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Attitude Determination For A Brazilian CubeSat Mission Using The Kalman Filter

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Abstract. *This work addresses the design and simulation of an Extended Kalman Filter for attitude determination in a CubeSat using the MATLAB/Simulink platform. The CubeSat is the concept proposed for the SERPENS II mission of the Brazilian Space Agency (AEB). The attitude instrumentation is: 3 solar sensors, 3 axes magnetometer and 3 axes gyroscope. The vector measurements of the magnetometer and solar sensor are combined with the method TRIAD to give one attitude measurement. The gyroscope propagates the attitude, via the integration of quaternion kinematics. These two attitude results are, then, fused via the Kalman filter. The attitude is referenced to the local vertical local horizontal (LVLH) reference frame. Simulations indicate that the approach can give reasonable results for a given amount of simulated noise.*

Keywords: *CubeSat, Kalman Filter, Attitude*

1. INTRODUCTION

According to (Bouwmeester and Guo, 2010), nanosatellites have mass between 1.0 and 10.0 kg. The CubeSat Project began in 1999 in California Polytechnic State University and Stanford University's Space Systems Development Laboratory (TheCubeSatProgram, 2014). The aim was to standardize small satellites to reduce costs and development. A 1U CubeSat is a 10 cm sided cube weighing up to 1.33 kg. So, satellites consisting of 2 or 3 cubes are called 2U and 3U respectively. For example, the maximum mass of a 3U CubeSat is 4 kg. If these definitions are considered, the 3U CubeSat would belong the nanosatellite class, since it weighs up to 4 kg.

SERPENS mission involves research and development of nanosatellites, being inserted in a set of activities carried on by Brazilian universities in the scope of strategies coordinated by Brazilian Space Agency (AEB). The first satellite of a series, SERPENS-1, was put into orbit in September 2015, retiring in March 2016. SERPENS-2 mission is currently being designed. The experiments to be carried on in SERPENS-2 are: X and Gamma rays detection; South Atlantic magnetic anomaly (SAMA) measurement and testing of a pulsed plasma thruster (PPT).

Each experiment proposed for the SERPENS-2 mission is related to the attitude. This work concerns with the attitude determination and control system (ADCS), more specifically, an algorithm for attitude determination. The attitude instrumentation is: 3 solar sensors, 3 axes magnetometer and 3 axes gyroscope. The attitude is referenced to the local vertical local horizontal (LVLH) reference frame (RF) (Curtis, 2005) because the peculiarities of the proposed experiments.

All the attitude sensors are subject to noises that may compromise the feedback control system. Essentially, the idea of the Kalman filter (KF) is to minimize the variance of the estimation error knowing the variance of the system noise (propagation) and measurement noise (correction). In this paper, propagation is done from the gyroscope, the correction is performed by the TRIAD method that calculates the attitude from measurements of vector sensors (solar sensor and magnetometer) (Markley, 1999).

The KF fuses the particular measurements (gyroscope, magnetometer and solar sensor) by means of a weighted average. This means that the greater the error of TRIAD method, the greater the preference given to the result of the gyroscope and vice versa. The filter, on one hand, limits the accumulated error in the integration of the quaternion kinematics (Sidi, 1997), preventing the error from increasing progressively. The filter also reduces the impact of the abrupt instantaneous variations of the magnetometer and solar sensor readings. In this sense, the algorithm provides the improvement of the attitude measurement by exploiting the contribution of complementary sensors.

The general algorithm proposed is well discussed in the literature, Hasan *et al.* (2016). The setup of attitude sensors in question can be seen in several CubeSats, being a commercial off the shelf (COTS) technology. However, in most CubeSat missions, they are seen as "black boxes", with the developers of the mission having small access to their peculiarities. Moreover, this is a technology that is not consolidated in Brazil.

The main contribution of this work is technical, rather than scientific, since the main focus is to apply consolidated methods and technology in order to increasing the understanding of the system. One of the main reasons for promoting this Brazilian CubeSat mission is: to provide subsidies for local researchers to develop their own technologies. Thus, the scope of the SERPENS II mission is used as background to develop this CubeSat attitude determination system. Unfortunately, the system being developed in this paper will not be used in the SERPENS II mission, which will make use of a COTS system. However, the learning obtained through modeling, simulation and prototyping will provide requirements for such a system. In addition, these results can provide the basis for developing a technology for a possible SERPENS III mission, or other Brazilian CubeSat missions.

The remaining of this paper is divided as follows: section 2.summarizes the SERPENS II mission; section 3.models the attitude of the CubeSat; section 4.defines the attitude determination algorithm; section 5.describes the details of a simulation and its results; section 6contains final remarks.

2. SERPENS MISSION

The requirements of the attitude determination and control system (ADCS) are discussed in detail in paper Brum *et al.* (2017). Here, a few words are addressed for contextualization.

