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ANALYSIS OF THE INFLUENCE OF THE CROSS-SECTIONAL GEOMETRY OF REINFORCING FIBERS ON THE EFFECTIVE PROPERTIES OF TRANSVERSELY ISOTROPIC MATERIALS

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Abstract. *This work presents a computational study about composite materials reinforced by aligned fibers, randomly distributed in the matrix. The focus of the analysis is the transverse plane to the direction of the fibers, characterized by an isotropic behavior, where the influence of the geometries of these reinforcing fibers on the effective properties of the plane is evaluated. The problem is addressed in the context of linear elasticity, with the Plane Stress State (PSS) hypothesis and homogenization procedures. Fibers with circular, square and triangular cross section are investigated for two fractions of reinforcement volume: 0.1 and 0.2. A computational algorithm is implemented to generate and analyze multiple models. A statistical analysis is performed to define a Representative Volume Element (RVE) of the material. The effective properties of the plan for each proposed geometry are also determined by statistical analysis on a series of numerical tests of the RVEs. By comparing the results it is shown that, for the reinforcement volume fractions studied, the properties of the matrix are dominant in the effective properties of the transverse plane to the fibers. It is concluded that the influence of the cross-sectional geometry of the reinforcing fibers on the effective elasticity modulus of the transverse plane to them is not significant. However, the effects of small Poisson's coefficient variations may be relevant in certain applications. Also, it should be noted that the maximum stresses developed at the interface between the fibers and the matrix have significantly different values for the different geometries.*

Keywords *composite materials reinforced by fibers; Representative Volume Element; effective properties.*

1. INTRODUCTION

Fiber-reinforced composite materials have high strength and rigidity relative to specific weight when compared to other conventional engineering materials. This characteristic boosted its use, initially, in the aerospace industry. However, with the current development of new production techniques, the use of these materials in different sectors of modern industry is growing. These high-performance materials have been used, for example, in the automotive, sports, prosthetic and biomedical devices, civil construction, and other industries.

The advantages obtained by composite materials are accompanied by large complexity from the point of view of the analysis of the mechanical behavior. Such materials are heterogeneous and commonly have many details incorporated into their microstructure, which requires a high computational cost in numerical simulations. For this reason, homogenization models are used to replace the heterogeneous environment by an equivalent homogeneous, where the effective properties of the macroscopic scale are derived from microscopic scale behavior [Pinho-da-Cruz et al., 2009].

There are several techniques for obtaining effective properties of heterogeneous materials, such as the Homogenization Mathematical Theory, analytical and semi-analytical methods such as Hashin-Shtrikman bands, the self-consistent model, the Mori-Tanaka Method, and many others. According to the studies of Oliveira et al., 2009, the homogenization of composites with this fiber-reinforced matrix configuration leads to a transverse isotropic macroscopic behavior, that is, in the plane of the fibers, the homogenized material can be treated as isotropic . Moreover, for elastic constituent materials, the behavior of the homogenized material in the plane can be completely represented by two independent elastic constants: the modulus of elasticity and the Poisson's coefficient.

The proposed work aims at a homogenization analysis in two dimensions, assuming Plane Stress State and using a computational approach with the Finite Element Method (FEM), to later evaluate the influence of different reinforcement geometries on the effective properties of the material in the transverse plane to the direction of the fibers.

2. MODELING AND NUMERICAL IMPLEMENTATION

The Figure 1 shows schematically the configuration of the material studied. The fibers are parallel aligned in direction x_3 and randomly distributed in the plane x_1, x_2. The problem is addressed in the context of linear elasticity, with the hypothesis of Plane Stress State (PSS) and procedures of homogenization (determination of effective properties). The evaluated plan is x_1, x_2.

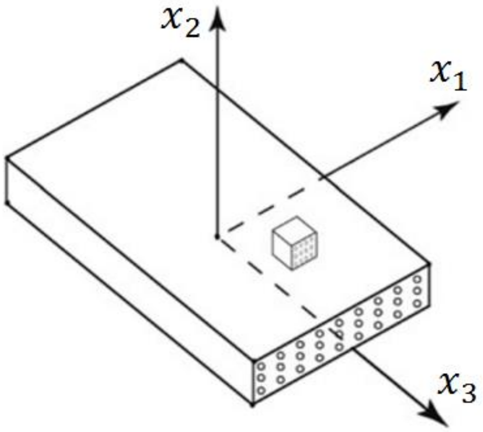

Figure 1. Schematically representation of the composite materials reinforced by aligned fibers, randomly distributed in the matrix.

