24th COBEM - 2017

24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-1335
RELIABILITY BASED DESIGN OPTIMIZATION USING FUZZY LOGIC

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Abstract. *Reliability and robustness are mathematical concepts intrinsically related to engineering system design and applied to take into account the inherent uncertainties that affect the performance of these systems. In this contribution, a new methodology to reliability-based design of mechanical system is proposed. In this approach, the uncertain parameters are modeled as fuzzy sets and evaluated in the design problem by means of a secondary cost function. The reliability is assessed by means of nested optimization algorithm and the associated problem is formulated in the multi-objective context. An inner loop is used to obtain the uncertain variables bounds and an outer loop determines a predefined reliability index within the obtained limits. The proposed approach is evaluated through numerical simulations of classical engineering system design. The obtained results are compared with a well-known approach and the efficiency of the methodology is verified.*

Keywords: *reliability, robustness, fuzzy sets, fuzzy logic, uncertainty analysis*.

1. INTRODUCTION

Uncertainties are inherent features of mechanical systems that are frequently associated with manufacturing errors, operational fluctuations, or simply due to lack of knowledge on the system performance. Uncertain variables can lead to extreme responses that reduce the performance and reliability of the system. In drastic scenarios, parameter fluctuations can lead to unsafe operating conditions or even, in some cases, to failure.

Usually, engineering systems are designed considering deterministic quantities (i.e., disregarding the effects of any uncertainty). Consequently, the resulting design can be sensitive to small parameter variations. The sensitivity to parameter deviations or operational fluctuations is associated to robustness (see Fig. 1). In a more complete design approach, the goal is to reduce the system sensitivity to parameters fluctuation and/or operational variations, increasing its robustness. The robustness analysis in a robust design problem (RD) is usually associated with an optimization procedure. Commonly, robust optimization approaches rely on stochastic techniques based on statistical moments (Beyer and Sendhoff, 2007; Gabrel *et al.*, 2014; Lima *et al.*, 2015; Tammareddi *et al.*, 2016) and the definition of a robust objective function. This approach leads to large combinatorial problems that impose some disadvantages (e.g., computational cost, knowledge on the probability distribution, among others).

While in robust design problem the goal is to reduce the design overall sensitivity to parameter deviations and/or operational variations, in reliability-based design (RBO) the emphasis is to ensure the achievement of predefined constraints related to design stability and/or system safety performance (Du and Chen, 2004) (see Fig. 2). As in RD, most RBO applications relies on stochastic approaches aiming in reduce failure probability.

RBO stochastic approaches are usually based on two strong assumptions (Cheng and Mon, 1993): the probability assumption in which is supposed that systems behavior can be fully represented by probability measures; and a binary-state assumption in which failure states are precise definitions (e.g., the system is either in safety or in failure state, without a transition state). However, in several applications, due to imprecise information, some uncertainties do not satisfy these assumptions, making the probability theory unsuitable to properly handle the system uncertainties.

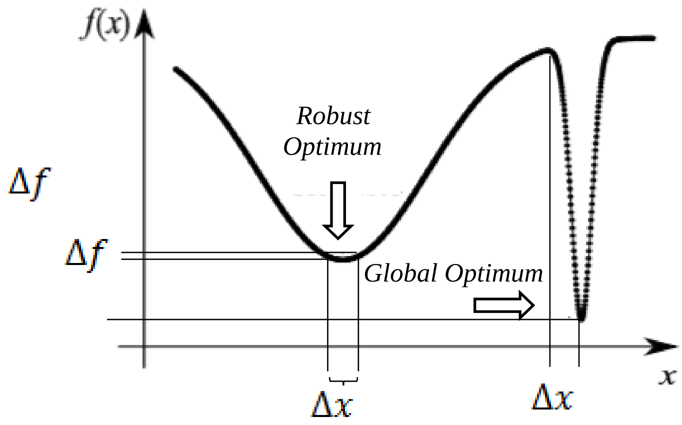

Figure 1. Concept of robustness.