The SERPENS-2 mission will consist of a 3U CubeSat to operate at a 400 km altitude orbit with inclination from 45 to 60 degrees. The attitude requirements are related to the mission objectives such as:

- Detumbling: process of stabilizing the angular rate of an uncontrolled tumbling. To control this effect, active control will be used.
- Payload: Atlantic magnetic anomaly measurement, X and Gamma rays detection, testing of a pulsed plasma thruster (PPT) are the experiments that SERPENS-2 will be carrying.

Interaction to the geomagnetic field can be used as control torque. According to (Gerhardt and Paolo, 2010), the magnetic control is passive if a magnetic bar is used, or active, using dipole moment control.

In this mission, it will be used the active magnetic control, that is the most suitable for a CubeSat with Attitude Control System (ACS). Gravity gradient, aerodynamic drag, radiation pressure and residual magnetic field of the electronics are the main perturbation torques. This way, the control law have to minimize these disturbances, that can be in the order of $10^{-8}\,Nm$.

In order to the mission be executed, attitude estimation in the LVLH (Local Vertical Local Horizon) frame is required. For the sake of simplicity, the only embedded attitude sensors are: 3-axes MEMS gyroscopes, solar sensors (attached to the solar panels) and a 3-axes magnetometer.

2.1 Control System

The actuators are 3 magnetorquers (3 axes), which are commanded by suitable power electronics, which allow current modulation for generating a variable magnetic dipole. These actuators will be used to point and stabilize the satellite. When magnetorquers interacts with Earth's magnetic field, they produce control torque. Because of its low cost, simplicity of manufacturing and integration, besides low volume, mass and energy consumption, this actuator is the most suitable for the CubeSat.

The attitude control has two operation modes: detumbling and tracking/stabilization.

After deployment, the satellite is likely to be tumbling, i.e. experiencing high angular rates, so, the detumbling control generates a magnetic momentum for damping the kinetic energy, reducing the angular velocity to an amount suitable for the next control mode.

In the detumbling mode, attitude estimation is not required. In the tracking/stabilization mode, feedback is required: the attitude shall be measured in the LVLH system, which is the main reason for an attitude determination system in the mission.

The objective of the feedback controller is to drive the satellite to a required attitude, such that the experiments can be performed (PPT test and X/Gamma ray detection).

For a thorough understanding of the control system, (Brum *et al.*, 2017) should be consulted. In that work, the attitude control is executed with idealized measurements, without inserting the attitude estimator.

3. ATTITUDE MODEL

The data in this section came from reference Brum *et al.* (2017). An illustration of its geometry is on Fig. 1.

The 3U CubeSat standardized dimension and mass are: y axis: 34.05 cm; x and z axis: 10 x 10 cm.

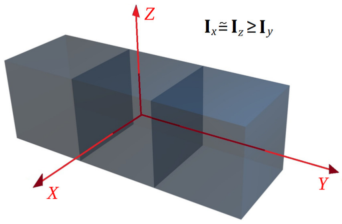

Figure 1: SERPENS-II, 3U structure and axes fixed to the body. From Brum *et al.* (2017).

A PPT thruster is included in SERPENS-II. It is positioned in the middle of the body, centered at the coordinates (0; 0; 0) in the body RF. According to (ClydeSpace, 2015) the thruster has dimensions 9 x 9 x 3 cm and mass less than 0.28 kg. Its inertia moment was calculated considering a 3 cm thick and homogeneous plate located at satellite center, where the thruster is aligned with $+z$ axis. Its propulsive forces accelerates de body in the z direction.

The whole satellite inertia matrix (I^b_{sat}) was calculated from a symmetric and homogeneous parallelepiped with 3.6 kg, plus the thruster inertia matrix, being given by:

$$\mathbf{I}^b_{sat} = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} = \begin{bmatrix} 0.0414\,kgm^2 & 0 & 0 \\ 0 & 0.0065\,kg/m^2 & 0 \\ 0 & 0 & 0.0414\,kgm^2 \end{bmatrix} \tag{1}$$

3.1 Coordinate Frames

The following reference frames (RFs) are adopted in this problem:

Inertial System - $O_iX_1X_2X_3$: a geocentric inertial system is considered. Its axis X_3 is coincident with the EarthâĂŹs rotation axis and the X_1 axis points towards the vernal equinox, which is defined by the intersection of the Equator plane with the Ecliptic plane. Axis X_2 completes the Cartesian right hand system.

Orbit System - O_oxyz: the same as LVLH frame, it has origin at the satellite center of mass with z axis (yaw) pointing to Earth center, y axis (pitch) pointing perpendicularly to the orbital plane, anti-parallel to orbital angular moment, and x axis (roll) parallel to satellite velocity vector, tangent to the circular orbit. When Euler angles are zero, this system axes coincide with the body system.

North East Down (NED system) - $O_{ned}x_ny_ez_d$: it has origin at satellite center of mass, x_n axis points in the direction of the geographic north, y_e axis points to east and z_d is directed to the Earth's center.

