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Statistical calibration of 2DOF vortex-induced vibration phenomenological model.

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Abstract. *The vortex-induced vibrations of floating structures play an important role in the design of offshore engineering. The accurate prediction of structural instability is extremely important due that the vortex shedding behind bluff bodies may lead to degradation of structural performance or even structural failure. Analytical VIV models can be represented by numerous approaches to modeling both the structure and fluid. The CFD (Computational Fluid Dynamics) approaches consists of solving the Navier-Stokes equations directly, mostly limited by heavily computational costs that many times very difficult to satisfy in the practical engineering. In this sense, semi-empirical models are an alternative approach, where the fluid dynamic forces acting on the structure are emulated by phenomenological equations and can be a very useful tool in wide industrial applications. The purpose this work is to present a phenomenological model of vortex-induced vibrations for spring-mounted rigid cylinder structures putting the parameter variability in the general context of Uncertainty Analysis doing first a sensitivity analysis for input empirical parameters of the model using the Adaptive Sparse Grid Stochastic Collocation method (ASGSC). After this, a backward parameter estimation analysis is done using a Bayesian technique to calibrate these empirical parameters, by means of exploring posterior density functions. Synthetic data were generated as reference simulating experimental data to show the calibration technique used. This kind of analysis can help to understand the behavior of the structure to critical situations as well the effects of varying empirical parameters in the response variables. The influence of these parameters and other coefficients that affect the dynamical response is analyzed and also discussed.*
Vortex induced vibrations, Reduced model, Calibration

1. INTRODUCTION

Vortex-Induced Vibration are motions induced on bodies interacting with an external fluid flow, it is caused by the vortex shedding behind bluff bodies and may lead to degradation of structural performance or even structural failure. In offshore structures such as pipes, risers and mooring lines, it is a particularly important. Exist several ways to predict the dynamic response of structures undergoing large amplitude vibrations induced by the surrounding flow. One of the most effective prediction method consists of solving the coupled fluid-structure system modeling the flow through Navier-Stokes equations where the structure, due to its rigidity, can be characterized by a simple oscillator with one or two degrees of freedom. The main problem is that the complexity involved in solving these systems generally leads to the use of simpler computational models, as a preliminary approach, due to the prohibitive costs of considering more complex systems in the preliminary phase of the project. Thus, an alternative, is to use a phenomenological model based on wake oscillators Gabbai and Benaroya (2005) that replace the vortex shedding mechanisms of the flow by simple models. In this work we present a model proposed in Postnikov *et al.* (2017) that captures important features of the VIV dynamics 2DOF. Another problem that appears is the reliability of those simulations many time is disrupted by the inexorable presence of uncertainty in the model data, such as inexact knowledge of system forcing, initial and boundary conditions, physical properties of the medium, as well as parameters in constitutive equations. These situations underscore the need for efficient uncertainty quantification methods for the establishment of confidence intervals in computed predictions, the assessment of the suitability of model formulations, and/or the support of decision-making analysis.

In this sense, traditional statistical tool for uncertainty quantification within the realm of Engineering is the Monte Carlo method, see Elishakoff (2003). This method requires, first, the generation of an ensemble of random realizations associated with the uncertain data, and then it employs deterministic solvers repetitively to obtain the ensemble of results. The ensemble results should be processed to estimate the mean and standard deviation of the final results. The implementation the Monte Carlo is straightforward, but its convergence rate is very slow (proportional to the inverse of the square root of the realization number) and often infeasible due the large CPU time needed to run the model in question. Another technique that has been applied recently is the so-called Stochastic Galerkin Method (SG), which employs Polynomial

Chaos expansions to represent the solution and inputs to stochastic differential equations, see Babuska *et al.* (2004). A non-intrusive method, referred to as Stochastic Collocation (SC), see Xiu and Hesthaven (2005), arises addressing as a combination of interpolation methods and deterministic solvers, like Monte Carlo where a deterministic problem is solved at each point of an abstract random space. Similarly to SG methods, SC methods achieve fast convergence when the solution possesses sufficient smoothness in random space. Thus when there are steeps gradients or finite discontinuities in the stochastic space, these methods converge very slowly or even fail to converge. In this work, we present an adaptive sparse grid collocation strategy with the aim of obtaining greater accuracy in nonlinear systems analysis. Specifically, it will be examined an interesting situation involving fluid-structure interaction in a model used as preliminary approach of Engineering projects.The analysis done takes into account uncertainties in the coupling parameters of the model. Thus, for the forward sensitivity analyzes, the problem is then formulated through the probabilistic approach where the uncertainties are characterized by a probability density function. Particular emphasis is placed on investigating uncertainty propagation in the nonlinear response of fluid-structure interaction, see Xiu *et al.* (2002). The Stochastic Collocation method is used to propagate uncertainties through the model offering an appropriate framework to tackle the external forces and uncertainties in the input data. Here, the fluid-structure interaction is modeled in a simple way focusing the assessment of an SC method as an effective tool for uncertainty quantification. Results are presented as errorbars for inline and crossflow the amplitude for a reduced velocity range.

