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RUBBER BUSHING HYPERELASTIC BEHAVIOR BASED ON SHORE HARDNESS AND UNIAXIAL EXTENSION

Debora Francisco Lalo
Marcelo Greco
Federal University of Minas Gerais
6627 Antônio Carlos Ave, building 1, Department of Structural Engineering - 4th floor, Belo Horizonte, MG 31270-901, Brazil
debora@dees.ufmg.br
mgreco@dees.ufmg.br

Abstract. *The elastomeric bushings are essential in the engineering field once they isolate vibration, reduce noise, accommodate oscillatory motions and allow axes misalignments. As the bushings are rubber-like materials, their constitutive relations are highly nonlinear and characterized by strain energy functions that need to be identified. Most of these models are referred as hyperelastic material models. In the present paper, an elastomeric bushing manufactured by Vibtech company is evaluated, and its stiffness response in a certain working load condition is determined through an uniaxial tensile test in which the parameters of Marlow model are defined by fitting the experimental tensile stress–strain data with the Finite Element Analysis. It is investigated the correlation between the identified results obtained from the prototype test and the virtual model obtained from uniaxial tensile experimental data. The final results are also compared with the Mooney-Rivlin constitutive model response based on the parameters extract from literature that can be ranged according to the rubber shore hardness.*

Keywords: *Rubber-like materials, Hyperelastic constitutive models, Uniaxial tensile test, Finite Element Analysis*

1. INTRODUCTION

Rubber-like materials are widely used in industry to manufacture tires, seals, belts, engine mounts and many other devices. To enhance the design of these products for a given working load condition, in general, numerical methods are used in addition with accurate constitutive laws.

Elastomers, which is a more descriptive name for rubbers, present a very complex mechanical behavior that exceed the linear elastic theory, exhibiting large deformations and viscoelastic properties such as stress softening (Naser *et al.*, 2005). For this reason, in the Finite Element Analysis (FEA) of rubbers some experiments should be performed in order to assess the material constitutive law (Drozdov, 2007).

To define the hyperelastic material behavior, a stress-strain response is required to determine the material parameters in the strain energy potential. The Mooney-Rivlin model is very popular in the modeling of large strain nonlinear behavior of incompressible materials, *i.e.* rubber and it can be viewed as a particular case of the polynomial model (Beda, 2013). Its correlation with Shore hardness can be found in the work of Lindley (1974) and Gobel (1974).

Once the stress-strain behavior of the engineering bushing was tested only based on the uniaxial extension, which is easily accessible, it was used the Marlow form, because in this case a strain energy potential is constructed so as to reproduce the test data exactly, with a reasonable behavior in other deformation modes (Simulia, 2013).

Rubber bushings can be manufactured from natural rubber, synthetic rubber, and even thermoplastics. The hardness of these materials is between 30 and 98 IHRD (International Rubber Hardness) measure equivalent to Shore hardness. According to Gobel (1974) the natural rubber is the one which has the best elastic properties.

2. HYPERELASTIC CONSTITUTIVE MODELS

Hyperelastic material models are used to define the mechanical behavior of elastomers, foams and many biological tissues. Once elastomers are materials that exhibit nearly incompressible behavior and often experience high strains in service, its strain states are usually complex. They are a mixture of tension, compression and shear with a very small amount of volume change.

For incompressible elastomers, the Poisson's ratio is nearly 0.5 and the main strain states are simple tension, pure shear and simple compression, which is generally represented by the biaxial extension (Miller, 2004).

Several mathematical constitutive theories of hyperelastic large-strain response in quasi-static conditions and without irreversible strain phenomena have been extensively researched since the works of Mooney (1940) and Rivlin (1948). The authors' approach is based on the strain invariants and in the concept of the material to be isotropic and incompressible. Additional documentation on this research topic including other theories and conditions can also be found in Valanis and Landel (1967), Treloar (1975), Ogden (1972; 1984), Gent (1992), Arruda and Boyce (1993), Drozdov and Dorfmann (2003) and Drozdov (2007).