The constituent materials are defined in a generic way. Assumptions are made that the matrix has no voids and that the fibers are perfectly bonded to the matrix. The properties of the constituent materials are described in Table 1.

Table 1 – Properties of constituent materials.

Properties	Fibers	Matrix
Modulus of elasticity *E*:	1.000 GPa	70 GPa
Poisson's ratio *v*:	0,45	0,33

The influence of microstructural heterogeneities on the stress field of the material can be interpreted at the macroscopic level as fluctuations around an average value. The macroscopic stress tensor is the spatial mean of the stress tensor in a Representative Volume Element (RVE) [Nemat-Nasser, 1999]. The average spatial operator is defined as:

$$\langle \cdot \rangle_\Omega \overset{\text{def}}{=} \frac{1}{|\Omega|} \int_\Omega \cdot \, \mathrm{d}\Omega \, , \tag{1}$$

where Ω denotes the domain of the material. An RVE is characterized as the smallest volume of the microstructure that allows to obtain, with certain precision, a macroscopic property of interest [Buroni, 2006]. It should also contain the correct proportions of the fiber and matrix phases, capable of representing the composite material and its constituents by volume.

In order to determine the macroscopic effective properties of a heterogeneous material one must compute the relationship between means, where the couplings between the micro and macro-scale are implicit.

$$\{\langle \boldsymbol{\varepsilon} \rangle\} = [\boldsymbol{S}^*]\{\langle \boldsymbol{\sigma} \rangle\} \, , \tag{2}$$

being $[\boldsymbol{S}^*]$ the compliance or flexibility matrix, $\boldsymbol{\varepsilon}$ the infinitesimal strain tensor and $\boldsymbol{\sigma}$ the stress tensor.

Using the Mean Field Theory, in the absence of body forces and applying the Green's Theorem, the mean stress field of a solid is taken to the contour. [Nemat-Nasser, 1999]:

$$\langle \sigma_{ij} \rangle_\Omega = \frac{1}{\Omega} \int_\Gamma \sigma_{ik} \, x_j \, n_k \, d\Gamma \, , \tag{3}$$

where σ_{ij} is the stress tensor, Ω is the domain of the material, x is the coordinate in the cartesian system, n is the normal vector at the surface, Γ the contour of the material, and $i, j \; e \; k \in \{1,2,3\}$.

The elastic problem can be solved computationally, applying the Finite Element Method (FEM), to determine the stress and strain microfields. The boundary conditions employed must be the same as those which would produce a uniform stress or strain field on a homogeneous solid [Buroni, 2006], Figure 2.

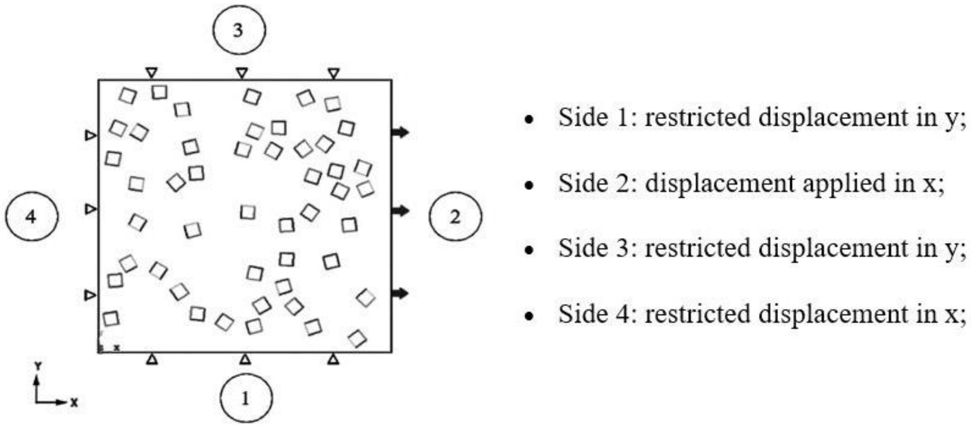

Figure 2. Contour conditions, identifying the sides assigned to the model.