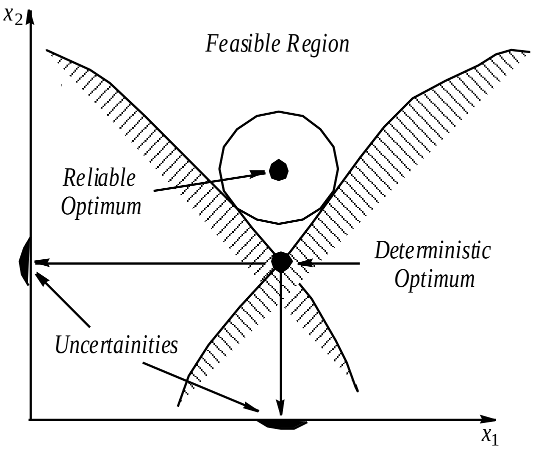

Figure 2. Concept of reliability (Lobato *et al.*, 2017).

Thus, in this contribution, a new methodology based on fuzzy logic (Zadeh, 1968) is proposed for RBO design problems of mechanical systems. The proposed methodology evaluates simultaneously the design problem and its associated reliability by defining a multi-objective optimization problem. A fuzzy metric is formulated to assess the system reliability and is handled in the RBO procedure as a nested optimization problem. An inner optimization loop is used to determine the uncertain variables limits and an outer loop evaluates the reliability metric within these limits. The performance of the proposed approach is evaluated by considering two different test cases, namely, a Highly Nonlinear Limit State (HNLS) problem and a cantilever beam design problem.

2. FUZZY SET THEORY

In this contribution, the uncertain information is modeled as fuzzy sets by using the fuzzy set theory as an alternative to the probabilistic approach. Fuzzy set theory was formulated by Zadeh (1968) aiming to characterize vagueness information and epistemic uncertainties. Fuzzy sets can be viewed as the counterpart of the Boolean notion in regular sets. Thus, in a fuzzy set an element can belong, not belong, or partially belong to the set and the pertinence of the element in the set is weighted by a membership function.

In fuzzy set theory, the membership function μ is a continuous closed interval [0, 1] that weights the pertinence of the element x with respect to the fuzzy set $\tilde{A}$. Values of $\mu_{\tilde{A}}(x)$ close to 1 indicates high compatibility of x to the set $\tilde{A}$. Uncertainties can be computationally modeled as fuzzy numbers by using the fuzzy set theory. Thus, the actual value of the uncertain parameter is unknown but limited in an interval weighted by the membership function.

For computational purposes, the uncertain information modeled as fuzzy sets can be discretized in subsets known as α-levels, that corresponds to continuous intervals defined by $A_{\alpha k}$ (see Eq. (1)). For convex fuzzy sets, each α-level $A_{\alpha k}$ corresponds to an interval $[x_{\alpha kl}, x_{\alpha kr}]$ (see Eq. (2)) that can be obtained by means of optimization procedures. The α-level α_0 defines the support of the fuzzy set, corresponding to the largest fuzzy set interval containing all considered realizations of the uncertain parameter.

$$A_{\alpha k} = \left\{ x \in X, \mu_{\tilde{A}}(x) \geq \alpha_k \right\} \tag{1}$$

$$x_{\alpha kl} = \min\left[x \in X \left| \mu_{\tilde{A}}(x) \geq \alpha_k \right. \right]$$
$$x_{\alpha kr} = \max\left[x \in X \left| \mu_{\tilde{A}}(x) \geq \alpha_k \right. \right] \tag{2}$$

In this sense, the fuzzy uncertainty analysis can be summarized as follows: for computational purposes, an input vector is generated, that corresponds to the uncertain parameters discretized by means of the α-level representation. Thus, each element of this vector is considered as an interval and optimization problems as presented in Eq. (3) are defined. A second step can be defined related to solving the optimization problems. These optimization problems consist in finding the maximum and minimum values of a given objective function regarding all a-levels of interest (Cavalini Jr *et al*, 2017).

$$z_{\alpha kl} = \min_{\mathbf{x} \in \mathbf{X}_{\alpha k}} f(\mathbf{x})$$
$$z_{\alpha kr} = \max_{\mathbf{x} \in \mathbf{X}_{\alpha k}} f(\mathbf{x}) \tag{3}$$

In this contribution, the aforementioned fuzzy analysis procedure was modified to be carried out only in the α-level α_0. This means that the uncertainty analysis is performed considering only the support interval of the associated fuzzy sets, which corresponds to the inner loop of the proposed RBO procedure.