2. MATHEMATICAL MODEL

The problem of this approach, when applied to real problems is that sometime lead in to highly cost models for UQ analysis or in preliminary phase os design. An alternative, is to use a phenomenological model based on wake oscillators Gabbai and Benaroya (2005) that replace the vortex shedding mechanisms of the flow by simple models. In this example we use a similar model as presented in Kreuzer (2008) and Rosetti *et al.* (2011) that captures important features of the VIV dynamics after a calibration of its parameters, taking into consideration the available experimental data. Due this reason, the calibration is an activity that leads with parameters subject to uncertainties that can be modeled using experimental data as references solution to analyse the parameters of our phenomenological model with the following equations model. Thus considering an elastically supported rigid circular cylinder shown schematically in Fig. 1.

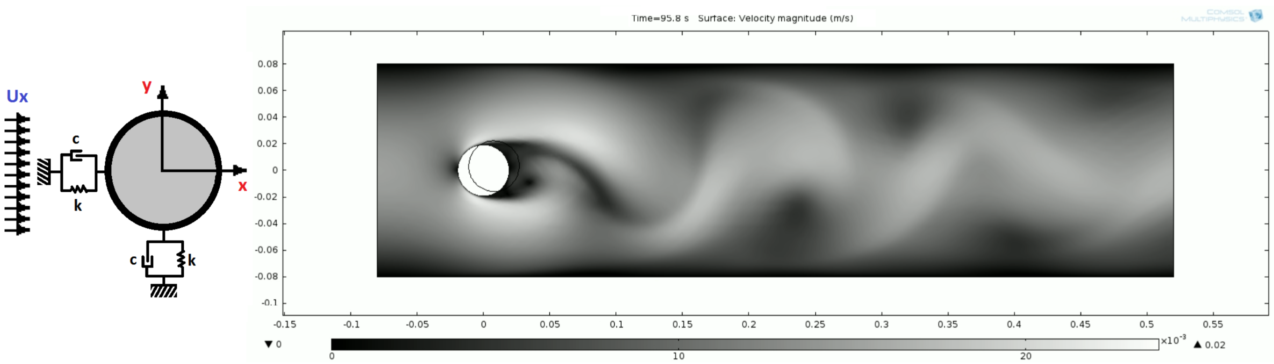

Figure 1: Computational setup in our study

Based on the classical Van der-pol equation, with empirical parameters, this system can emulate the fluid dynamics around the body to predict the behavior of the system. The scheme of the coupling wake and structure oscillator is represented by the following equations,

$$
\begin{aligned}
m\ddot{x} + r_s\dot{x} + hx &= \frac{1}{2}\rho_f C_L D|U_R|\dot{y} + \frac{1}{2}\rho_f C_D D|U_R|(U-\dot{x}) \\
m\ddot{y} + r_s\dot{y} + hy &= \frac{1}{2}\rho_f C_L D|U_R|(U-\dot{x}) + \frac{1}{2}\rho_f C_D D|U_R|\dot{y} \\
\ddot{w} + 2\varepsilon_x\Omega_F(w^2-1)\dot{w} + 4\Omega_F^2 w &= (A_x/D)\ddot{x} \\
\ddot{q} + \varepsilon_y\Omega_F(q^2-1)\dot{q} + \Omega_F^2 q &= (A_y/D)\ddot{y}
\end{aligned}
$$

where

$$|U_R| = \sqrt{(U-\dot{x})^2+\dot{y}^2}$$
$$C_L = \frac{C_{L0}q}{2}$$
$$C_D = C_{D_0}(1+Kq^2)+\frac{C_D^{fl}}{2}$$

with Where, the dimensions of the numerical model are $L = 1$m and $D = 0.1$ m. The aspect ratio of this system is therefore L/D = 0.4. In the structural oscillator, the structural mass in air is $m = 45.71$ kg; the structural damping coefficient is a given parameter estimated $\xi = 4.4$ and the stiffness constant is $k = 5.$ kN/m. The vortex shedding lift coefficient $C_{Lo} = 0.3$ is taken as in Facchinetti *et al.* (2004) and the reference drag is taken as $C_o = 0.7$, which was obtained from the experiments reported in Rosetti *et al.* (2011). The Strouhal number is equal to 0.2 and $C_{io} = 0.1$. The parameter $\gamma = 2.14$ and $A_x = 8$ and $A_y = 42$ correspond respectively to cross-flow and in-line amplification factors and ε_x and ε_y to the cross-flow and in-line damping respectively with K parameter that couples cross-flow and in-line motions. Being the numerical values the followings $A_x = 6$, $A_y = 12$, $\varepsilon_x = 0.3$, $\varepsilon_y = 0.15$ and $K = 0.05$ like in Rosetti *et al.* (2011).

