24th COBEM - 2017

ABCM Associação Brasileira de Engenharia e Ciências Mecânicas

24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-1513
VIBRATION CONTROL USING MULTIPLE PIEZOELECTRIC ABSORBERS

Jorge A. B. Gripp
Domingos A. Rade
Instituto Tecnológico de Aeronáutica, Department of Mechanical Engineering. São José dos Campos, SP, Brazil
gripp@ita.br , rade@ita.br

Abstract. *This paper discusses the modelling of piezoelectric absorbers using multiple transducers. In this technique, known as piezoelectric shunt damping, each piezoelectric transducer converts mechanical vibration energy into electrical energy, which is dissipated in an electrical circuit. Typically, a resonant circuit formed by a resistance and an inductance is used for that. Besides the circuit design, the geometry and location of the piezoelectric transducer is a fundamental factor of the piezoelectric absorber design. The location and shape of the piezoelectric transducers can contribute to improve or cancel the electrical charges produced during the transducer deflection. Therefore, the model of the piezoelectric absorbers is derived in this paper.*

Keywords: *Piezoelectric shunt damping, vibration control, smart materials, finite element method, damping.*

1. INTRODUCTION

Due to operational or environmental forces, mechanical structures are often subjected to vibration effects, which can cause mechanical failure, limit the precision of mechanical equipment, and create noise.

Over decades, a variety of active, passive and hybrid techniques intended for tackling problems of noise and vibration control of structures have been developed and acquired considerable degree of maturity (Vér and Beranek, 2005). However, current trend towards lightweight structures, as a direct consequence of the ever increasing demand for more efficient and environmentally friendly structures, leads to new challenges on vibration control technology, most frequently under constraints imposed on energy consumption, added weight, and installation and operation costs. In contrast, the traditional noise and vibration control techniques, such as the addition of viscoelastic materials with high loss factor and dynamic vibration absorbers increase the structural mass in a significant way.

As an alternative to passive damping materials, piezoelectric transducers together with appropriate electrical circuits have been proposed as vibration dissipation devices. Piezoelectricity is a property some materials have of converting mechanical energy into electrical energy and vice versa. Significant piezoelectric effect can be observed in ceramics such as lead zirconate titanate (PZT) or barium titanate ($BaTiO_3$) and polymers such as polyvinylidene fluoride (PVDF). Conventional vibration control using piezoelectric transducers is implemented mostly in an active arrangement: an actuator generates forces to annul the vibrations based on a signal from an accelerometer, velocity or strain sensor (Fig. 1c). Hybrid systems combining viscoelastic (Fig. 1(a)) and active control (Fig. 1(c)) were also proposed (Trindade and Benjeddou, 2002). The active systems require charge amplifiers (G), filters (LP), and converters (A/D and D/A) to transform electrical charges into voltage signals that are processed in the control loop. Although the effectiveness of this strategy has been confirmed in many studies (Aridogan and Basdogan, 2005; Preumont and Seto, 2008), it is recognized the existence of practical drawbacks, some of them inherent to any active control approach, such as energy consumption, stability issues, cost of necessary equipment (especially costly and massive power amplifiers).

As a substitute for this large electronic instrumentation, this paper presents a promising method named piezoelectric shunt damping (Fig. 1(d)) (Niederberger, 2005; Qureshi et al., 2014; Benjeddou, 2000; Erturk and Inman, 2008; Moheimani, 2003; Sales *et al*. 2013, Gripp *et al*., 2015). This technique uses a piezoelectric transducer to transform mechanical vibration energy into electrical energy, which is dissipated as heat in an electrical circuit. This article shows the piezoelectric absorber modelling as well as the integration of several transducers to optimize the dissipated vibration energy.

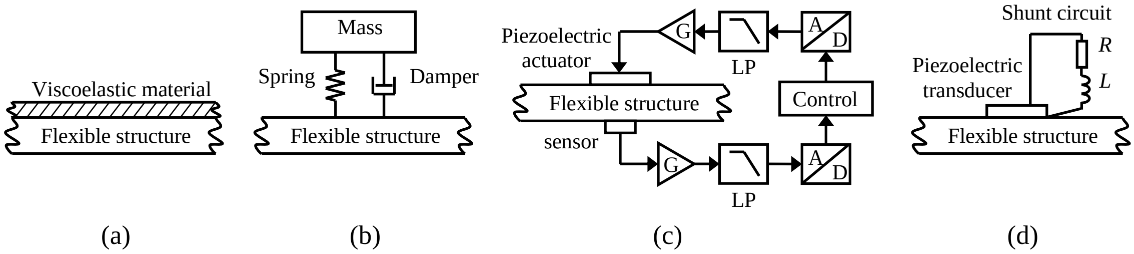


Figure 1. Some vibration control techniques. (a) Viscoelastic materials with high loss factor, (b) dynamic vibration absorber, (c) active vibration control, (d) piezoelectric shunt damping.

2. PIEZOELECTRIC SHUNT CIRCUITS

In the last 20 years, various types of electronic circuits have been used in association with piezoelectric transducers for the purpose of vibration attenuation. The key problem is to design simple circuits that efficiently dampens the structural motion. The shunt circuit is said to be autonomous (or passive) if it does not require external power supply (e.g. R-shunt) and semi-passive if the circuit operation needs external power supply but does not provide any power to the mechanical structure (e.g. SSDI). In both cases, the system stability is guaranteed. On the other hand, semi-active shunts can provide power to the system via, for instance, a negative capacitance circuit, which can become unstable depending on the component values.

A resistor connected to the piezoelectric transducer (Fig. 2(a)) provides the simplest means of achieving energy dissipation and thus vibration damping (Hagood and von Flotow,1991; Uchino and Ishii, 1988). Despite their low cost and simplicity, R-shunts are rarely used due to their low damping effectiveness.

