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ANALYSIS OF UNSTEADY FRICTION MODELS FOR ONE-DIMENSIONAL PIPE FLOW AT THE LIGHT OF THE SECOND LAW OF THERMODYNAMICS

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Abstract. *For decades, unsteady friction models have been developed to predict more accurately the water hammer phenomenon. However, the authors usually do not analyze whether their models do satisfy the Second Law of Thermodynamics (SLT), i.e., whether the rate of energy dissipation caused by the friction is in fact non-negative. This analysis is crucial to, if not validate, to rule out a proposed model. Therefore, this work aims to performing such analysis for two well-known unsteady friction models proposed in the literature. These models describe through different approaches the wall shear stress in unsteady slightly compressible viscous flow in complainant pipes, under laminar or turbulent regimes. For some models in particular, it can be analytically proved that they do unconditionally satisfy the SLT. However, it is not trivial to do so for these two models under investigation due to the complex analytical nature of their equations. Thus, in the lack of analytical tools, numerical simulations seem to be a feasible shortcut. The two selected models were implemented in the context of the Method of Characteristics (MOC) so that the overall rate of energy dissipation produced by the friction could be assessed. Then numerical simulations were carried out for a typical sudden valve closure in a reservoir-pipe-valve system for an isothermal process. The computed results reveal that one tested model produces negative dissipations, indicating that it apparently violates the SLT.*

Keywords: *water hammer, unsteady friction model, energy dissipation, unsteady fluid flow.*

1. INTRODUCTION

In hydraulic engineering, one-dimensional friction models are currently used in computational simulations to predict the pressure and discharge inside the pipeline after introducing changes in the fluid momentum (Fox, 1977; Wylie and Streeter, 1993). Depending on the system, the fluid friction helps to attenuate the potential damages (or even the destruction) in the pipeline due to the water hammer phenomenon. In other words, the better the models are, the more accurate are the prediction of the effects of the water hammer on the pipeline, thus preventing accidents due to the pressure fluctuation. Therefore, those models are an important feature to the design of pipelines conveying liquids and so are receiving an increasing attention in the past years.

In the last century, the first simulations were carried out by employing the quasi-steady friction model whose results did not describe precisely the phenomenon (Fox, 1977; Wylie and Streeter, 1993). Since the predicted values of pressure were overestimated, it has led to an oversized thickness of the pipe wall and, consequently, to higher costs of the final product. Aiming to circumvent such a problem, researchers have been developing new unsteady friction models in order to generate more economical solutions.

Different from the quasi-steady friction model that despises the rate of time effects, the new ones are capable to preview the complementary relaxation effect, due to the acceleration, that reduces faster the pressure gradient in accelerated flows (Zielke, 1968; Vardy and Brown, 2003, Kudzma, 2005; Urbanowicz and Zarzycki, 2012). These models are usually developed using the conservation principles of mass and linear momentum, without referring to another basic and quite important postulate of the Second Law of the Thermodynamics (SLT).

It is known that negative rates of energy dissipation are inadmissible, so an analysis using the SLT is relevant to rule out inconsistent models. For this reason, the objective of this work is to analyze this condition for two well-known unsteady friction models.

2. GOVERNING EQUATIONS

Assuming one-dimensional and isothermal flows with low Mach numbers of Newtonian slightly compressible fluids in elastic compliant pipes of circular cross-section, the governing equations, in Eulerian coordinates, can be written as:

$$\frac{\partial H}{\partial t} + \frac{a^2}{g}\frac{\partial v}{\partial x} = 0\,, \tag{1}$$

$$\frac{\partial v}{\partial t} + g\frac{\partial H}{\partial x} + \frac{4\tau}{\rho D} = 0\,, \tag{2}$$

$$d = \frac{4\tau v}{D} \geq 0\,, \tag{3}$$

for $(x,t) \in (0,L) \times (0,\infty)$, being L the pipe length. In Eqs. (1-3), H, v and τ are the dependent variables and stand for the piezometric head, the mean velocity and the wall shear stress. They are functions of the spatial position x, measured along the pipe centerline and the time instant t. The local acceleration gravity is represented by g.The undisturbed mass density of the fluid, ρ, and the unperturbed internal diameter of the pipe, D, are constants as well as the wave front speed, a, with which disturbances propagate in the medium.