The expressions for the modulus of elasticity and Poisson's coefficient effective for elastic materials transversely isotropic in PSS are developed from the following system of equations [Buroni, 2006]:

$$\begin{Bmatrix} \beta \\ 0 \\ 0 \end{Bmatrix} = \frac{1}{E^*}\begin{bmatrix} 1 & -\nu^* & 0 \\ -\nu^* & 1 & 0 \\ 0 & 0 & 2(1+\nu^*) \end{bmatrix}\begin{Bmatrix} \langle\sigma_{11}\rangle_\Omega \\ \langle\sigma_{22}\rangle_\Omega \\ \langle\sigma_{12}\rangle_\Omega \end{Bmatrix}, \tag{4}$$

$$E^* = \frac{1-\left(\frac{\langle\sigma_{22}\rangle_\Omega}{\langle\sigma_{11}\rangle_\Omega}\right)^2}{\beta}\langle\sigma_{11}\rangle_\Omega\,, \tag{5}$$

$$\nu^* = \frac{\langle\sigma_{22}\rangle_\Omega}{\langle\sigma_{11}\rangle_\Omega}, \tag{6}$$

where β is the strain due to the applied displacement, E^* is the effective elasticity modulus and ν^* is the effective Poisson's coefficient.

A computational algorithm is implemented for automation of the process of generating and analyzing multiple material samples with all necessary variations, such as matrix sizes, fiber quantities and formats, and random distribution of reinforcement geometries.

The Representative Volume Element is determined by repeating the homogenization procedure. Samples are consecutively tested with square geometry matrix, whose side measures 10 mm, 20 mm, 40 mm and 50 mm, respectively. Each sample is computationally tested 30 times and for each simulation a random distribution of the fibers within the matrix is considered.

Once the RVE size is determined, the fiber mesh is generated. Essentially, the RVE geometry is subdivided in the necessary number of random subregions representing the fiber phase, at the specified volume fraction. The FEM mesh is automatically generated taking into account the particular cross section geometry of the fiber being analyzed. The algorithm creates the subregions avoiding overlapping between them, and also forbidding the intersection of any fiber subregion with the RVE's boundary.

The FEM analysis was accomplished using a commercial finite element software (ANSYS) using quadratic two-dimensional plane stress hypothesis. The size of the elements was controlled so that there were 100 elements on each side of the RVE’s boundary.

The stress components obtained in the FEM analysis are numerically integrated, according to Equation (3), and the effective properties are determined by Equations (5) and (6).

To evaluate how the reinforcement geometries affect the effective properties of the material in the transverse plane to the fibers, reinforcements of circular, square and triangular shape are studied. The same value is maintained for the area of each reinforcement, regardless of the geometry, in order to take into account only the influence caused by the different forms of the fibers on the effective properties of the material. Fifty computational tests are performed for each geometry proposed on the RVEs of the two volume fractions studied of 0.1 and 0.2.

3. RESULTS AND DISCUSSION

A series of numerical simulations were performed for several samples of material in order to generate a sufficient amount of data to carry out a statistical analysis and, thus, determine the RVEs and the effective properties produced by the proposed geometries, in the transversal plane to the fibers.

3.1 Definition of RVE size

The size of the representative sample should be one that provides an invariant response (with some margin) for different distributions of heterogeneities [Buroni, 2006; Zohdi, 2007].

The results obtained for the effective elasticity modulus and the effective Poisson's coefficient for several samples with a reinforcement volume fractions of 0.1 are shown in Figures 3 and 4.