The hyperelastic constitutive models are able to describe the behavior of nearly incompressible materials that exhibit instantaneous elastic response up to large strains. They are expressed in terms of a strain energy density function (W) which defines the strain energy stored in the material per unit of reference volume. This function depends on the principal stretches or invariants of the strain tensor and is directly linked to the material's stress-strain relationship, which depends on a series of parameters (material constants). In order to determine these constants, it is required the nominal stress versus nominal strain data obtained from experimental tests to fit most models theoretical behavior available (Bortoli *et al.*, 2011).

For an isotropic and incompressible material, W can be expressed as a function of the strain tensor invariants or principal stretches:

$$W = W(\overline{I_1}, \overline{I_2}, \overline{I_3}) = W(\lambda_1, \lambda_2, \lambda_3) \tag{1}$$

where, the three invariants $(\overline{I_1}, \overline{I_2}, \overline{I_3})$ of the Green strain tensor are given in terms of the principle extension ratios λ_1, λ_2 and λ_3 by:

$$\overline{I_1} = {\lambda_1}^2 + {\lambda_2}^2 + {\lambda_3}^2 \tag{2}$$

$$\overline{I_2} = {\lambda_1}^2.{\lambda_2}^2 + {\lambda_2}^2.{\lambda_3}^2 + {\lambda_3}^2.{\lambda_1}^2 \tag{3}$$

$$\overline{I_3} = {\lambda_1}^2.{\lambda_2}^2.{\lambda_3}^2 \tag{4}$$

2.1 Polynomial model

The polynomial strain energy function of a hyperelastic material can be expanded as an infinite series of the first and the second deviatoric principal invariants $\overline{I_1}$ and $\overline{I_2}$. The polynomial form of strain energy function is a phenomenological model and it is given as follows (Shahzad *et al.*, 2015):

$$W = \sum_{i+j=1}^{N} C_{ij}.(\overline{I_1} - 3)^i.(\overline{I_2} - 3)^j + \sum_{k=1}^{N} \frac{1}{D_k}(J - 1)^{2k} \tag{5}$$

where C_{ij} are the material constants for a "N" number of terms, J is the total volume ratio and D_k relates the material incompressibility parameters.

A higher number of N may provide a better fit to the exact solution. However, on the other hand, it may cause numerical difficulty in fitting the material constants and requires enough data to cover the entire range of the aimed deformation. Therefore a very higher N value is not usually recommended.

The initial shear modulus "μ_0" and bulk modulus "K_0" are given by:

$$\mu_0 = 2.(C_{10} + C_{01})\ ; \qquad K_0 = 2/D_1 \tag{6}$$

For cases where the nominal strains are small or only moderately large (< 100%), the first terms in the polynomial series usually provide a sufficiently accurate model. Some particular material models (Mooney-Rivlin, neo-Hookean, and Yeoh forms) are obtained for special choices of C_{ij} (Simulia, 2016).

2.2 Mooney-rivlin model

The Mooney-Rivlin models are very popular to model large strain nonlinear behavior of incompressible materials. It is a phenomenological model and works well for moderately large stains in uniaxial elongation and shear deformation (Shahzad *et al.*, 2015).

It was proposed by Rivlin and Saunders (1951) and can also be viewed as a particular case of the polynomial form. The original first order Mooney-Rivlin model is equivalent to the polynomial form with $N = 1$ and it is represented by the following equation:

$$W = C_{10}.(\overline{I_1} - 3) + C_{01}.(\overline{I_2} - 3) + \frac{1}{D_1}(J - 1)^2 \quad (7)$$

Extending Eq. (7) to the second order, the five-term Mooney-Rivlin model is similar to the polynomial form when $N = 2$:

$$W = C_{10}.(\overline{I_1} - 3) + C_{01}.(\overline{I_2} - 3) + C_{20}.(\overline{I_1} - 3)^2 + C_{11}.(\overline{I_1} - 3).(\overline{I_2} - 3) + C_{02}.(\overline{I_2} - 3)^2 + \frac{1}{D_1}(J - 1)^2 \quad (8)$$

The parameters C_{ij} are generally determined from experimental data and they can be related to μ_0 in the same way as in the polynomial form.