Equations (1) and (2) represent respectively the principles of conservation of mass and linear momentum in the axial direction, whereas Eq. (3) stands for an alternative local version of the SLT known as Clausius-Duhem Inequality. Equation (3) expresses locally the overall rate of energy dissipation per unit volume, d, which in this case is solely restricted to the power expended by the wall shear stress. It establishes a distinction between impossible ($d < 0$) and possible thermodynamic processes ($d \geq 0$), being the equality representative of the reversible processes.

In an attempt to model unsteady friction in one-dimensional flows, several investigators have used an additive decomposition of the wall shear stress:

$$\tau = \tau_s + \tau_u\,, \tag{4}$$

in which τ_u represents the unsteady parcel of the wall shear stress and τ_s the steady one.

The steady parcel of the wall shear stress is usually expressed as follows:

$$\tau_s = \frac{f\rho v|v|}{8}, \tag{5}$$

where $f = f(\mathrm{Re}, \epsilon)$ is the well-known Darcy-Weisbach friction factor, in which Re stands for the Reynolds number ($\mathrm{Re} = |v|D/\nu$, being ν the fluid kinematic viscosity) and ϵ the relative roughness of the pipe. In spite of Eq. (5) holds essentially for steady state flows (both for laminar and turbulent regimes), it has been widely used for unsteady flows in the past, due to the absence of more accurate models at that time.

Up to the advent of the unsteady friction models, the simulation of transient fluid flows had been carried out by considering Eq. (4) with $\tau_u = 0$. For this specific choice, it is easy to see, by taking Eq. (5) in Eq. (3) the SLT is unconditionally satisfied throughout the entire domain $(x,t) \in (0,L) \times (0,\infty)$, inasmuch as $f > 0$.

3. UNSTEADY FRICTION MODELS

In pursuit of more accurate models to properly describe pressure and flow fluctuations in fluid transients in pipes, several distinct unsteady friction models have been proposed in the literature. Among them, we may cite two principal categories: the models based on the convolution integral of the local fluid acceleration and the friction relaxation model, originally proposed by Zielke (1968) and Kucienska (2004), respectively. By adhering to the additive decomposition expressed by Eq. (4) both categories of model have proposed different expressions for τ_u.

The convolution integral model (CIM) uses the following expression to evaluate the unsteady wall shear stress:

$$\tau_u = \frac{4\mu}{D}\int_0^t \frac{\partial v}{\partial t}(x,t')W(t-t',\mathrm{Re})\,dt'\,, \tag{6}$$

where $W(t;\mathrm{Re})$ is a positive weighting function, with distinct forms for laminar and turbulent regimes, which depends on the Reynolds number and t' is the time used in convolution integral. The weighting function is derived by invoking the transient two-dimensional fluid flow in the pipe, when the fluid is suddenly decelerated. Different analytical approximations, $\widetilde{W}(t;\mathrm{Re})$, have been proposed in the literature in the past decades (Zielke, 1968; Thrika, 1975; Zarzycki, 1994 and 2000; Vardy and Brown, 2003; Kudźma, 2005; Urbanowicz and Zarzycki, 2012):

$$W(t;\mathrm{Re}) \cong \widetilde{W}(t;\mathrm{Re}) = \sum_{i=1}^{N} W_i(t;\mathrm{Re}) \,, \tag{7}$$

$$W_i(t;\mathrm{Re}) = m_i \exp\left[-n_i\,(t/t^*)\right] , \tag{8}$$

where m_i and n_i are coefficients and $t^* = D^2/4\nu$.

By defining an intermediate variable:

$$y_i(x,t) = \int_0^t \frac{\partial v}{\partial t}(x,t')W_i(t-t';\mathrm{Re})dt' \,, \tag{9}$$

it results by virtue of Eqs. (6) and (9) that:

$$\tau_u(x,t) \cong \frac{4\mu}{D}\sum_{i=1}^{N} y_i(x,t) \,. \tag{10}$$

By adopting the Schohl (1993) approximation on a diamond grid from $t^{n-2} = (n-2)\Delta t$ to $t^n = t^{n-2} + 2\Delta t$, then a recursive expression can be written in Eq. (10) as follows:

$$y_i(x,t^n) \cong y_i(x,t^{n-2}) \exp\left[-n_i\,(2\Delta t/t^*)\right] + \ m_i \frac{(1-\exp\left[-n_i\,(2\Delta t/t^*)\right])}{n_i(2\Delta t/t^*)} \left[v(x,t^n) - v(x,t^{n-2})\right] , \tag{11}$$

where Δt is the time step.