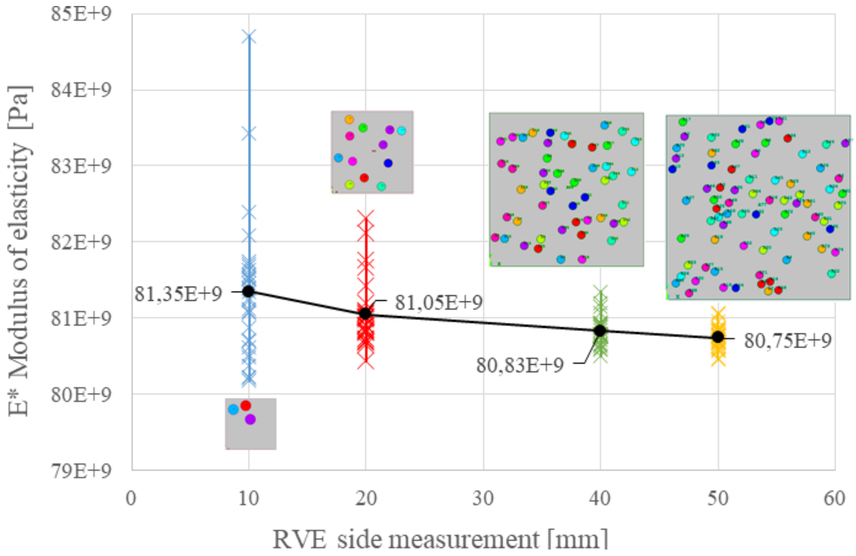

Figure 3. RVE's convergence curve for the effective elasticity modulus and reinforcement volume fraction of 0.1. Samples with different random distributions of the reinforcements and with consecutive increments of matrix size and number of reinforcements.

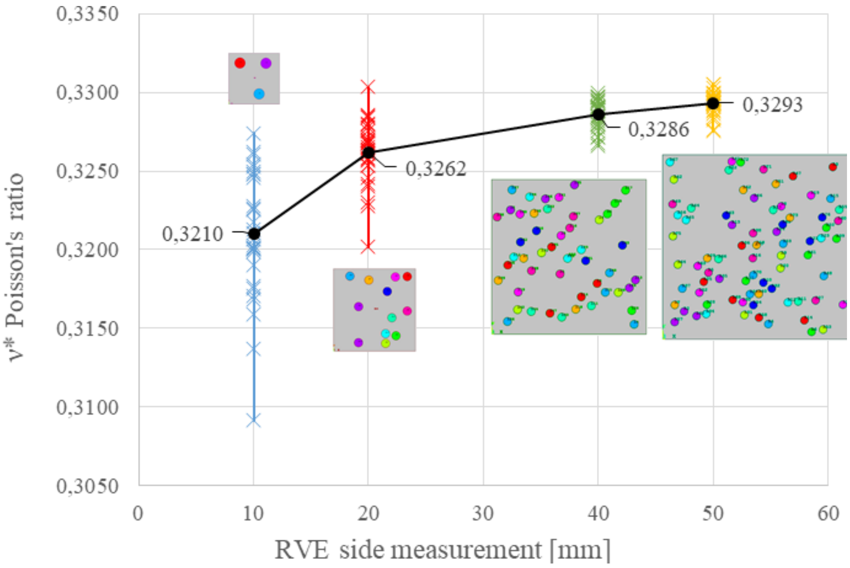

Figure 4. RVE's convergence curve for the effective Poisson's coefficient and reinforcement volume fraction of 0.1. Samples with different random distributions of the reinforcements and with consecutive increments of matrix size and number of reinforcements.

Likewise, the results obtained for the effective elasticity modulus and the effective Poisson's coefficient for several samples with a reinforcement volume fractions of 0.2 are shown in Figures 5 and 6.