More terms can be added to Eq. (8) to obtain a N^{th} order polynomial form, but usually they do not produce appreciable improvements (Sasso et al., 2008).

2.3 Marlow model

The Marlow form is a general first-invariant constitutive model proposed by Marlow (2003). It assumes that the strain energy potential is independent of the second deviatoric invariant $\overline{I_2}$. This model is only supported by Abaqus® commercial software and can be defined by providing test data which defines the deviatoric behavior by W_{dev}, and optionally, the volumetric behavior by W_{vol} if compressibility must be taken into account (Simulia, 2016), as it follows by Eq. (9).

$$W = W_{dev}.(\overline{I_1}) + W_{vol}(J_{el}) \quad (9)$$

where J_{el} is the elastic volume ratio obtained by an isotropic thermal expansion:

$$J_{el} = \frac{J}{(1 + \varepsilon^{th})^3} \quad (10)$$

where ε^{th} is the linear thermal expansion strain that is obtained from the temperature and the isotropic thermal expansion coefficient.

The interpolation and extrapolation of stress-strain data with the Marlow model is approximately linear for small and large strains. For intermediate strains in the range of 0.1 to 1.0 a considerable degree of nonlinearity may be observed. To minimize undesirable nonlinearity, the data points needs to be specified in the intermediate strain range (Simulia, 2016).

The model allows an exact matching of experimental data in such defined mode, which can be uniaxial, biaxial or planar. Therefore, once uniaxial stress-strain data for a material have been acquired, the response in the other strain modes is reasonable. If the volumetric behavior is a concern, it can be defined by specifying the lateral strains or providing the volumetric test data, in the same way that the effective Poisson's ratio can be imputed (Marlow, 2003).

2.4 Other classical constitutive models

Other classical constitutive models have been tested and compared with the previous ones. Their final equations are given below:

- Neo-Hooke model (Treloar, 1946)

$$W = C_{10}.(\overline{I_1} - 3) + \frac{1}{D_1}(J - 1)^2 \quad (11)$$

The neo-Hookean form can be thought of as a subset of the polynomial form for $N = 1$ and $C_{01} = 0$. Thus, μ_0 and K_0 are given by:

$$\mu_0 = 2.C_{10}\,; \qquad K_0 = 2/D_1 \quad (12)$$

- Ogden model (Ogden, 1972)

$$W = \sum_{i=1}^{N} \frac{\mu_i}{\alpha_i}.(\bar{\lambda}_1^{\alpha_i} + \bar{\lambda}_2^{\alpha_i} + \bar{\lambda}_3^{\alpha_i} - 3) + \sum_{k=1}^{N} \frac{1}{D_k}(J - 1)^{2k} \quad (13)$$

where μ_i and α_i are real material parameters, being positive or negative and satisfying the condition for μ_0 and K_0 given by:

$$\mu_0 = \frac{1}{2}\sum_{i=1}^{N} \mu_i . \alpha_i \ ; \qquad K_0 = 2/D_1 \tag{14}$$

- Yeoh model (Yeoh, 1993)

$$W = \sum_{i=1}^{N} C_{i0} . (\overline{I_1} - 3)^i + \sum_{k=1}^{N} \frac{1}{D_k} (J - 1)^{2k} \tag{15}$$

The Yeoh model is commonly considered with $N = 3$. Thus, μ_0 and K_0 are given by:

$$\mu_0 = 2. (C_{10}) \ ; \qquad K_0 = 2/D_1 \tag{16}$$

- Arruda Boyce model (Arruda and Boyce, 1993)

$$W = \mu \sum_{i=1}^{5} \frac{C_i}{\lambda_L^{2i-2}} . (\overline{I}_1^i - 3^i) + \frac{1}{D}\left(\frac{J^2 - 1}{2} - \ln J\right) \tag{17}$$

where λ_L is the limiting network stretch, and the constants C_i are defined as:

$$C_1 = \frac{1}{2}; \qquad C_2 = \frac{1}{20}; \qquad C_3 = \frac{11}{1050}; \qquad C_4 = \frac{19}{7000}; \qquad C_5 = \frac{519}{673750} \tag{18}$$

- Gent model (Gent, 1996)

$$W = -\frac{E I_m}{6} . \ln\left(1 - \frac{\overline{I_1} - 3}{I_m}\right) + \frac{1}{D}\left(\frac{J^2 - 1}{2} - \ln J\right) \tag{19}$$

where the constant "E" is the initial elastic modulus, which for incompressible materials, is $3\mu_0$, and I_m is the limiting value of $(\overline{I_1} - 3)$, analogous to λ_L for Arruda-Boyce.