On the other hand, in the friction relaxation model (FRM), the unsteady wall shear stress is computed according to:

$$\frac{\partial \tau_u}{\partial t} + \frac{\tau_u}{k_\theta} = \frac{\partial \tau_s}{\partial t} + k_T \rho a \frac{\partial v}{\partial t} \,, \tag{12}$$

where k_θ is the relaxation time and k_T is a coefficient. Even though Kucienska (2004) has derived the time-evolution equation for the unsteady wall shear stress given by Eq. (12) by appealing to the extended irreversible thermodynamics, the author has not attempted to check whether the SLT would be satisfied.

To numerically approximate the solution of the model given by Eq. (12) in the context of the Method of Characteristics (MOC), by using a diamond grid, the numerical approximation proposed by Kucienska (2004) based on the wall shear stress at the previous (or last) time step is used. Within this context, τ_u in Eq. (12) is approximated as follows:

$$\tau_u(x,t^n) \cong \tau_u(x,t^{n-2})\exp[-2\Delta t/k_\theta] + k_T\rho a\left[v(x,t^n) - v(x,t^{n-2})\right] , \tag{13}$$

in which $\tau_u(x, t^n = 0) = 0$ and the coefficient k_v is evaluated as:

$$k_T = \begin{cases} k_T^d \,, & \text{if } \mathrm{sign}[v(x,t^n) - v(x,t^{n-2})] < 0 \\ k_T^a \,, & \text{if } \mathrm{sign}[v(x,t^n) - v(x,t^{n-2})] \geq 0 \end{cases} , \tag{14}$$

where k_T^d and k_T^a are constants and refer to the values of the coefficient k_T when the flow is locally decelerating and accelerating, respectively.

As it is expected, it can be seen that in both models $\tau_u = 0$ as the steady state is reached. However, in contrast to the steady state wall shear stress, it is not easy to prove analytically that the SLT is unconditionally satisfied when the unsteady wall shear stress is taken into account. To circumvent such a difficulty, we appeal to numerical simulations of the unsteady friction models to compute the rate of energy dissipation in order to verify whether the inequality given by Eq. (3) is or is not violated, at least for some particular problems.

4. COMPUTATIONAL PROCEDURE

Whatever category of unsteady friction model is considered, the resulting governing equations given by Eqs. (1), (2) and (10) or by Eqs. (1), (2) and (13), along with Eqs. (4) and (5), form a system of partial differential equations of the hyperbolic type. As long as in both models the characteristics curves are straight lines in the $x - t$ plane, the MOC with specified constant time interval proves to be a great computational procedure for obtaining numerical approximations to the initial-and-boundary value problem formed by these hyperbolic partial differential equations. Besides of being easily implemented in a computational code, it allows the imposition of the boundary conditions in an easy and very accurate fashion. It is also capable to cope with sharp gradients or even discontinuities of the dependent variables.

By considering Eq. (2), along with Eqs. (10) and (13), a general equation for the balance of linear momentum equation, contemplating both transient friction models, can be written as:

$$\frac{\partial v}{\partial t} + g\frac{\partial H}{\partial x} + \frac{4\tau_s}{\rho D} + \frac{4\tau_u}{\rho D} = 0\,. \tag{15}$$

With the proper application of the MOC on the diamond grid in Fig.1, the system of partial differential equations given by Eqs. (1) and (15), along with Eq. (5), is transformed into an equivalent system of ordinary differential equations:

$$\frac{dH}{dt} + \frac{a}{g}\frac{dv}{dt} + \frac{fv|v|}{2gD} + \frac{4a\tau_u}{\rho g D} = 0\,, \qquad \text{along} \qquad \frac{dx}{dt} = +a\,, \tag{16}$$

$$\frac{dH}{dt} - \frac{a}{g}\frac{dv}{dt} - \frac{fv|v|}{2gD} - \frac{4a\tau_u}{\rho g D} = 0\,, \qquad \text{along} \qquad \frac{dx}{dt} = -a\,. \tag{17}$$