3. FUZZY RELIABILITY ANALYSIS

In deterministic design analysis, the solution of the problem is given at a constraint boundary, as shown in Fig. 2. However, if any perturbation in the system emerges, the violation of some design constraint is unavoidable and the system failure is observed. Therefore, in a RBO design analysis a reliability index is formulated to accomplish reliability and feasibility.

Fuzzy limit state functions ($FLSF_j$) are defined in fuzzy reliability analysis with respect to the vector of fuzzy variables as inequality constraints ($g_j(\boldsymbol{x})$), in which the difference between structural stress (S_j) and structural strength (R_j) is evaluated. These *FLSFs* defines critical surfaces ($g_j(\boldsymbol{x})=0$) that separates the variables space in two domains: a failure domain, in which $S_j > R_j$ and, consequently, $g_j(\boldsymbol{x})>0$; and a safety domain, where $S_j < R_j$, resulting in $g_j(\boldsymbol{x})<0$. As the *FLSFs* are treated as fuzzy variables, the fuzzy uncertainty analysis presented previously can be applied, redefining the *FLSFs* as intervals with respect to the α-level α_0. Thus, a reliability index can be defined as shown in Eq. (4), where η_j is obtained considering exclusively the support of the fuzzy limit state function $FLSF_j$. $FLSF_{jR}$ and $FLSF_{jL}$ corresponds to the upper and lower bounds of the $FLSF_j$ support, respectively.

$$\eta_j = \frac{FLSF_{jL}}{FLSF_{jR} - FLSF_{jL}} \tag{4}$$

Regarding the proposed reliability index, positive values of η_j indicates an unavoidable failure state, while negative values of η_j can indicate either a safety or a transition state. However, if $\eta_j \leq -1$ the system is in a safety state. The assessment of the proposed reliability index corresponds to the outer loop in the proposed RBO approach. The *FLSFs* limits ($FLSF_{jR}$ and $FLSF_{jL}$) are obtained in the inner loop of the proposed procedure. It is important to mention that the design problem becomes a multi-objective optimization process. Figure 3 summarizes the flowchart of the proposed procedure.

