and K_{22} (each one a fourth order sub-matrix) and the superscript index represents the element number. The K_b corresponds t

o the matrix of Eq. (4). The blue nodes represent the addition of the bearing stiffness in the global stiffness of system.

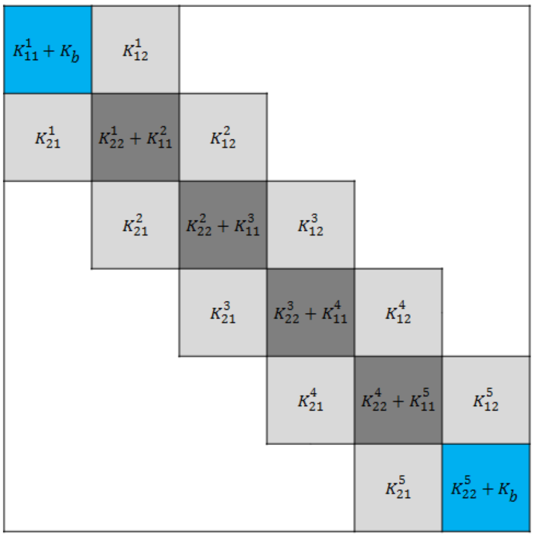

Figure 2 – Representation of global matrix mounting

Here, the ball bearing damping matrix (C) is analogous to the ball bearing stiffness matrix - Eq. (3) and (4) - and represents the entire damping added in the system. In this work, the damping is considered as proportional one, according with is presented in Ewins (2000). The addition of the bearing damping coefficients follows the same principle shown in Fig. 2. However, the system damping matrix is zero, i.e., the matrix C has the same order as the matrix K, but there are only values corresponding to the added by damping of the bearings.

3.1.1 Bearing Mathematical Model – Angular Stiffness

The mathematical model for the ball bearing angular stiffness used in this paper was based in the model presented in Zhang (2013),

$$K_{ang} = K_n \sum_{j=1}^{Z} \frac{\delta_n^{1.5} r_i R_j \sin^2 \psi_j}{\left(f_i - 0.5\right) D + \delta_n}, \tag{5}$$

where ψ_j (angular position of jth ball in the pitch circle) is

$$\psi_j = \frac{2\pi}{Z}\left(j-1\right), \tag{6}$$

and K_n is the contact stiffness of the ball with the races, δ_n is the translational displacement of the race, D is the ball diameter, r_i is the orbit radius of the inner race groove curvature center, f_i is the groove curvature coefficient, and R_j is the moment arm of the force imposed on the bearing inner ring.

These coefficients were obtained according to Guay (2015). The mathematical development is based in the theory developed by Hertz (1882) and in the advances made by Jones (1946) and Hamrock and Anderson (1983). Then, the relation between stiffness K_n and deformation δ_n is

$$Q = K_n \delta_n^{3/2}, \tag{7}$$

where Q is the normal load applied in the balls.

4. OPTIMIZATION PROCESS

The nonlinear optimization process consists in fitting the numerical unbalance response to an equivalent experimental response to obtain the parameters of the ball bearings of interest.

In this work, the optimization problem is defined as

$$\min\ f\left(x\right): R^6 \rightarrow R, \tag{8}$$

where x is the design vector,

$$x = \left\{k_{\varphi\varphi}, k_{\theta\theta}, c_{\varphi\varphi}, c_{\theta\theta}, m_U, \phi\right\} \tag{9}$$

where $k_{\varphi\varphi}$ and $k_{\theta\theta}$ represents the ball bearing angular stiffness, $c_{\varphi\varphi}$ and $c_{\theta\theta}$ represents angular damping associated, and m_U and ϕ are the modulus and phase of the unbalance of the system.