3. UNIAXIAL STRETCHING TESTS

Simple tension experiments are widely used for elastomers. It corresponds to an elongation in one direction accompanied by free contraction in the other two Cartesian directions. To achieve a pure strain state, the specimen must be much longer in the stretching direction than in the other dimensions (Miller, 2004).

According to ASTM D412 (2013) measurements for tensile stress, tensile strength and ultimate elongation should be made on specimens that have not been pre-stressed, and rubbers should be compared only when tested under the same conditions. The periods of extension and recovery must be controlled in order to avoid residual deformation and obtain comparable results.

The experimental uniaxial tests have been performed at Vibtech based on the standard ASTM D412 (2013). The rubber hardness, which makes up the bushing component, is 55 SH. The specimens analyzed were submitted to three mechanical traction tests in an electromechanical machine Kratos K-500S MP, under a 500 mm/min deformation rate up to a load of 500 kgf.

The samples adopted in the experimental tests have been cut from a 2mm thick flat sheet, with a width of 6mm on the reduced section where the distance between each clamp is 25.4mm. The standard tensile testing machine with the dumbbell specimen connect by the grips is illustrated as in Fig. 1.

Figure 1: Rubber tensile strength test performed at Kratos K-500S MP

In this experimental test the length was comprised between the instrument clamps. Once the pure tension strain state occurs away from the clamps, the specimen straining cannot be measured close to them. The uniaxial stress-strain curve obtained from the test data is depicted in Fig. 2.

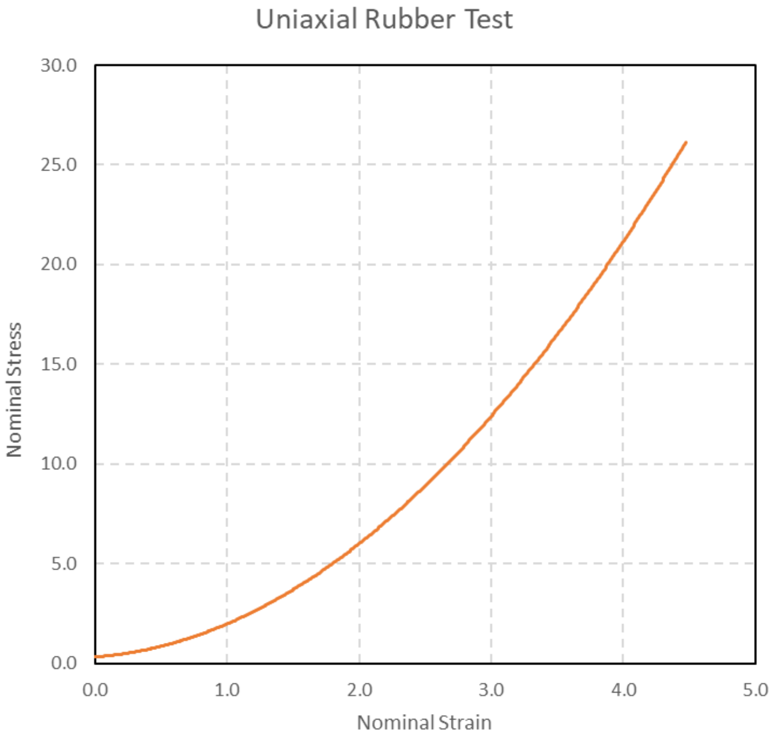

Figure 2: Uniaxial stress-strain curve for material data fitting obtained by test data performed at Vibtech

4. COMPUTATIONAL PROCEDURE AND RESULTS

The finite element method is a popular tool for designing elastomeric components. This numerical method is able to approximate the stress-strain behavior of rubber components based on a theoretical material model.