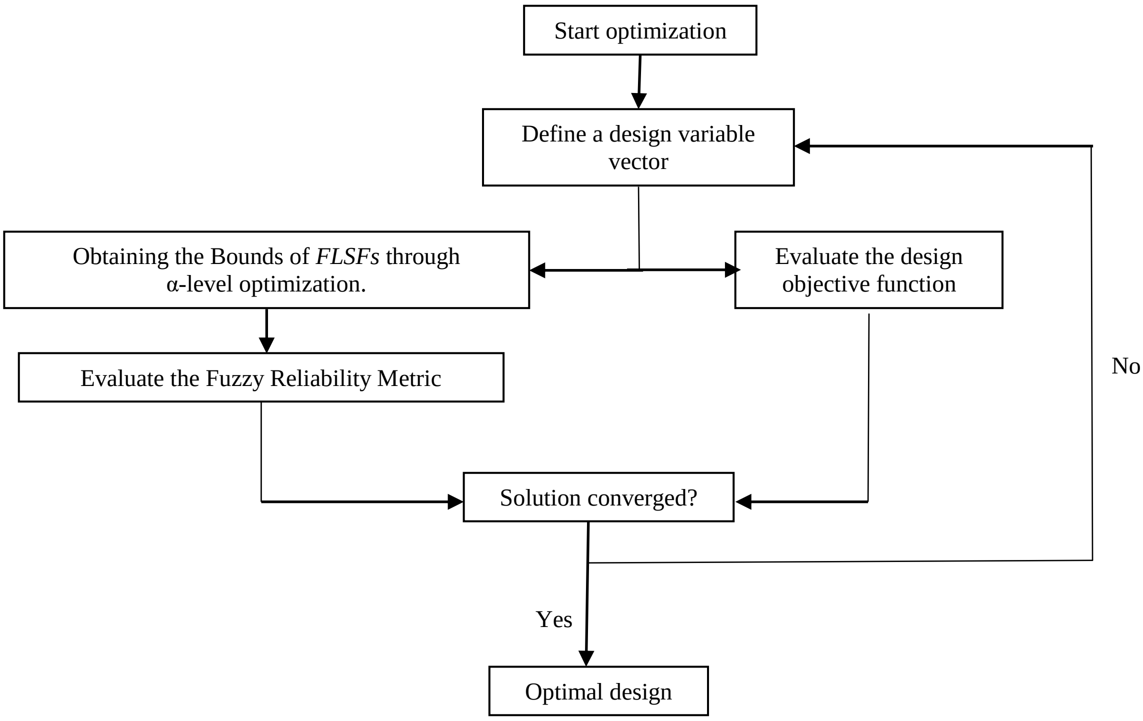

Figure 3. Flowchart of the proposed procedure.

4. NUMERICAL RESULTS

4.1 Highly Nonlinear Limit State

This RBO problem was proposed and solved by Aoues and Chateauneuf (2009), and it can be mathematically expressed as given in Eq. (5).

$$FLSF = \ln(x_{f_1}) - x_{d_1} x_{d_2} x_{f_2}$$
$$\begin{cases} \min x_{d_1}^2 + x_{d_2}^2 \\ \min \eta\left(FLSF\right) \end{cases} \tag{5}$$

In this case, the random variables of the original problem were converted into fuzzy triangular variables by using the procedure proposed by Pota *et al.* (2013). In this approach, the support intervals are [3.8165, 6] and [1.9485, 4.0287] for the variables x_{f1} and x_{f2}, respectively.

Figure 4 presents the Pareto's curve obtained by using the proposed reliability fuzzy approach. The multi-objective optimization problem was solved by using the compromise programing technique associated with the Sequential Quadratic Programming (SQP) algorithm (Vanderplaats, 2005). The Pareto's curve was generated considering different weights in each cost function, in which the corresponding mono-objective optimization problems were carried out to obtain the target values for each cost functions.

Table 1 presents the obtained results considering the minimum safety possibility (design variables values that ensures η = -1). It is important to note that the proposed approach was also able to find configurations belonging to each domain: safety, failure, or transient. Additionally, a safety configuration was obtained (η = -1.416) for the same cost function value (f = 3.65) obtained in Lobato et al. (2017), in which the IRA-DE was applied (target reliability R of 98.9830% corresponding to β=2.32). The minimum safety configuration results in a lower cost function value (f = 2.10) when compared with the probabilistic response (f = 3.65 in IRA-DE), indicating that the proposed methodology was less conservative. Also, evaluating the bounds of the *FLSF* in each response, it can be inferred that the proposed approach was able to obtain a more robust solution due the smaller variation of the *FLSF* function (see Table 1).

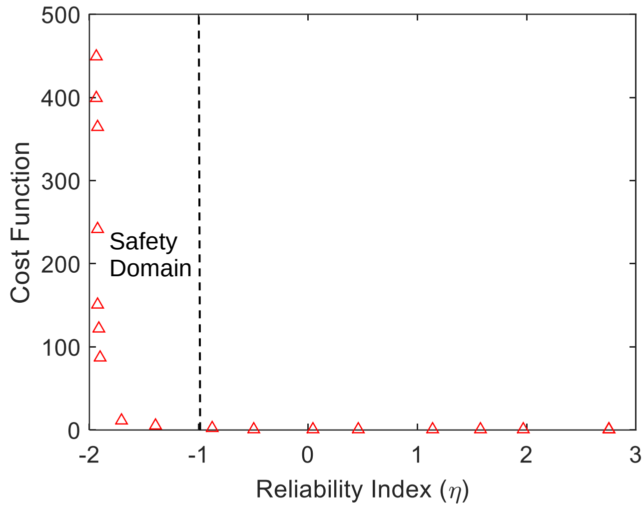

Figure 4. Pareto's curve of the highly nonlinear limit state problem.