Body System - $O_bx_1x_2x_3$: it has origin at satellite center of mass and its axes are coincident with vehicle principal inertia axes, as illustrated in Fig. 1.

According to (Tewari, 2007), considering the attitude of a coordinate frame relative to another frame, a general orientation can be obtained by using successive rotations about the axes of the RF. The largest number of such rotations needed to uniquely specify a given orientation, called rotational degrees of freedom, is three. The sequence of axial rotations is very important in this representation. The Euler angles and the axes of sequential rotations can be defined using $(\psi)_3$, $(\theta)_2$ and $(\phi)_1$, which denotes a rotation of $(OXYZ)$ by angle ψ about OZ, resulting in the intermediate orientation,$(OX'Y'Z')$, followed by a rotation by angle θ about OY', resulting in $(OX''Y''Z'')$, and then a final rotation by angle ϕ about OX'', to produce the new orientation, $(OX'''Y'''Z''')$. Fig. 2 shows the orientation of the Euler angles. This specific sequence is called 3-2-1.

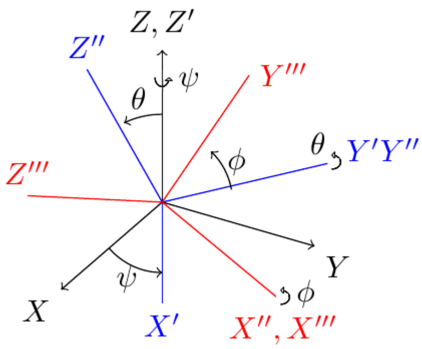

Figure 2: Euler angles: sequence of standard rotations.

The rotation matrix for the rotation sequence $(\psi)_3$, $(\theta)_2$ and $(\phi)_1$ is:

$$\mathbf{R} = \mathbf{R}_1(\phi)\mathbf{R}_2(\theta)\mathbf{R}_3(\psi) \\ \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ (\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi) & (\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi) & \sin\phi\cos\theta \\ (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) & (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) & \cos\phi\cos\theta \end{bmatrix} \tag{2}$$

A rotation matrix, also called direction cosine matrix (DCM). The attitude between orbital and body systems is given by the DCM $\mathbf{R}_o^b$. Rotations in the ZYX sequence are used to transform orbital to body system, composing the $\mathbf{R}_o^b$ matrix, from which quaternions and Euler angles can be calculated.

Other attitude transformations are of interest in this work: $\mathbf{R}_i^o$ from the inertial to orbital; $\mathbf{R}_i^{ned}$ from the inertial to NED; $\mathbf{R}_{ned}^o$ from the NED to orbital.

3.2 Attitude Kinematics

Euler angles are useful for interpretation and visualization, but quaternions are more suitable for computational issues. So, quaternions are used to process the attitude kinematics.

The various attitude representations: DCM, Euler angles and quaternions can be converted from one to another using well defined equations, Tewari (2007).

If $\mathbf{q}_o^b$ is the quaternion that represents the attitude of the body RF with respect to the orbit RF, the differential Eq. 3 describes the evolution of the satellite attitude in terms of its angular velocity. This is a general attitude kinematics equation that can be found in textbooks such as Tewari (2007).

$$\dot{\mathbf{q}}_o^b = \frac{1}{2}\boldsymbol{\Omega}(\boldsymbol{\omega}_{ob}^b)\mathbf{q}_o^b \tag{3}$$

where $\boldsymbol{\omega}_{ob}^b = [\omega_1\ \omega_2\ \omega_3]^T$ is the angular velocity vector of the body with respect to the orbital RF written in the body RF. $\boldsymbol{\Omega}(\cdot)$ is the antisymmetric matrix of the vector product:

$$\boldsymbol{\Omega}(\boldsymbol{\omega}_{ob}^b) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \tag{4}$$

When using a gyroscope to obtain the angular velocity of the body, one shall note that this device provides its measurements with respect to the inertial space, but written in the body RF: $\boldsymbol{\omega}_{ib}^b$. If $\boldsymbol{\omega}_{io}^o$ is the angular velocity of the orbital RF with respect to the inertial RF written in the orbital RF, one has that:

$$\boldsymbol{\omega}_{ob}^b = \boldsymbol{\omega}_{ib}^b - \mathbf{R}_o^b\boldsymbol{\omega}_{io}^o \tag{5}$$

3.3 Attitude Dynamics

Attitude dynamics shall be written with respect to an inertial frame. In this paper, the satellite is assumed as a rigid body in such a way that the Euler equation for the attitude dynamics can be considered (see some classical textbook as Sidi (1997) or Tewari (2007)):

$$\mathbf{I}_{sat}^b\dot{\boldsymbol{\omega}}_{ib}^b = \mathbf{M}_c^b + \mathbf{M}_d^b - \boldsymbol{\omega}_{ib}^b \times \mathbf{I}_{sat}^b\boldsymbol{\omega}_{ib}^b \tag{6}$$

where $\times$ is the vector product operator. $\mathbf{M}_c^b$ and $\mathbf{M}_d^b$ are control and disturbance torques, respectively, written in the body RF.