Thereby the objective function used in optimization process is defined as

$$f\left(x\right) = d^T \cdot d \tag{10}$$

where d is a error vector and is defined as

$$d_i = Ue_i - Uc_i, \quad i = 1 \ldots n \tag{11}$$

and Ue is the experimental unbalance response, Uc is the numerical unbalance response and n is the number of points of both curves. For each iteration in the optimization process, the follow operation occurs

$$K_b = K_b + \Delta K_b, \tag{12}$$

where ΔK_b is a test value for ball bearing angular stiffness. Likewise, the same is done to damping matrix, C, and for the modulus and phase of the unbalance (m_U and ϕ),

$$\begin{aligned} C &= C + \Delta C \\ F_U &= m_U \Omega^2 e^{i\phi} + \Delta m_U \Omega^2 e^{i\Delta\phi}, \end{aligned} \tag{13}$$

where F_U is the unbalance force and Ω is the rotational speed.

The optimization process is stopped when a convergence criterion is achieved, thus $f(x)$ is minimized. The last ΔK_b, ΔC, Δm_U and $\Delta\phi$ found are considered the angular stiffness, angular damping and the modulus and phase of unbalance of the system, respectively.

The radial bearing stiffness (k_r) will be not used in optimization process because of rotor dynamics characteristics, as the critical speeds, do not are influenced by radial bearing stiffness, as can be observed in the critical speed map for radial bearing stiffness (Fig. 3), that shows the critical speeds as a function of the radial bearing stiffness.

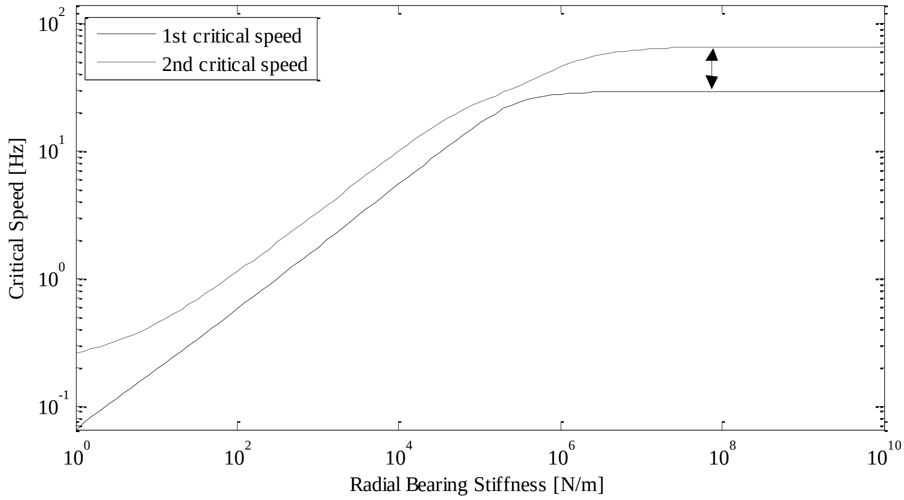

Figure 3 – Critical Speed Map for radial bearing stiffness

The radial bearing stiffness calculated by eq. (2), that will be presented in table 1, is in the upper plane region (pointed by arrows) where the rotor dynamics characteristics are governed by the shaft, regardless of the radial bearing stiffness.

5. EXPERIMENTAL SET-UP

The unbalance responses seen in Fig. 1 were obtained experimentally using the rotor presented in Fig. 4.