Table 1: Results associated with the nonlinear limit state problem.

	f	x_{d1}	x_{d2}	$FLSF_L'$	$FLSF_R'$	η'
IRA-DE	3.65	1.3512	1.3518	-6.0198	-1.7686	-1.416
Proposed	2.10	0.7368	1.2473	-2.3629	$-7.22x10^{-7}$	-1

4.2 Cantilever Beam Problem

This test case was previously investigated by Lobato et al. (2017), Qu and Haftka (2004), and Kuo-Wei and Gautama (2014). The problem consists to find the minimum beam cross-sectional area presented in Fig. 5 considering the cost function shown in Eq. (6).

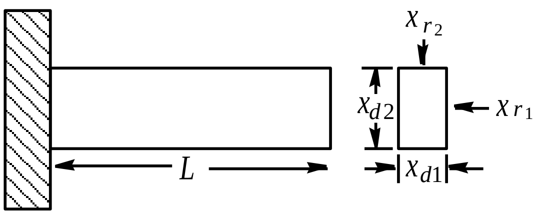

Figure 5: Cantilever beam problem (Lobato *et al.*, 2017).

$$\min_{x_d} \; x_{d1}x_{d12} \tag{6}$$

where x_{d1} and x_{d2} are the width and height of the beam, respectively, defined as the design variables with lower and upper limits equal to 0 and 10, respectively.

The two loads applied at the free end of the beam, the Young's modulus, and the yield strength are considered as random parameters with normal distribution. Similar to the previous problem, the random variables were converted into fuzzy triangular variables by using Potta *et al.*, (2013) procedure. The stochastic and fuzzy data associated with the cantilever beam problem are presented in Table 2.

The length L of the cantilever beam is 254 cm and the tip displacement has to be smaller than the allowable displacement d_o = 7.724 cm. The original definition of the first and second failure modes are redefined into the fuzzy problem given by Eq. (6).

Table 2: Stochastic and fuzzy data used in the cantilever beam problem.

	Mean value	Standard deviation		Nominal value	Lower limit	Upper limit
x_{r1} (lb)	500	100	x_{f1} (lb)	500	333	760.8
x_{r2} (lb)	1000	100	x_{f2} (lb)	1000	882.5	1217
x_{r3} (psi)	$29x10^6$	$1.45x10^6$	x_{f3} (psi)	$29x10^6$	$26.35x10^6$	$30.97x10^6$
x_{r4} (psi)	40000	2000	x_{f4} (psi)	40000	36040	43755

$$FLSF_1 = \frac{600x_{f2}}{x_{d1}x_{d2}^2} + \frac{600x_{f1}}{x_{d1}^2 x_{d2}} - x_{f4}$$

$$FLSF_2 = \frac{4L^3}{x_{f3}x_{d1}x_{d2}} \sqrt{\left(\frac{x_{f2}}{x_{d2}^2}\right)^2 + \left(\frac{x_{f1}}{x_{d1}^2}\right)^2} - d_o \tag{7}$$

$$\begin{cases} \min \ x_{d1}x_{d2} \\ \min\max\left[\eta'_{FLSF1} \quad \eta'_{FLSF2}\right] \end{cases}$$

The cantilever beam problem is composed by two distinct failure modes, each with a corresponding fuzzy limit state function ($FLSF_1$ and $FLSF_2$). In the proposed fuzzy procedure, the reliability analysis is treated as additional cost functions to the design problem. Therefore, in the cases which presents multiple limit state functions, additional cost functions would be included to the multi-objective problem. In order to overcome this limitation, the worst corresponding reliability index (maximum η) is minimized. Thus, the values of the reliability index associated with other limit state functions will always be smaller than the currently evaluated η value (the safety condition is obtained for all limit state functions).