The control torque $\mathbf{M}_c^b$ is generated by the ACS. Disturbance $\mathbf{M}_d^b$ can be caused by gravity gradient, solar radiation pressure, atmospheric drag, residual magnetic moments, etc. In this paper, only gravity gradient is considered.

4. ATTITUDE DETERMINATION

The simplest form of determining an attitude are: (1) direct computation from two vector observations; (2) integration of the attitude kinematics, e.g. Eq. 3, from the gyroscope measurement.

4.1 Vector Measurements - TRIAD

In the class of attitude determination from vector observations, at least two measurements shall be taken in the body RF and two counterparts shall be available in the RF with respect to which the attitude is being computed. One of the most simple is the TRIAD (Three Axes Attitude Determination), Markley (1999), Tanygin and Shuster (2007). This method uses only two observation vectors. From sensor weighting it's possible to perform symmetric or asymmetric calculations.

If $\mathbf{b}_1$ and $\mathbf{b}_2$ are non collinear vectors measured in the body RF, while $\mathbf{r}_1$ and $\mathbf{r}_2$ are their counterparts in a given RF $\mathcal{R}$, by definition one has that:

$$\mathbf{b}_1 = \mathbf{A}\mathbf{r}_1, \;\; \mathbf{b}_2 = \mathbf{A}\mathbf{r}_2 \tag{7}$$

where $\mathbf{A}$ is the DCM matrix of the body with respect to the RF $\mathcal{R}$.

Using this algorithm, the attitude can be calculated in three different ways. It uses the former two pairs. A third pair is constructed: $\mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$, $\mathbf{b}_3 = \mathbf{b}_1 \times \mathbf{b}_2$

The first form of TRIAD, called TRIAD-1, where the first and third vectors are used, can be calculated by the following equation.

$$\mathbf{A_1} \doteq \mathbf{b_1}\mathbf{r_1^T} + \mathbf{b_3}\mathbf{r_3^T} + (\mathbf{b_1} \times \mathbf{b_3})(\mathbf{r_1} \times \mathbf{r_3})^T \tag{8}$$

The second form, called TRIAD-2, uses the second and third vectors:

$$\mathbf{A_2} \doteq \mathbf{b_2}\mathbf{r_2^T} + \mathbf{b_3}\mathbf{r_3^T} + (\mathbf{b_2} \times \mathbf{b_3})(\mathbf{r_2} \times \mathbf{r_3})^T \tag{9}$$

TRIAD-1 and TRIAD-2 treat the measures in a non-symmetric way. TRIAD-1 emphasizes the first vector measurement, $\mathbf{b}_1$, while the other emphasizes the vector $\mathbf{b}_2$. The third way of calculating TRIAD treats both measurements symmetrically. For this, two more pairs of unit vectors are defined. Vectors $\mathbf{r}_+$ and $\mathbf{r}_-$ are given below, b_+ and b_- are described similarly.

$$\mathbf{r}_+ \doteq \frac{(\mathbf{r_2} + \mathbf{r_1})}{|\mathbf{r_2} + \mathbf{r_1}|} = \frac{(\mathbf{r_2} + \mathbf{r_1})}{\sqrt{2(1 + \mathbf{r_1}\mathbf{r_2})}}, \;\; \mathbf{r}_- \doteq \frac{(\mathbf{r_2} - \mathbf{r_1})}{|\mathbf{r_2} - \mathbf{r_1}|} = \frac{(\mathbf{r_2} - \mathbf{r_1})}{\sqrt{2(1 - \mathbf{r_1}\mathbf{r_2})}} \tag{10}$$

Thus, r_+ and r_- are perpendicular to each other the same way b_+ is perpendicular to b_-. Finally:

$$\mathbf{A_3} \doteq \mathbf{b}_+\mathbf{r}_+^{\mathbf{T}} + \mathbf{b}_-\mathbf{r}_-^{\mathbf{T}} + (\mathbf{b}_+ \times \mathbf{b}_-)(\mathbf{r}_+ \times \mathbf{r}_-)^T \tag{11}$$

4.1.1 References for Satellite Attitude

The TRIAD method is general. However, in the specific case of this paper, the RF $\mathcal{R}$ is the orbit RF (LVLH). The two vector observations in the body system are: the measurements of a 3-axis solar sensor and 3-axis magnetometer.

The reference vector $\mathbf{r}_1$ and $\mathbf{r}_2$ are the normalized vector of the sun position with respect to the Earth and Earth's magnetic field.

The Earth's reference magnetic field is given in the NED RF. The unit vector of the direction of the Sun in relation to the Earth, is taken in the inertial RF. In order for the TRIAD method determine the attitude in the LVLH system, the solar and magnetic reference vector must be converted to the LVLH system. This is done by the appropriate matrices, which depend on the orbital parameters and the satellite position at the time of interest.