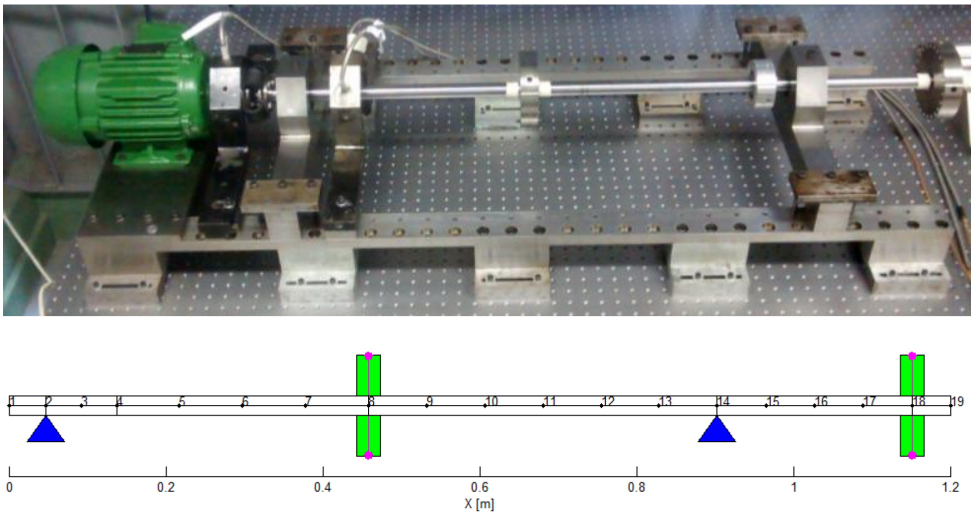

Figure 4 – Experimental rotor (above) and its finite element model (below).

The experimental rotor shown in Fig. 4 is composed of a steel shaft (SAE 52100), with length of 1200 mm and diameter of 25 mm. The inertia was represented by two steel discs, with thickness of 30 mm and diameter of 130 mm, located at 458 mm and 1150 mm from the motor (nodes 8 and 18, in Fig. 4). Two types of ball bearings were used: deep groove ball bearing (SKF 6205) and self-aligning ball bearing (SKF 2205 ETN9). In one setup for the measurement of unbalance response, the self-aligning ball bearing was used in two positions; in another setup, one bearing was replaced by the deep groove ball bearing, producing the effect verified in Fig. 1.

The unbalance responses were performed by a cost-down measurement, which consisted in raising the rotor speed up to 6000 rpm, and, after the speed stabilization, measure the responses as the rotor is decelerated. The measured speed range was between 520 and 5150 rpm.

In order to perform the unbalance responses tests, it were used two displacement transducers (PS 1002, Vibrocontrol), a signal conditioner (GS 5001, Vibrocontrol), a data acquisition module (model 3160-B-042, Bruel & Kjær), as well as software for collecting data (Pulse Lab Shop, Bruel & Kjær).

6. RESULTS

The numerical simulation was performed by the software Rotordin 9.0 (made in MATLAB codes), which has been developed by the GVIBS group since last decade. The matrices were developed by finite elements method (FEM), with a mesh of 25 elements of beam type, using the Timoshenko's two nodes method.

The geometry of the axes and disks was mentioned in section 5. Both axes and disks are made of steel (SAE 52100) with a modulus of elasticity (Young) of 210.109 N/m and a density of 7850 kg/m³. The bearing data are described in table 1. The bearing positions refer to how far is the bearing of the motor position.

Table 1. Ball bearing data

	Ball bearing type		Bearing position	
	Deep Groove (6205)	Self alignment (2205 ETN9)	Position 01	Position 02
Nº of balls	9	22	47 mm	902 mm
Ball diameter (mm)	7.9385	8.50		
Bearing thickness (mm)	15	18		
k_r (N/m)	$4.85.10^7$	$9.92.10^7$		
k_{ang} (N/rad)	2.48.10³			

The values of radial (k_r) and angular (k_{ang}) stiffness were obtained applying the Eq. (2) and (5), where was previously stated that $k_b = k_{xx} = k_{yy}$ and $k_{ang} = k_{\varphi\varphi} = k_{\theta\theta}$. The values of proportional viscous damping were obtained by peak-amplitude method, where was obtained a value of 9.9 Ns/m.

The optimization process obtained the follow results for the angular stiffness and damping coefficients,

$$\text{Stiffness} \rightarrow \begin{cases} k_{\varphi\varphi} = 2.05 \cdot 10^3 \text{ N/m} \\ k_{\theta\theta} = 1.98 \cdot 10^3 \text{ N/m} \end{cases}$$

$$\text{Damping} \rightarrow \begin{cases} c_{\varphi\varphi} = 9.00 \cdot 10^{-11} \text{ N.s/rad} \\ c_{\theta\theta} = 2.63 \cdot 10^{-1} \text{ N.s/rad} \end{cases}.$$

Using these values, the unbalance response was calculated in a numerical simulation, which was compared with the experimental results for a system with deep groove ball bearing, as can be seen in Fig. 5.