The Pareto's curve obtained for the cantilever beam problem is presented in Fig. 6. Tables 3 and 4 show the results of the proposed fuzzy approach associated with the minimum safety possibility analysis and the ones which matches with the responses obtained by using the IRA-DE approach (considering a target reliability of 99.8651%; β=3).

Note that the probabilistic response (IRA-DE) consists of a transient response of the fuzzy approach for the first failure mode (η'_{FLSF1} = -0.71), which means that the system could be either in safety or in failure mode for this mode. Regarding the second failure mode, the response results in an unavoidable failure state (η'_{FLSF2} = 0.045). The minimum response ensured a safety state for both failure modes (η'_{FLSF1} = -1.54 and η'_{FLSF2} = -1; see Tab. 4). However, the value of the cost function increases as compared with the IRA-DE approach (f = 9.52 to f = 20.95; see Tab. 3). Also, as in the previous test case, the analysis of the *FLSF* bounds in each response indicates that the proposed approach obtained more robust solutions (see Tab. 4).

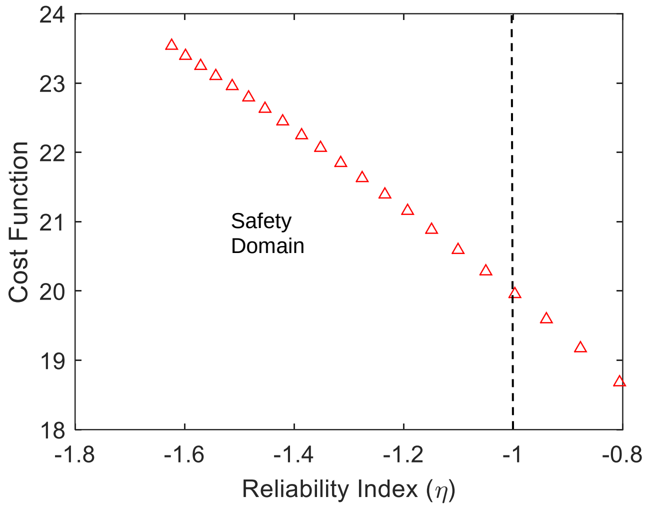

Figure 6. Pareto's curve of the cantilever beam problem.

Table 3: Optimization results associated with the cantilever beam problem.

	f	x_{d1}	x_{d2}
IRA-DE	9.52	2.4488	3.8877
Proposed	20.95	4.7831	4.3804

Table 4: Reliability results associated with the cantilever beam problem.

	$FLSF'_{1L}$	$FLSF'_{1R}$	η'_{FLSF1}	$FLSF'_{2L}$	$FLSF'_{2R}$	η'_{FLSF2}
IRA-DE	-40966	16747	-0.71	2.114	48.74	0.045
Proposed	-45813	-16152	-1.54	-4.205	$1.9x10^{-8}$	-1

5. FINAL REMARKS

In this contribution, a new formulation is proposed for the reliability based design of mechanical systems. The performance of the methodology was evaluated in two test cases and the obtained results were compared with a classical approach. In general, the obtained results demonstrated that the proposed methodology is configured as an interesting alternative to solve reliability problems, but that needs to be evaluated in other test cases. In the presented test cases, the differences observed on the values of the objective functions can be justified due to kind of treatment considered in each approach. It is important to emphasize that the proposed approach can effectively assess reliability without the definition of a probability density function (*PDF*) or a possibility distribution (fuzzy membership functions), requiring only the definition of support intervals. This represents the main advantage of this approach, since most of proposed methods to solve the reliability problem require the definition of type of distribution considered for each random variable.

6. ACKNOWLEDGEMENTS

The authors are thankful to Brazilian Research Agencies FAPEMIG, CAPES and CNPq (INCT-EIE) for the financial support provided in this research effort. The authors are also thankful to companies CERAN, BAESA, ENERCAN, and Foz do Chapecó for financial support through the R&D project *Robust Modeling for the Diagnosis of Defects in Generating Units* (02476-3108/2016).

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