The component geometry was imported from the Computer Aided Design (CAD) system. Due to the rotational symmetry of the model it was considered the axisymmetric modeling. Axisymmetric elements convert a 3-D problem into a 2-D problem, making it a smaller model with faster execution and easier post-processing.

The FEA was performed using the commercial software Abaqus®. The metal parts were modeled by using the 3-node linear triangle axisymmetric elements (CAX3) and the rubber pad by using the 4-node bilinear quadrilateral axisymmetric elements with the hybrid formulation and constant pressure (CAX4H). The basis of the hybrid elements is that the purely hydrostatic stress component can be treated as an independent variable; otherwise very small change in displacement generates large changes in hydrostatic stress.

The implementation assume that the elements are located exclusively in x, y plane, where the x-axis relates to the radial direction, and the 2-D model will be rotated around the y-axis always considering the $x = 0$ position. The final model with its refined mesh and boundary conditions is shown in Fig. 3. A surface-to-surface contact algorithm was used to enforce the contact between the elastomer and metal parts. The normal behavior was modeled as hard-contact by the direct method, while linear friction behavior was assumed for the tangential behavior, using the pure penalty method.

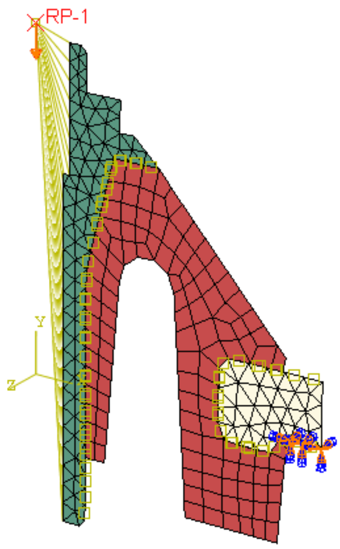

Figure 3: Rubber spring finite element model (axial section)

The relationship between the Shore-Hardness of rubber and the related first order Mooney-Rivlin with two coefficients is listed according to Tab. 1. These values do not cover all types of rubber, they are just a more or less matching based on uniaxial tension test for some rubbers with Poisson' ratio very close to 0.5, that is, nearly incompressible. For this reason, it can be used as a reference (Altidis and Warner, 2005).

Table 1: Mooney-Rivlin model with two coefficients. Adapted from Altidis and Warner (2005).

Shore-A	Young's Modulus (E)	Shear Modulus (G)	C_{10}	C_{01}
[°]	[N/mm^2]	[N/mm^2]	[N/mm^2]	[N/mm^2]
55	3.207	0.956	0.382	0.096
58	3.811	1.089	0.436	0.109
60	4.268	1.185	0.474	0.118
65	5.616	1.465	0.586	0.147
70	7.289	1.839	0.736	0.184

The correlation between uniaxial test data and theoretical curve fitted for Mooney-Rivlin and Marlow models is shown in Fig. 4. Since the Marlow model best fit the curve, it was used in the component simulation through FEA, then the material behavior could be defined based on experimental data. The biaxial and planar shear behaviors could also be estimated, and they are described according to Fig. 5.