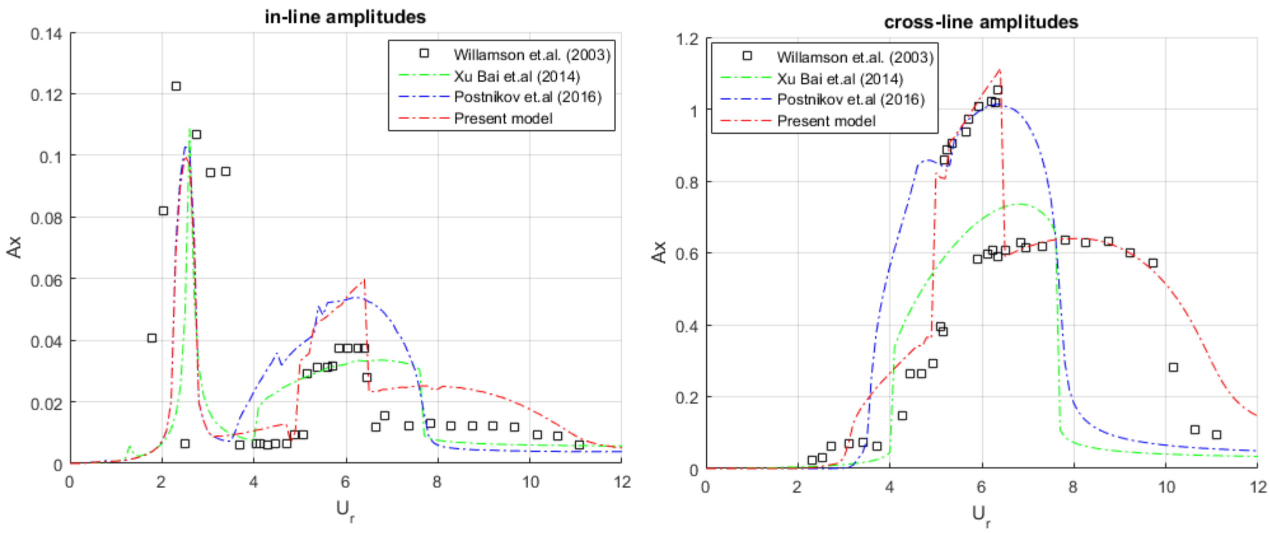

Figure 2: Comparison and experimental amplitudes for cross-flow and in-line versus reduced velocity. Experimental from: N.Javtis C.H.K, Williamson