In a real attitude determination system, it is necessary to generate the reference vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ from some reliable source. The sun position is obtained in the J2000 inertial reference frame using UTC time. The Earth's reference magnetic field is obtained from the IGRF model [1].

4.1.2 Sensor Models

This section describes how the magnetometer and sun sensor are simulated, representing the measurements that should be present when attached to the body of a satellite in orbit.

Solar sensor: In this paper, the sun position is obtained in the J2000 inertial RF (ECI), using the code of reference Carrara (2015). This vector is normalized to generate the reference solar vector in the inertial system: $\mathbf{e}_s^i$.

Next, the vector $\mathbf{e}_s^i$ is converted from the inertial system to the LVLH system, from the rotation matrices, as shown below:

$$\mathbf{e}_s^b = \mathbf{R}_i^b \, \mathbf{e}_s^i \tag{12}$$

$$\mathbf{R}_i^b = \mathbf{R}_o^b \, \mathbf{R}_i^o \tag{13}$$

Where $\mathbf{e}_s^b$ is the sun vector in the body RF, $\mathbf{R}_i^b$ is the DCM from inertial RF fixed on Earth to the body RF, $\mathbf{R}_i^o$ is the DCM from Earth's inertial system to LVLH and $\mathbf{R}_o^b$ is the attitude matrix of the body with respect to the LVLH.

[1] International Geomagnetic Reference Field: https://www.ngdc.noaa.gov/IAGA/vmod/igrf.html

Magnetic sensor: the value $\mathbf{m}^{ned}$ of the Earth's magnetic field, at the instantaneous point of the orbit, is obtained from the IGRF model. A function from Carrara (2015) is used. This value is given in the NED system, which is converted to the LVLH system:

$$\mathbf{m}^b = \mathbf{R}^b_{ned}\,\mathbf{m}^{ned} \tag{14}$$

$$\mathbf{R}^b_{ned} = \mathbf{R}^b_o\,\mathbf{R}^o_{ned} \tag{15}$$

Where $\mathbf{m}^b$ is the magnetometer vector measurement in the body RF, $\mathbf{R}^b_{ned}$ is the DCM from NED RF to the body RF and $\mathbf{R}^o_{ned}$ is the DCM matrix from NED RF to the LVLH RF.

The Simulink model developed for the simulations of this paper computes the DCM matrices from orbital parameters according to the ref. Coelho (2016).

Measurements $\mathbf{e}^b_s$ and $\mathbf{m}^b$ are idealized. In order do represent errors, these results are added to white Gaussian noises. The Simulink's limited-band white noise block is used for this purpose.

4.2 Integration of Attitude Kinematics

The integration of the attitude kinematics is the numerical solution of the differential equation that relates the attitude with angular velocity, from a given initial condition. This method is sometimes called *inertial navigation*, because it involves the use of an inertial sensor (the gyroscope).

Each attitude parametrization (Euler angles, quaternions, DCM, etc) has a specific structure for its equation of kinematics. In this paper, the quaternion formulation is considered, that is related to the Eq. 3.

A gyroscope determines its measurements with respect to the inertial space, but written in the body RF, that is: $\boldsymbol{\omega}^b_{ib}$. Where this is the notation defined in section 3.2.

The algorithm for determines the attitude from the gyroscope measurements is, then:

1: Define an initial condition for the quaternion: $\mathbf{q}^b_o(0) = \mathbf{q}_0$;

2: Insert the gyroscope measurement in Eq. 5, in order to obtain the body's angular velocity with respect to the LVLH RF;

3: Solve Eq. 3.2 using the result of step [2];

4: Compute the attitude matrix and Euler angles from the quaternion.

In the step 1, note that a suitable initial condition is necessary. Generally, the result of a vectorial method such as TRIAD is used for this purpose.

In the step 2, it is necessary to know the attitude matrix, but it is one of our unknowns. This can be solved using the result of this matrix computed in the last iteration of the algorithm. Note that the orbital angular rate is also necessary.

In the step 3, the differential equation is solved numerically, using as the initial condition the result of the last iteration.

Gyroscope simulation: the angular velocity value is obtained from the rotation dynamics, Eq. 6, (which depends on the control loop). But, this result is idealized.

In order to represent measurement errors in the gyroscope, the results of the attitude dynamics are added to white Gaussian noises. The Simulink's limited-band white noise block is used for this purpose.

4.3 Kalman Filter

The inertial navigation method of section 4.2has a strong problem: the measurement errors of the gyroscope are integrated. For long periods of operation, the integration of the errors tend do cause divergence.

On the another hand, the vector methods, such as TRIAD, generate measurements that tend to have strong oscillations, depending on the severity of the noises in the solar sensor and magnetometer.