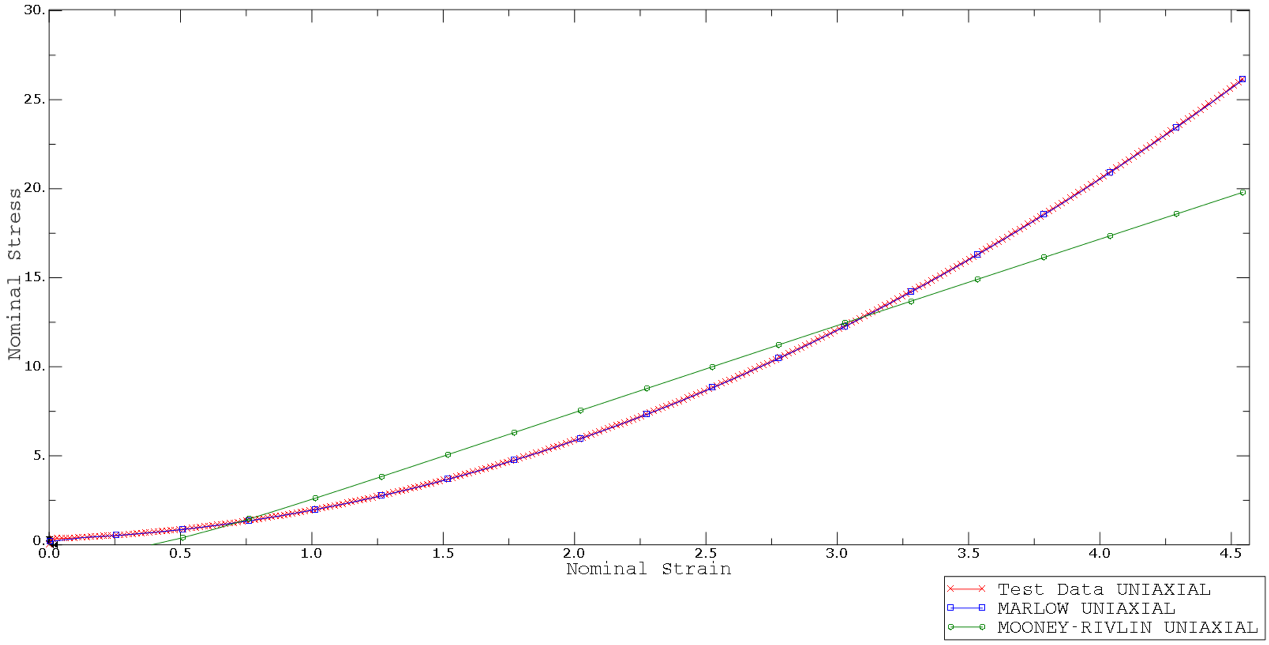

Figure 4: Material curve fitting based on uniaxial experimental data

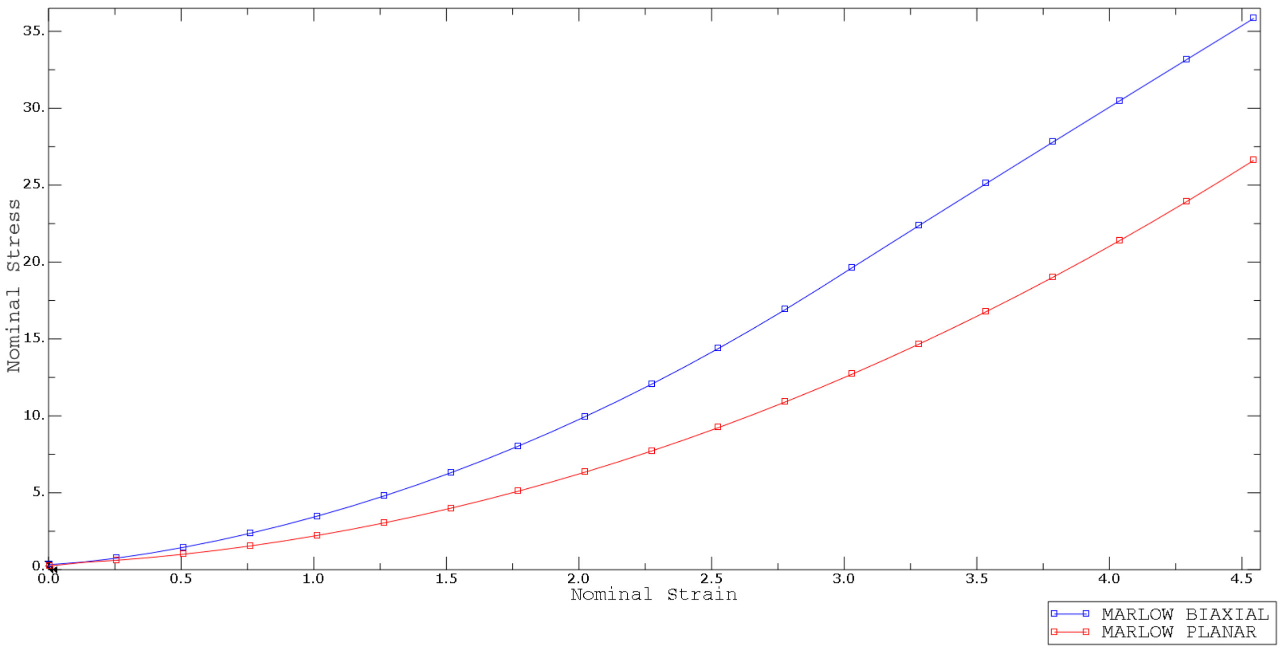

Figure 5: Estimated behavior for biaxial and planar shear deformation modes – Marlow model

The final results showed that the rubber section undergoes large strains but it still keeps the reasonable shape. According to Fig. 6 it is possible to compare the undeformed (a) and deformed shape (b). The maximum displacement value occurs around the metal axle, more exactly in the point where the concentrated load is applied.