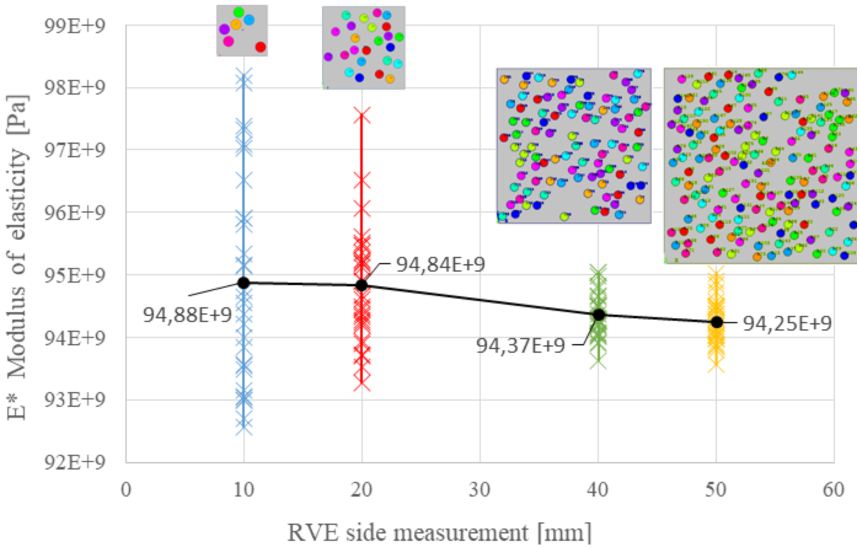

Figure 5. RVE’s convergence curve for the effective elasticity modulus and reinforcement volume fraction of 0.2. Samples with different random distributions of the reinforcements and with consecutive increments of matrix size and number of reinforcements.

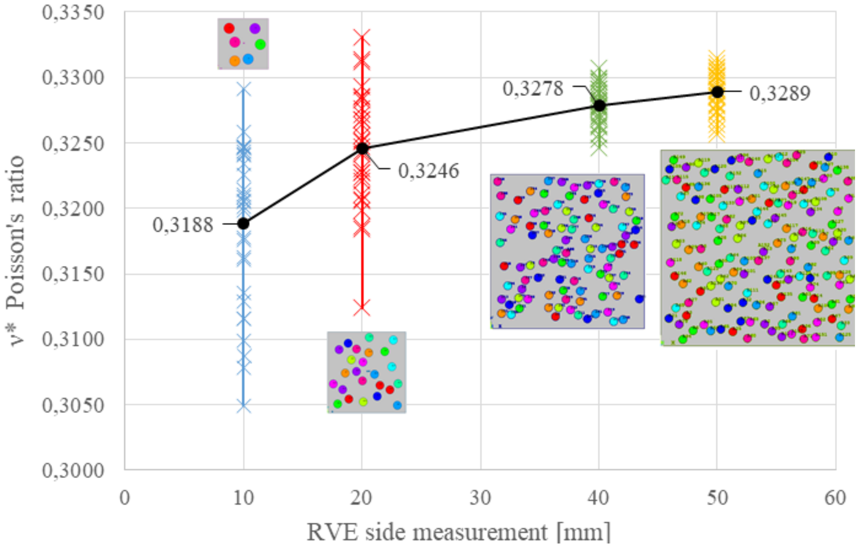

Figure 6. RVE’s convergence curve for the effective Poisson's coefficient and reinforcement volume fraction of 0.2. Samples with different random distributions of the reinforcements and with consecutive increments of matrix size and number of reinforcements.

Then, for the two reinforcement volume fractions studied, the sample with matrix of 40 mm of side length is established as RVE of the material. In the case of fraction 0.1, the percentage of standard deviation in relation to the mean of the results is 0.22% for the effective elasticity modulus and 0.28% for the effective Poisson’s coefficient. For fraction 0.2, the percentage of standard deviation in relation to the mean of the results for the effective elasticity modulus is 0.37% and for the effective Poisson’s coefficient is 0.46%.

3.2 Analysis of the influence of the cross-sectional geometry of the reinforcing fibers

In order to evaluate how the reinforcement geometries affect the effective properties of the material in the transversal plane to the fibers, reinforcements of circular, square and triangular shape were studied. In the case of square and triangular reinforcements, care was taken to orient them in a random way in relation to the plane, seeking to keep valid the hypothesis of isotropy.

For each geometry analyzed, the effective properties obtained by the numerical simulations oscillate around a central value and the results are evaluated statistically. For the three geometries the obtained results history has the same characteristic illustrated in the graphs of Figures 7 and 8.