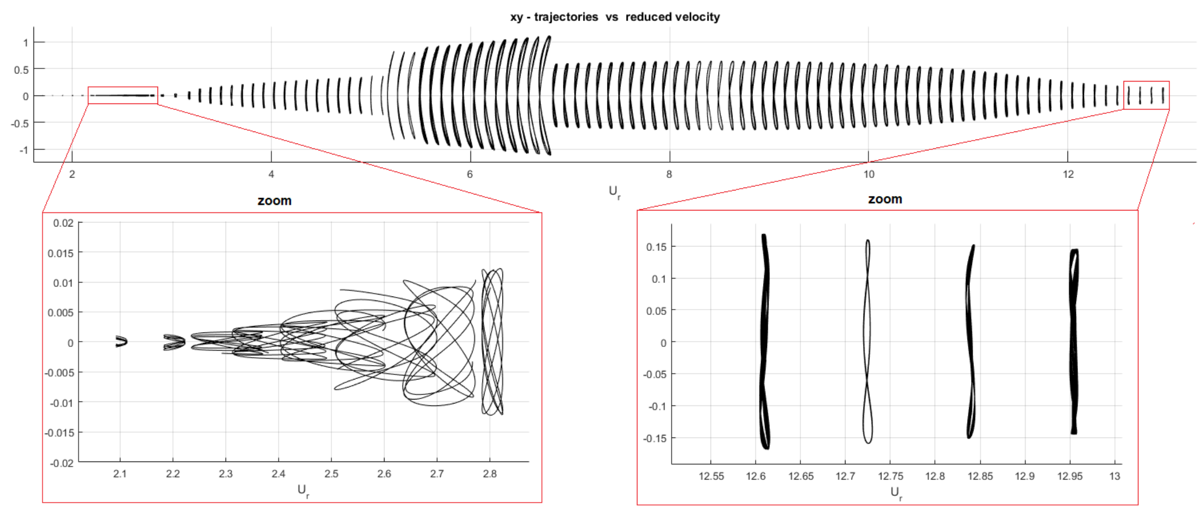

Figure 3: numerical solution for increasing reduced velocity

3. SENSITIVITY ANALYSIS

3.1 Adaptive sparse grid stochastic collocation method

The main idea of this method is approximate the multidimensional stochastic space building a interpolation function on a set of collocation points $\{\mathbf{Y}_i\}_{i=1}^{M}$ in the stochastic space $\Gamma \subset \mathbb{R}^M$. The method, similarly to Monte Carlo methods, requires only the solution of a set of decoupled equations, allowing the model to be treated as a black box and solved it with existing deterministic solvers. The multidimensional interpolation can be built through either full-tensor product of 1D interpolation rule or by the so called sparse grid interpolation based on the Smolyak algorithm. This algorithm provides a way to construct interpolations functions based on minimal number of points in multidimensional space, extending in a easy way the univariate interpolation to the multivariate case. Hence, considering a smooth function $f : [-1, 1]^N \rightarrow \mathbb{R}$, for the $1D$ case $(N = 1)$, f can be approximated by the following interpolation formula $\mathcal{U}^i(f)(y)$,

$$\mathcal{U}^i(f)(y) = \sum_{j=1}^{m_i} f(\mathbf{Y}_j^i)a_j^i, \tag{1}$$

in the set of support nodes,

$$X^i = \mathbf{Y}_j^i | \mathbf{Y}_j^i \in [0, 1] for j = 1, \ldots, m_i, \tag{2}$$

where, $i \in \mathbb{N}$, $a_i(\mathbf{Y}_j^i) \in C[0, 1]$ are the interpolation basis functions and m_i is the number of elements of the set X^i. This equation is a simple weighted sum of the value of the basis functions for all collocations points in the sparse grid, being an approximation to the solution of the equations of the system. From this equation, it is possible calculate easily the useful statistics of the solution for example, the mean of the random solution can be evaluated as follow:

$$\mathbb{E}[u(x)] = \sum_{|i| \leq q} \sum_{j \in B_i} w_j^i(x). \int_\Gamma a_j^i(\mathbf{Y}) d\mathbf{Y}, \tag{3}$$

where denoting $\int_\Gamma a_j^i(\mathbf{Y}) d\mathbf{Y} = I_j^i$ we can write,

$$\mathbb{E}[u(x)] = \sum_{|i| \leq q} \sum_{j \in B_i} w_j^i(x).I_j^i, \tag{4}$$

the mean is an arithmetic sum of the product of the hierarchical surpluses and the integral weight at each interpolation point. To obtain the variance of the random solution we can be calculate first,

$$u^2(x, \mathbf{Y}) = \sum_{|i| \leq q} \sum_{j \in B_i} v_j^i(x) a_j^i(\mathbf{Y}), \tag{5}$$

and then,

$$\mathbb{V}ar[u(x)] = \mathbb{E}[u^2(x)] - (\mathbb{E}[u(x)])^2 = \sum_{|i| \leq q} \sum_{j \in B_i} v_j^i(x).I_j^i - (\sum_{|i| \leq q} \sum_{j \in B_i} w_j^i(x) I_j^i)^2. \tag{6}$$

The method allows us to obtain an approximation of the solution dependent random variables and also easily extract the mean and variance analytically as well its probability density function (PDF) by simple sampling of this function, leaving only the interpolation error Xiu and Hesthaven (2005). SÃş, when the the smoothness condition in the stochastic space is not fulfilled it is possible to use adaptive strategies to improve de interpolation function in the stochastic space. The basic idea here is to use hierarchical surpluses $w_j^i(x)$ as an error indicator to detect the smoothness of the solution and refine the grid around the discontinuity region and less points in the region of smooth variation. This method proposed in Ma and Zabaras (2009), automatically detect the discontinuity region in the stochastic space and refine the collocation points in this region. Therefore, by adding the neighbor points, we add support nodes from the next interpolation level, so the magnitude of the hierarchical surplus satisfies $|w_j^i \geq \varepsilon|$. If this criterion is satisfied, one only add the $2N$ neighbor points of the current point to the sparse grid. It is noted that the definition of level of the Smolyak interpolation for the ASGC method is the same as that of the conventional sparse grid even if not all point are included. A more detailed explanation of the method and algorithm can be found in, Ma and Zabaras (2009).

Following we present the results involving fluid-structure interaction model. The choice of the analyzed situations was guided by the challenge that potentially can bring to the SGC even as the adaptive form ASGC, as robust tools applied to Uncertainty Quantification. The adaptive sparse grid interpolation and integration schemes of this section were generated using functions in the TASMANIAN toolkit Toyanov (2016).