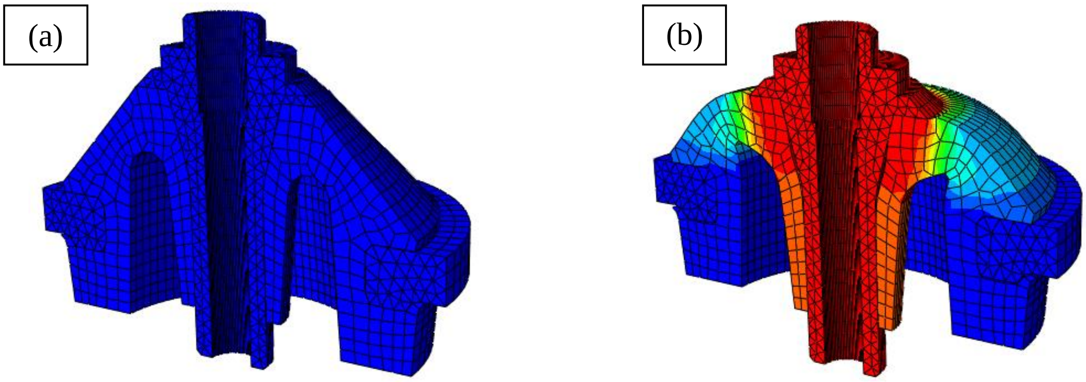

Figure 6: Deformation under axial load (a) undeformed shape; (b) deformed shape.

Figures 7 and 8 show the bushing stiffness comparison with the prototype experimental data and finite element models obtained according to Mooney-Rivlin constants for different shore hardness and Marlow model which was constructed by uniaxial strength test.

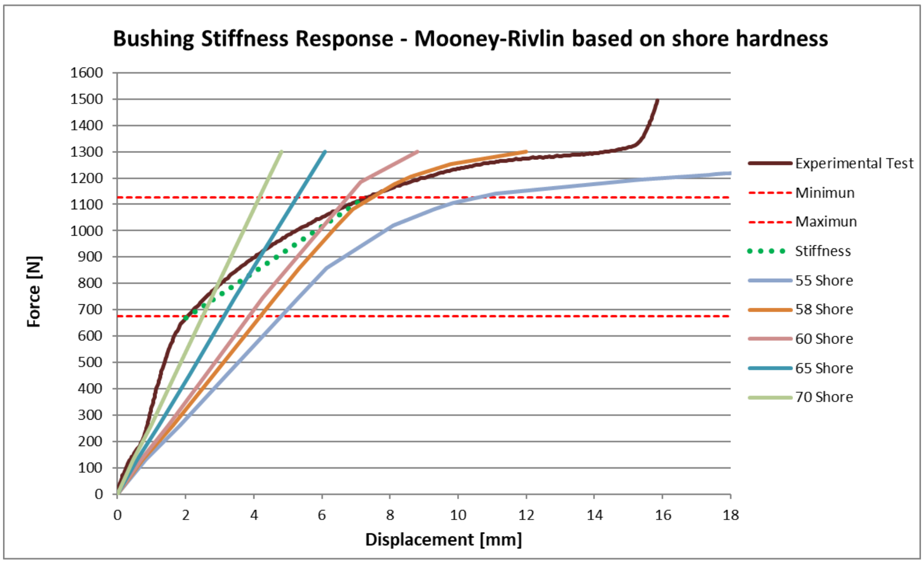

Figure 7: Bushing behavior comparison with experimental data and Mooney-Rivlin models based on shore hardness.