The stochastic Filtering approaches, by their turn, can perform the fusion of data originated from different approaches. By fusing the two methods above, the TRIAD method can prevent the divergence of gyroscope integration, while the last can smoothen the TRIAD oscillations. Then, the improvement of measurement is provided by the help of complementary sensors.

When a measurement is obtained by the stochastic filtering (fusion) of two different methods, it is called **estimation**.

The most popular method of stochastic filtering is the Kalman filter (KF). The idea of the KF is to minimize the variance of the estimation error from the knowledge of the variances of the process noise (gyroscope) and measurement noise (TRIAD). The filter fuses the particular measurements by means of a weight average, this means that the greater the error of TRIAD method, the greater the preference given to the result of the gyroscope and vice versa.

This method has two phases: the **prediction**, that calculates de state vector one step ahead, and the **correction**, which corrects the prediction by a current reading of another sensor.

That way, it is necessary a sensor that estimates the variable one step forward from the previous value and a sensor that determines variables from a current observation. In this paper, the prediction step is performed by the attitude kinematics model as described in section 4.2The correction is performed by the TRIAD, as described in the section 4.1

The estimation generated by the KF is the best measurement possible when the errors of all sensor are white gaussian noise with zero mean and uncorrelated, with all functions are linear. Although most real cases are not exactly the ideal case, the Kalman Filter still provides a better result than the result for separate sensors. To solve the non-linear question, the Extended Kalman filter (EKF) is used, which performs non-linear approximations.

The extended Kalman filter is widely spread in the literature. Below, the equations of the EKF are shown in detail. They were taken from (Silva and Cruz, 2016).

Consider that the nonlinear equation for the prediction dynamics is:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) + \mathbf{G}(\mathbf{x}, t)\mathbf{w}(t) \tag{16}$$

where x is the state of the system (quaternion), u is the entry variable (angular velocity), $\mathbf{w}$ is the noise of the gyroscope that has a variance matrix Q (3x3).

The measure equation of the TRIAD is:

$$\mathbf{z}(t) = \mathbf{h}(\mathbf{x}) + \mathbf{v}(t) \tag{17}$$

where $\mathbf{z}$ is the measurement (quaternion calculated by TRIAD), $\mathbf{v}$ is the noise of the TRIAD that has a variance matrix R (4x4). So, $\mathbf{w}$ is the process noise and v is the measure noise.

Some conventions of the Kalman Filter can be observed in the Tab. 1.

Table 1: Conventions for the Kalman Filter

T_s	Sampling time
$t_k = kT_s$	Sampling instant, $k = 0, 1, 2, ...$
$\hat{x}_k^-$	Predicted state in the $t_k = KT_s$ instant
$\hat{x}_k^+$	Corrected state (predicted) in the $t_k = KT_s$ instant
P_k^-	Prediction of the covariance matrix of the estimation error at t_k
P_k^+	Correction of the covariance matrix of the estimation error at t_k

The phases of the EKF are summarized below.

Prediction - Calculation of the state vector one step ahead:

$$\hat{\mathbf{x}}_{k+1}^- = \hat{\mathbf{x}}_k^+ + \int_{t_k}^{t_{k+1}} \mathbf{f}(\mathbf{x}, \mathbf{u}, t)dt \tag{18}$$

Error's covariance matrix:

$$\dot{\mathbf{P}} = \mathbf{F}_k\mathbf{P}(t) + \mathbf{P}(t)\mathbf{F}_k^T + \mathbf{Q}_k'' \tag{19}$$

Equation 19 is known as Riccati's Equation, where:

$$\mathbf{F}_k = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_k^+} \tag{20}$$

$$\mathbf{Q}_k'' = \mathbf{G}(\hat{\mathbf{x}}_k^+, t_k)\mathbf{Q}(t_k)\mathbf{G}^T(\hat{\mathbf{x}}_k^+, t_k) \tag{21}$$

Equation 20 is the linearization of the kinematics, Eq. 21 is the "formating" of the process noise distribution matrix. The prediction one step ahead is then:

$$\mathbf{P}_{k+1}^- = \mathbf{P}_k^+ + \int_{t_k}^{t_{k+1}} \dot{\mathbf{P}}(t)dt \tag{22}$$

For the correction process, it is necessary to define de Kalman Gain, $\bar{K}_k$.

$$\bar{\mathbf{K}}_k = \mathbf{P}_{k+1}^-\mathbf{H}_k^T[\mathbf{H}_k\mathbf{P}_{k+1}^-\mathbf{H}_k^T + \mathbf{R}_k]^{-1} \tag{23}$$

where $\mathbf{H}_k$ is the linearization of the output function:

$$\mathbf{H}_k = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_{k+1}^-} \tag{24}$$

Here, it is possible to calculate the correction itself:

$$\hat{\mathbf{x}}^{+}_{k+1} = \hat{\mathbf{x}}^{-}_{k+1} + \bar{\mathbf{K}}_k[\mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}^{-}_{k+1})] \tag{25}$$