The dependency of the empirical coupling parameter Ax and A_y on the frequency ratio δ, including a jump, might entail abrupt changes on the structure response. Whenever the shedding frequency approaches the natural frequency of the structure, the coupled system enters in the lock-in regime. In this case, the amplitude of the structural response during achieves a maximum and, thus, this is considered a critical mode of vibration. A schema of the lock-in is depicted in Figs. 2, where the maximum amplitude in the steady state regime is plotted as a function of the reduced velocity U_r in the range $[0, 12]$. To avoid problems, computational models are built aiming at identifying the narrow band of the reduced velocity corresponding to lock-in. Hence, in this example we investigate the impact of uncertainties on the predictions of those analyses. Thus. assuming the following probabilistic models for the parameters:

$$A_x = \overline{A_x}(1 + \delta_1 \xi_1) \tag{7}$$

$$A_y = \overline{A_y}(1 + \delta_2 \xi_2) \tag{8}$$

where $\overline{A_x}$, $\overline{A_y}$ are overbars are the expected values of the empirical coupling parameters. with $\delta_i = 0.1$ variation over de mean and ξ_i independent and identically distributed uniform random variables, i.e:$A_x = U[7.8, 8.2]$ and $A_y = U[41.2, 42.8]$. $U[., .]$ represents the uniform distribution interval and where overbars denote the mean (expected) values, δ_i the percentile variation over de mean and ξ_i independent and identically distributed uniform random variables taking values into $[-1, 1]$.

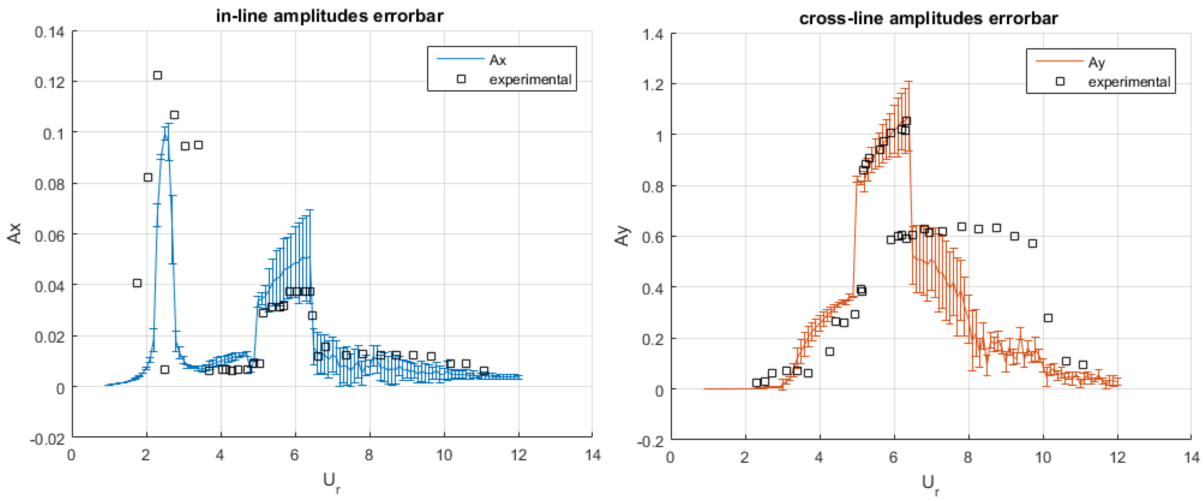

Figure 4: cross flow and in-line prediction bars for amplitude in steady state

4. PARAMETER ESTIMATION

The Bayesian inference has become recently a valuable tool for estimation of parametric and structural uncertainties of physical systems constrained by differential equations. Sampling techniques, such as Markov chain Monte Carlo (MCMC), have frequently employed in Bayesian inference [19,33,50]. However, MCMC methods [28,58,59] are, in general, computationally expensive, because a large number of forward model simulations is needed to estimate the PPDF and sample from it. In this sense, the use of phenomenological models of complex systems can help solve MCMC simulations for applications that would require the solution of models prohibitively large. Thus, a strategy to improve the efficiency of MCMC simulations is using substitution models, such as the one presented in this paper, which has already been developed used in a wide variety of problems, see [49] and the reference in it. The practice of substitution modeling seeks to approximate the response of an original function (outputs of the model or PPDF in this work), which is usually computationally expensive, by a cheaper substitute to execute. The PPDF can then be evaluated by sampling the substitute directly without direct model runs. Compared to conventional MCMC algorithms, this approach is advantageous which significantly reduces the number of runs of the advanced model with a desired accuracy. So, for applications where modeling and measurement error ε_i we employ the statistical model,

$$\gamma_i = f(Q) + \varepsilon_i, \quad i = 1, ..., n,$$

where $f(Q)$ denotes the parameter-dependent model response and γ_i, ε_i, and Q are random variables representing measurements error, and parameters.