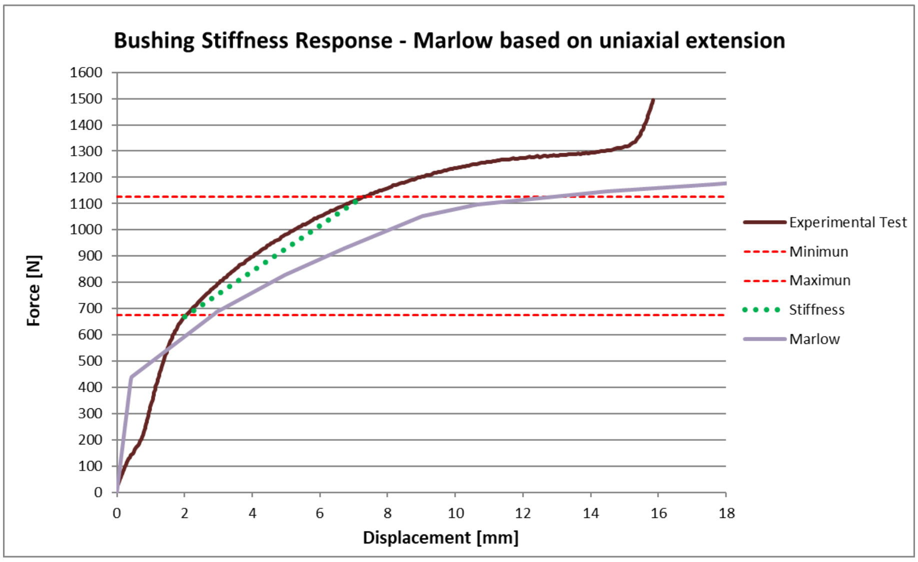

Figure 8: Bushing behavior comparison with experimental data and Marlow model based on uniaxial extension.

The prediction of the axial stiffness for most industrial applications is a key design parameter in the mechanical behavior calibration of the component. In this case, the axial stiffness is calculated in a determined range in which the bushing is more requested during the working-load conditions. Taking into account the minimum and the maximum ranges established for the stiffness calculation, it is possible to verify and compare the results obtained for each model through Tab. 2.

The studies proposed in the present paper are good for estimated approximations for performance and are generally adequate for design purposes. If more complex geometry effects must be considered, the basic equations relating only forces and deflections must be modified.

Table 2 – Bushing stiffness response for experimental data and finite element models.

Model	Axial Static Stiffness
Experimental Test	86.75
MR – 55 SH	80.56
MR – 58 SH	145.52
MR – 60 SH	150.37
MR – 65 SH	214.82
MR – 70 SH	265.36
Marlow	45.57

5. CONCLUSIONS

Through the results presentation it is possible to verify that the model which presented the best approximation in terms of experimental stiffness corresponds to Mooney Rivlin with a shore hardness of 55, which refers exactly to the rubber hardness used in the evaluated bushing. On the other hand, despite the curves show almost parallel inclinations, they do not overlap, what differentiates the behavior response of each one of them. In this case, the Marlow model obtained by the uniaxial test presented values closer to the experimental prototype curve, but with a certain difference in the slope, which distances its response with respect to stiffness. It is noted that as the hardness increases the stiffness response for the Mooney Rivlin model also increases. Once the evaluated component has different deformation modes other than uniaxial, more experimental tests considering more states of strain should be executed, such as planar shear and equibiaxial stretching, thus a more accurate calibration can be obtained.

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