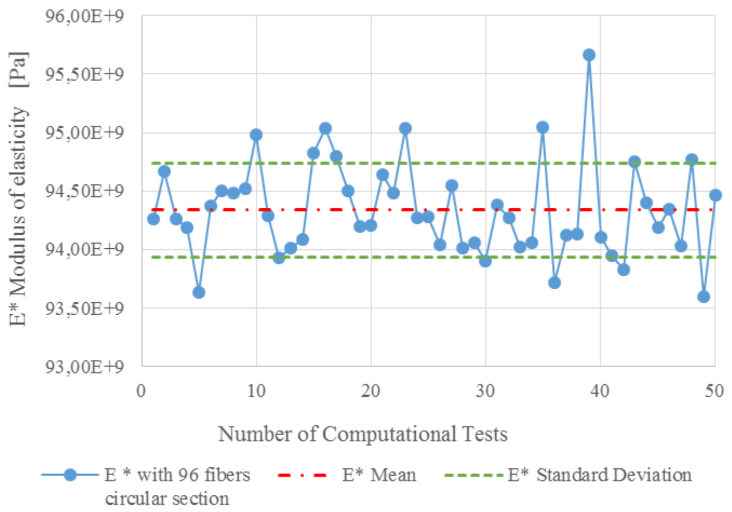

Figure 7. Effective elasticity modulus for RVE with reinforcement volume fraction of 0.2 and circular cross-section fibers. In each assay the fibers are randomly distributed.

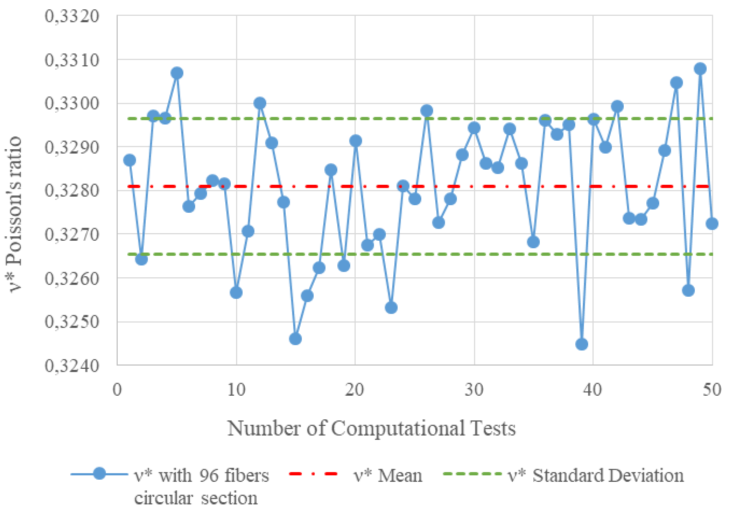

Figure 8. Effective Poisson's coefficient for RVE with reinforcement volume fraction of 0.2 and circular cross-section fibers. In each assay the fibers are randomly distributed.

The Figure 9 shows a comparison between the results obtained for the effective elasticity modulus in the transverse plane to the fibers.

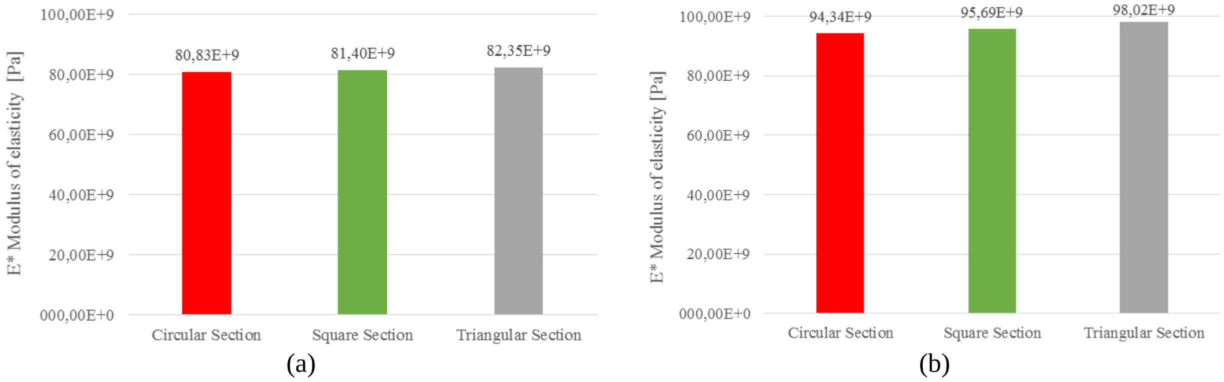

Figure 9. Results for the effective elasticity modulus of the reinforcement forms analyzed. (a) reinforcement volume fraction of 0.1. (b) reinforcement volume fraction of 0.2.