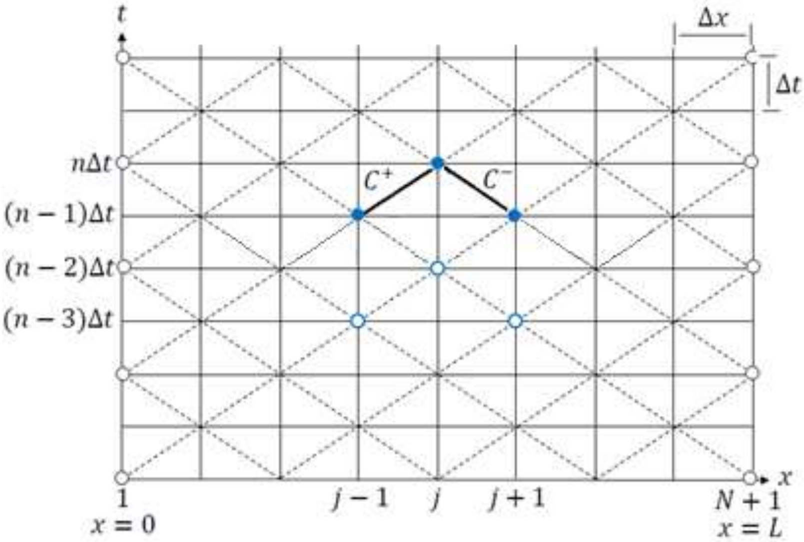

Figure 1: Diamond-shaped staggered grid for the application of the MOC.

By introducing an uniform spatial discretization $\Delta x = L/N$, where N is the number of divisions of the pipe whose length is L, then $N + 1$ nodes are generated with $x_j = (j - 1)\Delta x$, for $j = 1, \ldots, N$+1. By choosing $\Delta t = \Delta x/a$, a uniform (rectangular as well as diamond) grid of characteristics is formed in the $x - t$ plane (see Fig. 1) at different time levels $t^n = n\Delta t$, with $n = 0, 1, 2, 3, \ldots$.. By denoting ξ_j^{n-1} and ξ_j^{n-2} known approximations of $\xi(x = x_j, t = t^{n-1})$ and $\xi(x = x_j, t = t^{n-2})$, respectively, the integration of Eqs. (16) and (17) along their respective characteristic lines, when using the approximation of the Karney and Mc Innis for the quasi-steady wall shear stress, gives rise to the following finite difference equations for the compatibility equations C^- and C^+:

$$C^+\!: \; H_j^n = C_P - B_P v_j^n\,, \qquad \text{along} \qquad \frac{dx}{dt} = +a\,, \tag{18}$$

$$C^-\!: \; H_j^n = C_M + B_M v_j^n\,, \qquad \text{along} \qquad \frac{dx}{dt} = -a\,, \tag{19}$$

where:

$$C_M = H_{j+1}^{n-1} - v_{j+1}^{n-1}\left(\frac{a}{g} + (1-\varepsilon)R_s f_{j+1}^{n-1}\left|v_{j+1}^{n-1}\right|\right) - R_u\left\{\theta\left(L_j^{n-2} - V_j^{n-2}v_j^{n-2}\right) + (1-\theta)[\tau_u]_{j+1}^{n-1}\right\}, \tag{20}$$

$$B_M = \left(\frac{a}{g} + \varepsilon R_s f_{j+1}^{n-1}\left|v_{j+1}^{n-1}\right| - \theta R_u V_j^{n-2}\right), \tag{21}$$

$$C_P = H_{j-1}^{n-1} + v_{j-1}^{n-\kappa}\left(\frac{a}{g} + (1-\varepsilon)R_s f_{j-1}^{n-1}\left|v_{j-1}^{n-1}\right|\right) - R_u\left\{\theta\left(L_j^{n-2} - V_j^{n-2}v_j^{n-2}\right) + (1-\theta)[\tau_u]_{j-1}^{n-1}\right\}, \tag{22}$$

$$B_P = \left(\frac{a}{g} + \varepsilon R_s f_{j-1}^{n-1}\left|v_{j-1}^{n-1}\right| + \theta R_u V_j^{n-2}\right), \tag{23}$$

in which $\theta \in [0,1]$, $\varepsilon \in [0,1]$, $R_s = \Delta x/2gD$ e $R_u = 4\Delta x/\rho g D$.