The likelihood function $\pi(\upsilon|q)$ incorporates information provided by the samples and constitutes the mechanism through which data informs the posterior density as detailed in Ralph (2014) and Tran *et al.* (2016) The Bayesian frame-

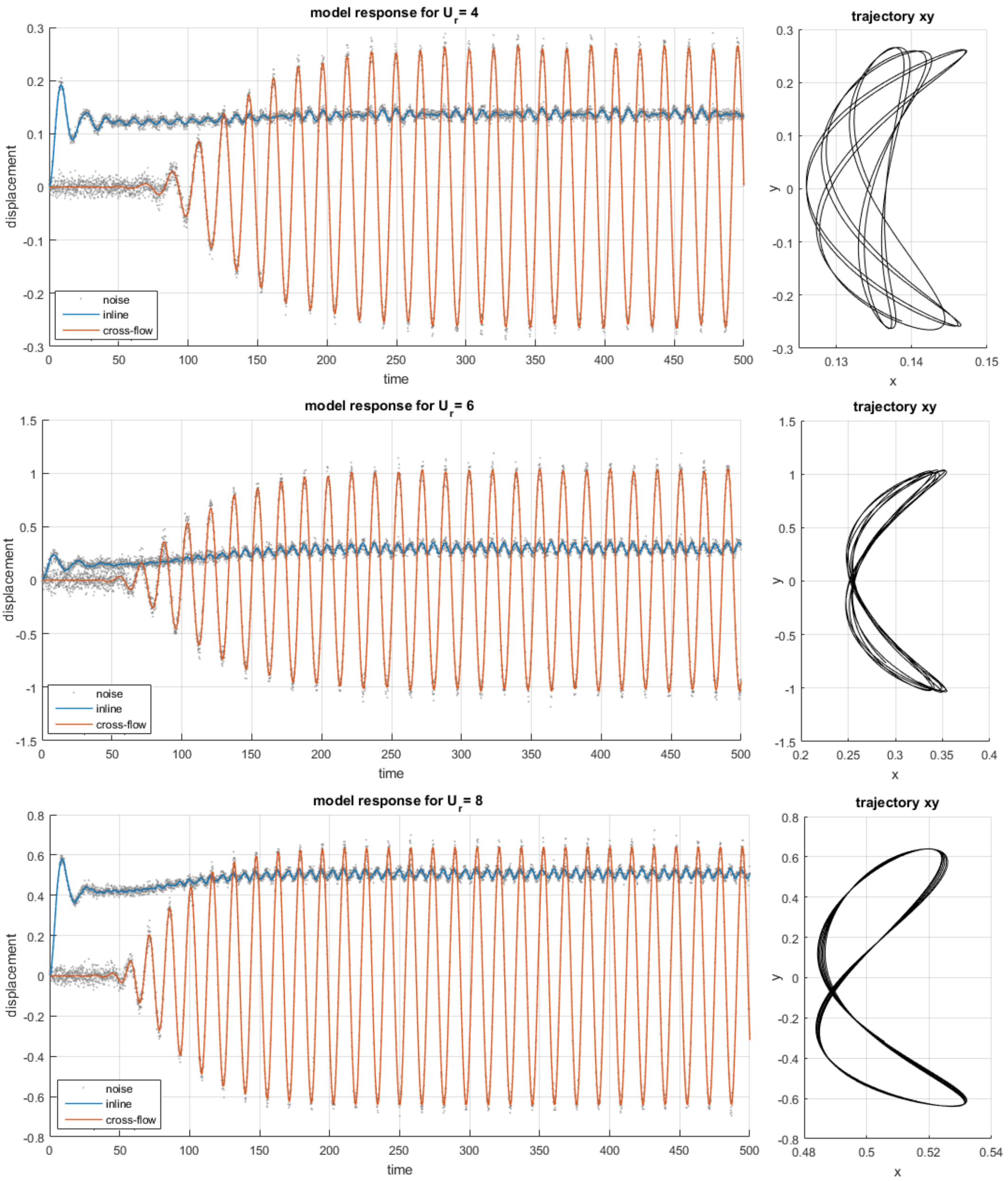

Figure 5: Left:Inline and crossflow model response, syntetic data. Rigth: trajectories for reduced velocity U_r (4,6 and 8)

work can be stated as follows: given measurements υ_{obs}, we find the posterior density $\pi(q|\upsilon)$. The likeihood function for this analises will be assumed with $\varepsilon_i \sim N(0, \sigma^2)$ where σ^2) is fixed.

$$\pi(\upsilon|q) = \frac{1}{(2\pi\sigma^2)^2} e^{-SS_q}/2\sigma^2$$

where

$$SS_q = \sum_{i=1}^{n} (\varepsilon_i - f_i(q))^2$$

the sum of squares error. Through bayesian inference techniques, given measurements υ_{obs} we can find the posterior density $\pi(q/\upsilon_{obs})$, where $\pi(q/\upsilon_{obs})$ provides the complete distribution of parameters Q based on the observations υ that can be estimated. For our model we assume ths coupling parameters $Ax = 8$ and $Ay = 42$ as unknown parameter to be estimated, for this we generate synthetic data with $\sigma = 0.01$ for the in-line and cross flow output data as shown in Figure 5.