For the two reinforcement volume fractions, the triangular cross-section fiber results in the largest effective elasticity modulus, followed by the square section fiber and finally the circular section fiber. The variation between the largest and smallest modulus of elasticity produced for the reinforcement volume fraction of 0.1 is approximately 1.90% and for the reinforcement volume fraction of 0.2 the difference is about 3.90%.

Similarly, the Figure 10 shows the comparison between the results obtained for the effective Poisson's coefficient. In this case, the values determined by the homogenization methodology adopted are lower than the Poisson's coefficient of the matrix material. This fact may be due to the high stiffness attributed to the fiber material and deserves a more detailed study. For the cases addressed in this work, the largest difference observed between the Poisson's coefficient of the matrix and the effective Poisson's coefficient calculated is less than 2.00%. Regarding the influence of the fiber cross section geometry, for the reinforcement volume fraction of 0.1 the variation between the highest and the lowest effective Poisson's coefficient is approximately 0.63%, produced by the triangular and circular sections, respectively. For the fraction of 0.2 the difference is about 1.30%, generated by the same forms of reinforcement.

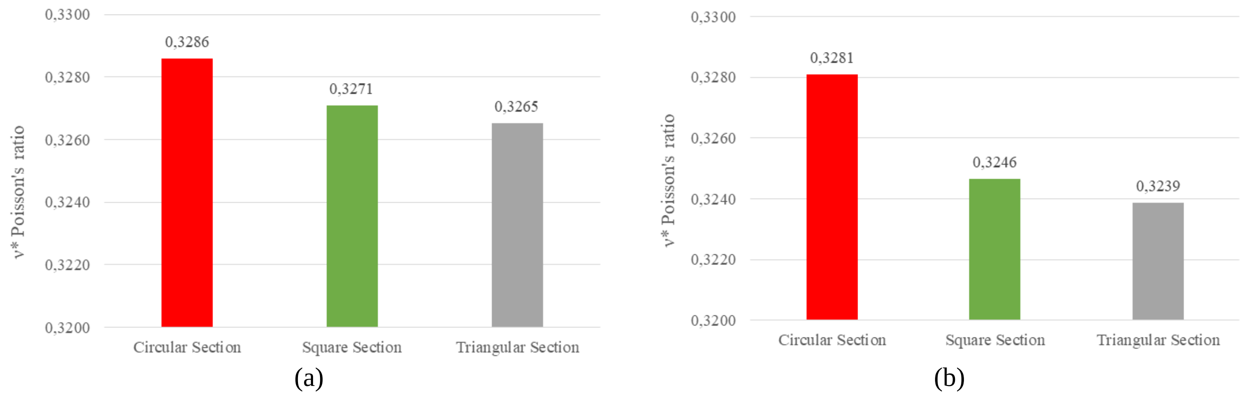

Figure 10. Results for the effective Poisson's coefficient of the reinforced forms analyzed.
(a) reinforcement volume fraction of 0.1. (b) reinforcement volume fraction of 0.2.

The results obtained are consistent with the results of Mortazavi, 2013, where the effects of the geometry (shape) of inclusions on the effective elasticity modulus of isotropic composites of two phases with low filler contents were evaluated by FEM, Mori-Tanaka Method and strong contrast modeling.

4. CONCLUSIONS

By comparing the obtained results it was demonstrated that:

• For the reinforcement volume fractions studied, the properties of the matrix are dominant in the effective properties of the transverse plane to the fibers, since the results were closer to the properties of the matrix than to the properties of the fibers.

• The effective Poisson's coefficient may be lower than the Poisson's coefficient of the matrix material, when the material of the fibers is sufficiently rigid.

• The influence of the cross-sectional geometry of the reinforcing fibers on the effective elasticity modulus of the transverse plane to them is not significant. However, the effects of small Poisson's coefficient variations may be significant in certain applications.

• Although it was not the objective of this work, it should be noted that the maximum stress observed at the interface between the fibers and the matrix showed significant differences. In the fibers of square section the maximum stress was superior to twice the maximum stress in the interface between the fibers of circular section and the matrix. For the fibers of triangular cross-section geometry the maximum stress at the interface was greater than three times the maximum stress between the circular section fibers and the matrix.

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