In Eqs. (20) and (22), $[\tau_u]_{j+1}^{n-1}$ and $[\tau_u]_{j-1}^{n-1}$ are computed based on the following equations for CIM and FRM:

$$[\tau_u]_{j\mp1}^{n-1} \cong L_{j\mp1}^{n-3} + V_{j\mp1}^{n-3}\left[v_{j\mp1}^{n-1} - v_{j\mp1}^{n-3}\right], \tag{24}$$

where:

$$L_{j\mp1}^{n-3} = \begin{cases} \dfrac{4\mu}{D}\displaystyle\sum_{i=1}^{N} Q_i[y_i]_{j\mp1}^{n-3}, & \text{for CIM models} \\ [\tau_u]_{j\mp1}^{n-3}\exp[-2\Delta t/k_\theta], & \text{for FRM models} \end{cases}, \tag{25}$$

$$V_{j\mp1}^{n-3} = \begin{cases} \dfrac{4\mu}{D}Z, & \text{for CIM models} \\ k_T\rho D, & \text{for FRM models} \end{cases}, \tag{26}$$

$$[\tau_u]_j^n \cong \begin{cases} \dfrac{4\mu}{D}\displaystyle\sum_{i=1}^{N} Q_i[y_i]_j^{n-2} + \dfrac{4\mu}{D}Z\left[v_j^n - v_j^{n-2}\right], & \text{for CIM models} \\ [\tau_u]_j^{n-2}\exp[-2\Delta t/k_\theta] + k_T\rho D\left[v_j^n - v_j^{n-2}\right], & \text{for FRM models} \end{cases}, \tag{27}$$

in which $[y_i]_j^n$, Z, Q_i and k_T are computed as:

$$[y_i]_j^n \cong Q_i[y_i]_j^{n-2} + m_i\frac{(1-Q_i)}{n_i(2\Delta t/t^*)}\left[v_j^n - v_j^{n-2}\right], \tag{28}$$

$$Z = \sum_{i=1}^{N} m_i\frac{(1-Q_i)}{n_i(2\Delta t/t^*)}, \tag{29}$$

$$Q_i = \exp[-n_i(2\Delta t/t^*)], \tag{30}$$

$$k_T = \begin{cases} k_T^d, \text{if } \operatorname{sign}\left[v_{j\mp1}^{n-1} - v_{j\mp1}^{n-3}\right] < 0 \\ k_T^a, \text{if } \operatorname{sign}\left[v_{j\mp1}^{n-1} - v_{j\mp1}^{n-3}\right] \geq 0 \end{cases}. \tag{31}$$

By solving the linear system defined by Eqs. (18) and (19) for H_j^n and v_j^n, we obtain to the interior points:

$$v_j^n = \frac{C_P - C_M}{B_P + B_M}, \qquad \text{for } 2 \leq j \leq N, \tag{32}$$

$$H_j^n = \frac{B_M C_P + B_P C_M}{B_P + B_M}, \qquad \text{for } 2 \leq j \leq N. \tag{33}$$

At the boundaries, either the positive or the negative characteristic line is used along one adequate boundary condition to determine ξ_1^{n+1} and $\xi_{N=1}^{n+1}$ for $\xi \in \{H, v, \tau_u\}$.

5. RESULTS AND DISCUSSION

To access the rate of energy dissipation associated with the unsteady friction models described before, we have considered a typical reservoir-pipe-valve installation in which fast transients are generated by valve slam. In such a system, a liquid flow in steady state with an initial velocity $v(x, t = 0) = 5$ m/s from a constant pressure reservoir (where $p(x = 0, t \geq 0) = 60$ bar) at upstream towards a valve positioned at downstream. The pipe is horizontal, has an internal diameter $D = 20$ mm and a total length $L = 50$ m. The liquid has a kinematic viscosity $\nu = 1.0$ cSt at the flowing temperature of 293 K, rendering an initial Reynolds number $\text{Re} \cong 10^5$. The wave front speed in the medium was $a = 1347$ m/s. As a result of an instantaneous valve closure, $v(x = L, t > 0) = 0$ and a transient is generated.

The computational procedure is then used to compute the rate of energy dissipation per unit volume d, in addition to the pressure, velocity, wall shear stress at every point $(x = x_j, t = t^n)$ of the grid of characteristics, for $j = 1, \dots, N$+1 and $n = 0,1,2, \dots$, with $x_1 = 0$ and $x_{N+1} = L$. For the FRM, it has been assumed that $k_\theta = 0.0014$ s and $k_T^a = 4.0 \times 10^{-4}$ if $\partial V/\partial t \geq 0$ and $k_T^d = 2.0 \times 10^{-2}$ otherwise (Kucienska, 2004). For the CIM, the weighting function in Eqs. (7) and (8) has been considered, using the coefficients of Urbanowicz and Zarzycki (2012) in Table 1:

Table 1: Coefficients of the Urbanowicz e Zarzycki (2012).