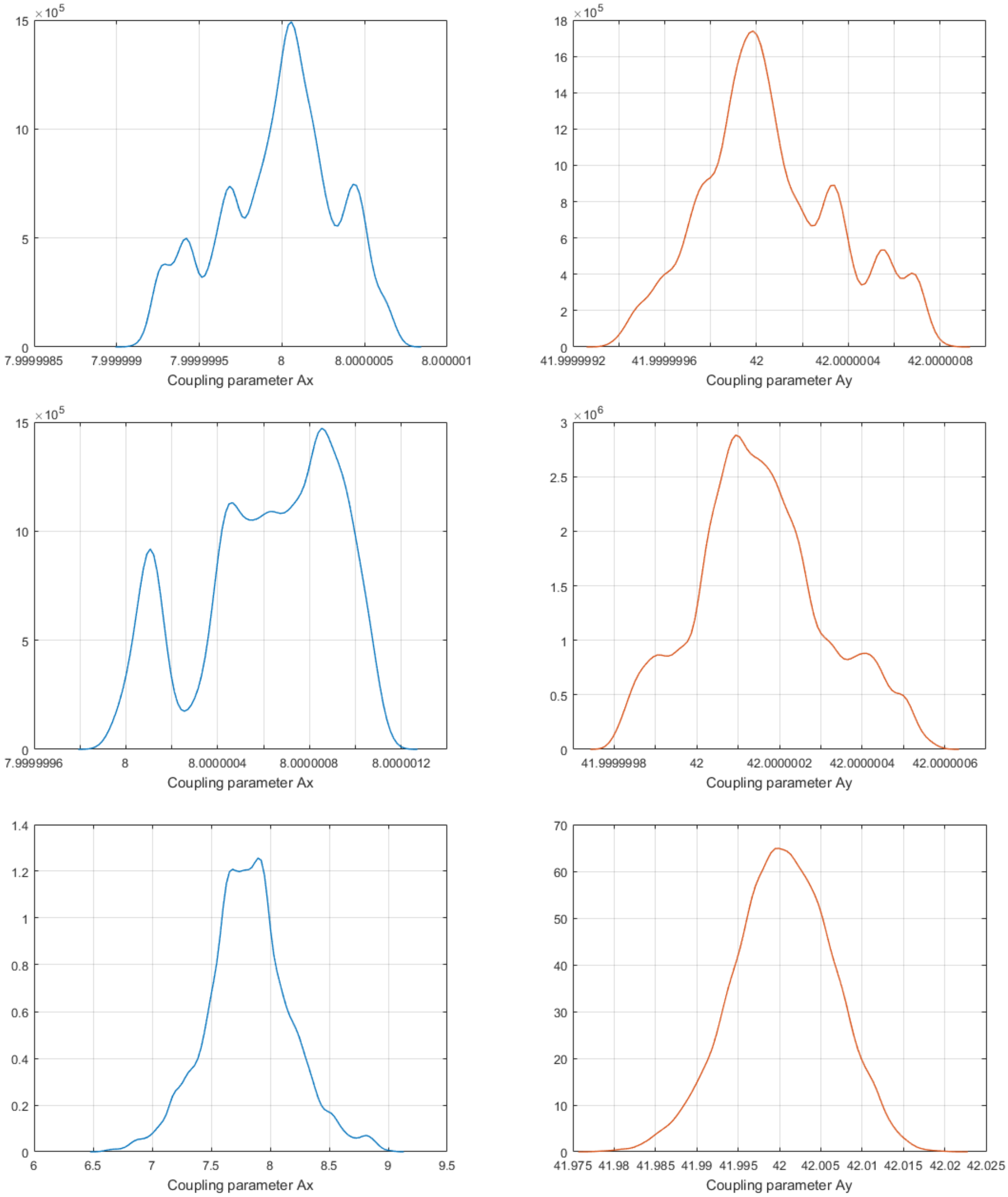

Figure 6: Left:Densities estimated for Ax coupling parameter. Rigth:Densities estimated for Ay coupling parameter, for reduced velocity U_r (4,6 and 8)