Where $\mathbf{z_k}$ is the value read by TRIAD and $\mathbf{h}(\hat{x}^{-}_{k+1})$ the approximated value by prediction (gyroscope). The last stage is the correction of the error's covariance matrix estimate:

$$\mathbf{P}^{+}_{k+1} = \mathbf{P}^{-}_{k+1} - \bar{\mathbf{K}}_k\mathbf{H}_k\mathbf{P}^{-}_{k+1} \tag{26}$$

5. SIMULATION AND RESULTS

The simulator is composed of Simulink blocks developed in the work Coelho (2016). The core routines of the majority of Simulink blocks came from the MATLAB toolbox of reference Carrara (2015). The model contains: orbit propagation, kinematics and attitude dynamics, attitude transformations, Earth magnetic field model (IGRF), position of the Sun and magnetic attitude control.

5.1 Simulation

In Fig. 3, the Simulink model used in the simulation is illustrated.

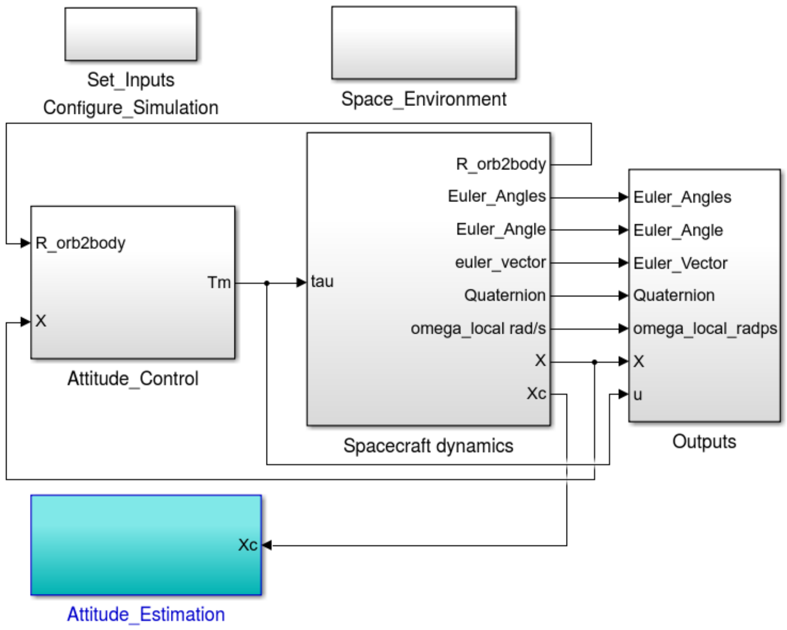

Figure 3: Block diagram of the simulation.

The attitude control described in (Brum *et al.*, 2017) is used. The data from attitude estimation were not used to close the control loop, since the objective was to evaluate the performance of the estimator, without considering, at the moment, the interaction of the same with the control, which will be investigated in future work.

Concerning the operation of the control and attitude determination tasks. In these simulations, the application of the control and the determination of attitude are done without interruption. However, on a real mission, the attitude can not be measured and controlled at the same time, since the magnetic field of the actuator interferes with the attitude measurement made by the magnetometer. In these actual systems, the control signal is turned off sporadically so the magnetometer can perform measurements during this interval. Because the attitude calculation is done in a discrete time, in a practical system, the execution of the control and measurement tasks can be done in an interleaved way, during the sampling interval, with minimum loss as stipulated in the ideal case.

The orbit parameters are: altitude $h = 400.000$ km, eccentricity $e = 0$ (circular orbit), inclination $i = 50^o$, Right ascension of ascending node raan $= 0$, Argument of perigee $\omega = 0$. The orbit period is 92.56 minutes.

Three alternatives for the determination of the attitude were investigated: TRIAD-3 algorithm, integration of the attitude kinematic from gyroscope data, and EKF.

5.2 Results

Graphical results are shown in Fig. 4 and 5. In Fig. 4, black, gray and blue lines represent the responses of the Euler angles given by simulation (the reference value), gyroscope (inertial navigation) and TRIAD-3, respectively. The noises simulated in the gyroscope generate a poor result that tends to divergence as time increases. On the other hand, the noises simulated in the magnetometer and solar sensor make the TRIAD-3 results to become very oscillatory.

On Fig. 5, fortunately, the results given by the EKF are better behaved. The EKF can reduce the severity of the oscillations of the TRIAD3 result, also preventing the divergence of the gyroscope integration. This is exactly the behavior expected by a Kalman filter: the improvement of the measurement of the two distinct methods.