i	m_i	n_i	i	m_i	n_i
1	0.06054	0.000671	13	55.603	7226.1
2	0.09698	0.00838	14	97.138	22686.2
3	0.17971	0.04504	15	175.825	72226.7
4	0.31240	0.1790	16	307.176	226796
5	0.56562	0.6457	17	551.342	720015
6	0.98348	2.159	18	954.362	2234661
7	1.77243	7.088	19	1727.71	7050737
8	3.08626	22.563	20	3171.2	22553627
9	5.57348	72.215	21	5899.4	74840660
10	9.7254	227.12	22	11013	253286747
11	17.591	723.19	23	19923	856109205
12	30.723	2270.23	24	37929	2893640000

Before assessing these preliminary results, several numerical simulations were run to find out an adequate time step in order to ensure convergence in both models. This time step was of the order of $\Delta t \cong 0.007$ ms. The rate of energy dissipation as a function of the time at the valve ($x = L$) is then plotted in Fig. 1 for the convolution integral model and for the friction relaxation model. If the predictions of the pressure variation as a function of the time at the valve were compared with experimental data, one would see that both models produce very accurate results. However, when we observe the responses in Fig. 2, we can see that the FRM predicts some discrete and highly concentrated spikes of negative rate of energy dissipations at some time instants, providing some evidence that it can be violating the SLT.

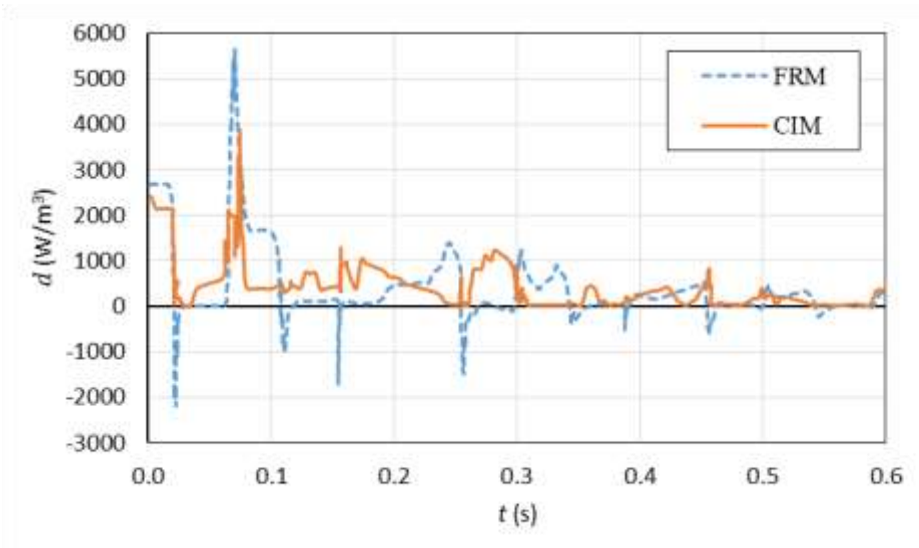

Figure 2 – Rate of energy dissipation per unit volume as a function of the time at $x = L$ for the two models.

The fact that the CIM has passed this test, while the FRM has apparently failed, may have a plausible justification. In spite of both methods have been derived via a logical and physical based framework, different from the CIM, the FRM had their constitutive coefficients adjusted empirically. This aspect might per se explain the observed results, at least to a certain extent.

6. CONCLUSIONS

The Second Law of Thermodynamics (SLT) plays an important role, and so should be invoked, when developing constitutive models. According to it, a coherent constitutive model when taken into account in a particular problem can not produce negative rate of energy dissipations, regardless of the initial and boundary conditions. In this paper, we have investigated by means of numerical simulations for a particular problem of fluid transients generated by valve slam in a reservoir-pipe-valve installation, two distinct unsteady-state friction models: the convolution integral model (CIM) and the friction relaxation model (FRM). The obtained results revealed that the former presented non-negative rate of dissipation. However, it has been observed that the FRM fails to satisfy such condition, at least for some discrete and highly concentrated time instants. These observations suggest that the model may be violating the SLT. Although we have shown that the CIM does not violate the SLT, we cannot also claim that this model will unconditionally satisfy it. Either an analytical proof is presented, or many other problems should be numerically simulated to definitively validate this statement. Other tests and models are currently under investigation and their results are going to be available soon in order to provide a better and more precise panorama about this question.

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