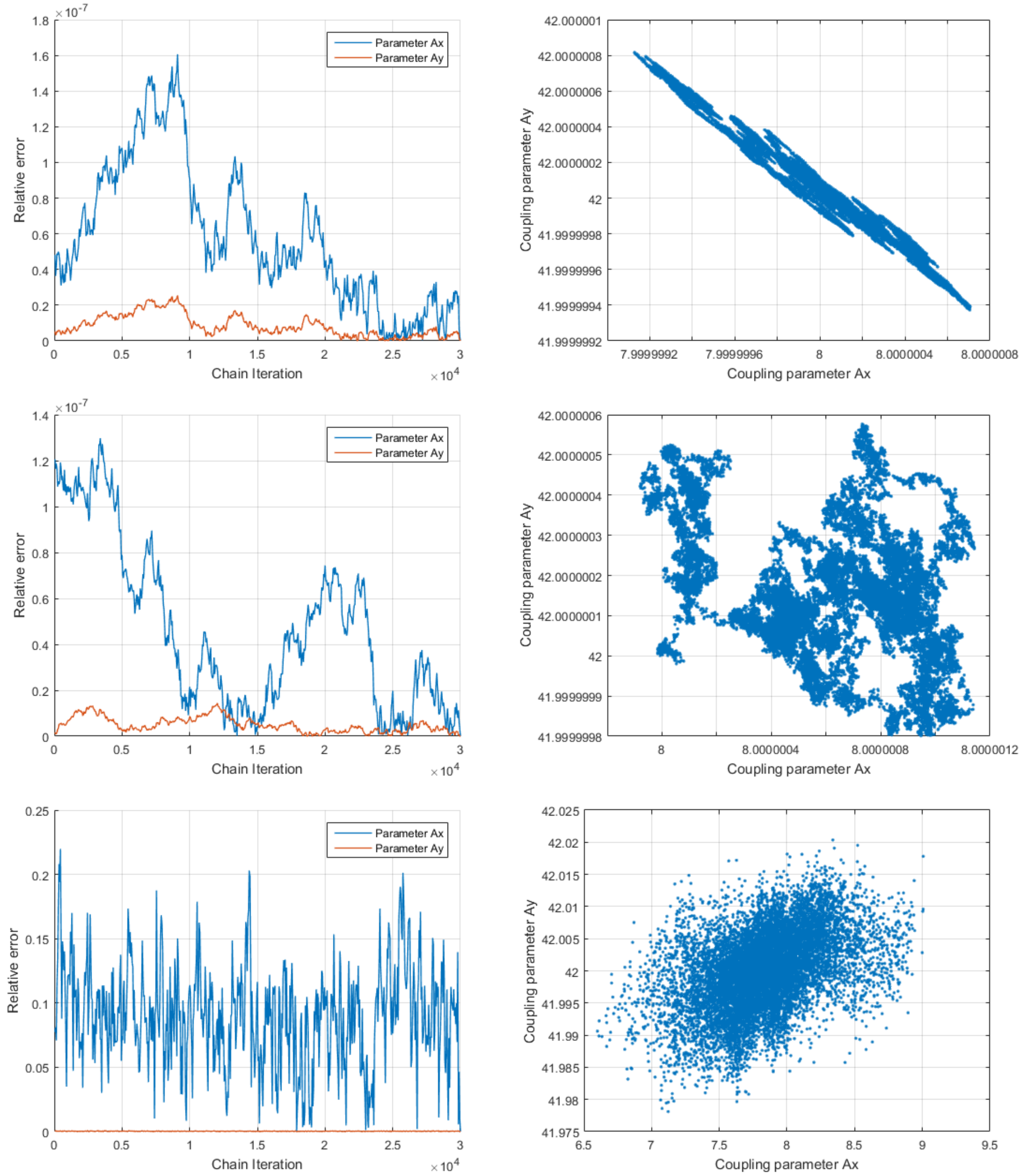

Figure 7: Left:relative error of path Ax ans Ay coupling parameters. Rigth:Correlation of parameters,for reduced velocity U_r (4,6 and 8)

5. CONCLUSIONS

In this work, we have explored the capabilities sensitivity analyses and calibrations techniques using a subrrogate model that deals with the vibrational response of submerged structures excited by vortex detachments in the surrounding flow. We adopted a popular engineering model to predict this dynamics, with special attention to the lock-in phenomena. Two of the parameters used by this modeling were considered uncertain, inasmuch as they often are obtained by means of model calibration based on field observations or physical experiments. An Adaptive Sparse Grid Collocation Method was used to estimate the statistical moments. This non-intrusive method, allow convert any deterministic code into a code that solves the corresponding stochastic problem. Compared with the Monte Carlo Simulation method, the ASGC method presents a significative reduction in the number of experiments required to achieve the same level of accuracy. The results obtained, show that it is possible refine the grid locally identifying automatically non smooth regions in the stochastic space achieving the same accuracy and reducing significatively the cost by the use of less collocations points in smooth regions of the stochastic space. Due to that the majority of engineering problems varying rapidly in only some dimensions, remaining much smoother in other dimensions and in general it have more stochastic dimensions.After this analyses we try calibrate two coupling parameter of parameters in the surrogate model using Bayesian technique. The results obtained are preliminary and need to be analyzed deeper. Future work will include analysis of sensitivity for others parameters as well the use of experimental data to validate and calibrate the models.

6. ACKNOWLEDGEMENTS

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