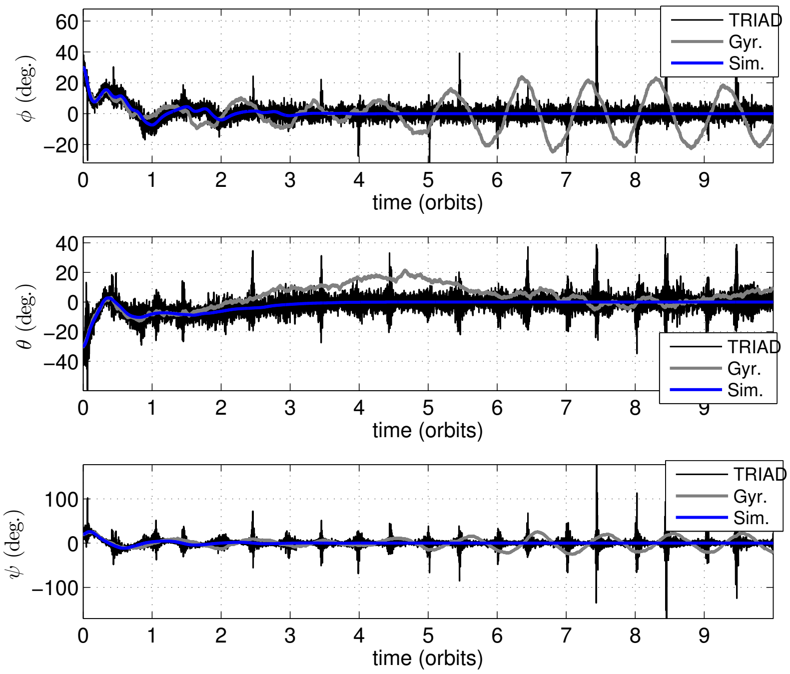

Figure 4: Euler angles: simulated (ideal), TRIAD3 computation and “Gyroscopic” computation (inertial navigation).

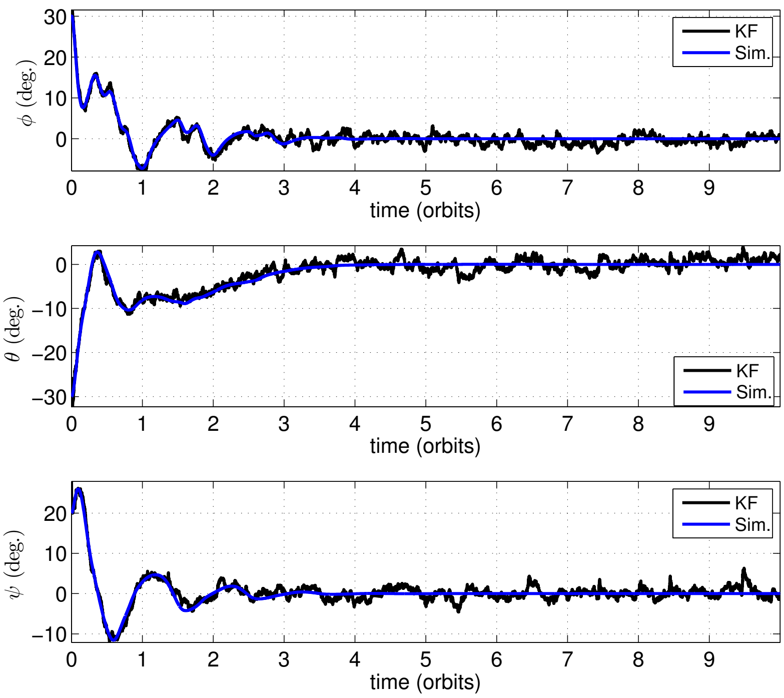

Figure 5: Euler angles: simulated and estimated by the EKF.

Numerical evaluation of the errors is performed. The error is computed taking the difference between the "x angle" calculated (x_{calc}) by the given method (TRIAD or EKF) and the respective simulated (ideal) value (x_{sim}): $e_x(t) = x_{calc}(t) - x_{sim}(t)$. The mean absolute error of the given variable x is: mean$|e_x| = \frac{1}{T}\int_0^T |e_x(t)|dt$.

Table 2 show the mean absolute errors (MAEs) given by the EKF and TRIAD-3 along the simulation. The MAEs given by the EKF are below 1^o, while that given by the TRIAD3 are roughly between 3 and 4.7 times greater.

Table 2: Mean absolute errors in attitude determination.

	Kalman Filter	TRIAD3
mean$\|e_\phi\|$	0.8463^o	2.5279^o
mean$\|e_\theta\|$	0.9546^o	3.5691^o
mean$\|e_\psi\|$	0.9850^o	4.6087^o

The numeric and graphical results show the importance of the EKF in the scenario under test. It can drastically improve the results of the attitude sensors, via the fusion of their measurements.

6. CONCLUSION

The extended Kalman filter was implemented for the attitude estimation of a CubeSat in the LVLH reference frame. Even in the presence of strong non linearity, the filter could reduce the errors of the TRIAD method (magnetometer and solar sensor) and attitude propagation (gyroscope) alone. However, the attitude estimator is still not integrated with the control system, requiring further work.

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8. RESPONSIBILITY NOTICE

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