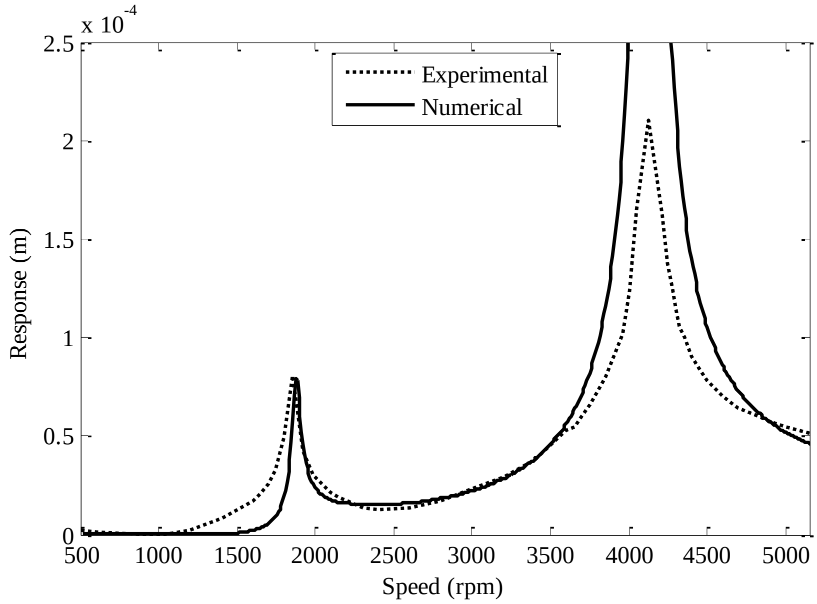

Figure 5 – Unbalance response of a rotor with deep groove ball bearing (experimental and numerical)

It can be noticed that the values obtained have a difference of about 17% in relation to the values theoretically obtained for the angular stiffness. This difference between the identified angular stiffness and that obtained analytically may have been due to the simplicity of the analytical model or the flexibility of the bearing box. This can occur if the bearing is sufficiently flexible, then what is identified is the stiffness of a bearing assembly associated in series, which reduces stiffness. However, due to the robustness of the bearing box used experimentally, the first justification seems more plausible.

Even so, despite this numerical difference, the value obtained analytically can be considered satisfactory for an initial identification, taking into account the numerical result presented. To obtain more accurate results, it is also necessary an improvement in the mathematical model used for the parameters identification, considering that phenomena such shaft bow and possible residual deformations in the shaft are not yet implemented.

As shown in Fig. 5, in spite of the values find to the critical speeds are satisfactory, the comparison between numerical and experimental curves are still not good at some points of the curves, with emphasis on the regions around the critical speeds. This may be due to problems related to the identification of the damping of the system, which is difficult to obtain with precision, in addition to other problems with the model as mentioned above.

Concerning the angular damping, although small, the damping coefficient has its importance in the precision of unbalance response of rotors with deep groove ball bearings. In some optimization environment in which these coefficients were neglected, the precision of coefficients identification was poor.

7. CONCLUSIONS

In this paper the influence of the angular rigidity present in deep groove ball bearing in the unbalance response of a rotor was observed. Theoretical models were presented to obtain the radial and angular stiffness of ball bearings, being tested in a numerical model to obtain a numerical unbalance response.

In the proposed methodology, the angular stiffness values were evaluated by comparing them with stiffness values obtained through a non-linear optimization process. The comparison between the optimal values and the experimental one showed satisfactory results, but there is a need for improvement in the mathematical model used in optimization process, in order to obtain more accurate results.

The results of this work may allow for this research group to use, in the future and with more accurate estimation parameters, this mathematical model in order to estimate the angular stiffness of the ball bearing.

8. ACKNOWLEDGEMENTS

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