Effective means of achieving vibration attenuation are circuits formed by the association of a resistor and an inductor, as shown in Fig. 2(b). These circuits are also known as resonant shunts, since, when associate to a piezoelectric transducer, which is modeled as a capacitor, for a typical second order system whose behavior is analogous to that of single-degree-of-freedom mechanical system and, as such, presents a resonant behavior near its natural frequency. The RL shunt circuit was first introduced by Forward (1979) to reduce mechanical vibrations in optical systems. Operating analogously to a viscously damped dynamical vibration absorber (Fig. 1(b)) described by Den Hartog (1985), given the inherent capacitance of the piezoelectric transducer, the two other parameters of the resonant circuit can be tuned to the natural frequency of the mechanical structure for achieving minimum vibration amplitudes in the vicinity of that frequency (Hagood and von Flotow, 1991).

While single-mode shunts can only be tuned to one frequency, multi-mode shunts to dampen several structural modes at the same time. Figure 2(c) shows a multimode circuit known as current-flowing (Behrens *et al.*, 2003). In this technique, each branch i has a series LC network tuned to approximate a short circuit condition at the target resonance frequency ω_i and an open circuit at other natural frequencies values, such as $\omega_i = 1/\sqrt{\hat{L}_i C_i}$. In this case, each branch is approximately independent from the others, and L_i and R_i are adjusted in the same way as for single-mode RL shunts.

Another method conceived to cancel out the transducer capacitive impedance consists in using a negative capacitance circuit (Manzoni *et al.*, 2012; Park and Park, 2003). Similarly to the RL shunt circuit, a negative capacitance has a positive imaginary impedance. Obviously, negative capacitances cannot be found in physical capacitors. However, they can be emulated using active circuits, containing, as constitutive elements, an operational amplifier, a capacitor, and resistors as shown in Fig. 2(d). One advantage of the negative capacitive circuit is that it has the ability to suppress structural vibrations in large frequency bands (Park and Baz, 2005).

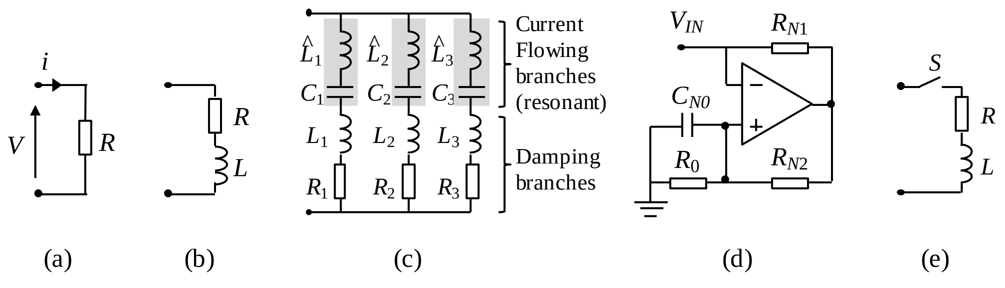


Figure 2. (a) R-shunt. (b) RL-shunt. (c) Multi-mode RL-shunt. d) Negative capacitance $C_N = -C_{N0}\, R_{N2}/R_{N1}$. (e) SSDI.

Ideally, the performance of the shunt circuit requires include robustness against system parameter variations and stability. In particular, resonant shunts are typically used to dampen structural vibration in narrow frequency bands, but their performance drops severely with a mistuning of the electrical resonance frequency (Andreaus and Porfiri, 2007). This drawback can be circumvented by adopting procedures enabling online tuning of the inductance L, as proposed by Niederberger *et al*. (2004) and Gripp *et al*. (2015).

Nonlinear circuits can also be used for vibration damping, by inverting repeatedly the piezoelectric voltage synchronously with the structure motion (Qureshi *et al.*,2014; Neubauer *et al*., 2012; Kelley and Kauman, 2017). Since the shunt circuit is turned on and off synchronously with the structure vibration period, this technique can adapt to different excitation frequencies. In SSDI (synchronized switched damping on an inductance), a small inductance is used to quickly invert the electrical charge at the electrodes (Fig. 2(e)). For this purpose, a switch is closed for precisely one half of the electrical period time, after which it is immediately opened so that the charge is kept on the electrodes. The switch is triggered by the vibration of the structure itself, so that the inversion occurs at the moments of maximum deformation of the piezoelectric transducers. It then generates rectangular-shaped voltage and force signals which counteract the vibration velocity and hence dissipates energy.

3. PIEZOELECTRIC TRANDUCER GEOMETRY AND POSITIONING

Besides the circuit design, the geometry and location of the piezoelectric transducer are fundamental factors to be considered in the design of vibration control systems based on shunt circuits. For illustration, consider the numerical simulation of a rectangular PZT transducer bonded on the surface of a fully clamped aluminum plate shown in Figure 3(a). Figure 3(b) depicts the transverse displacement fields corresponding to the first eight vibration mode shapes. It can be seen that some of these modes present, in the area covered by the piezoelectric transducer, regions with opposite curvature signals. Considering that the signals of the electric charges are determined by the signals of the strains which, on their turn, are related to the signals of bending curvatures, this implies that the electric charges will be fully or partially cancelled out. In such cases, the shunt circuits will have little or no effectiveness for the dampening of vibration amplitudes pertaining to the modes in question. This fact is confirmed by the frequency response function shown in Fig. 3(c), which shows the amplitudes of the voltage response due to a transverse force applied to the plate. It can be clearly seen that resonance amplitudes, indicating the participation of the vibration modes in the response, do not appear for all the modes within the frequency band of interest.

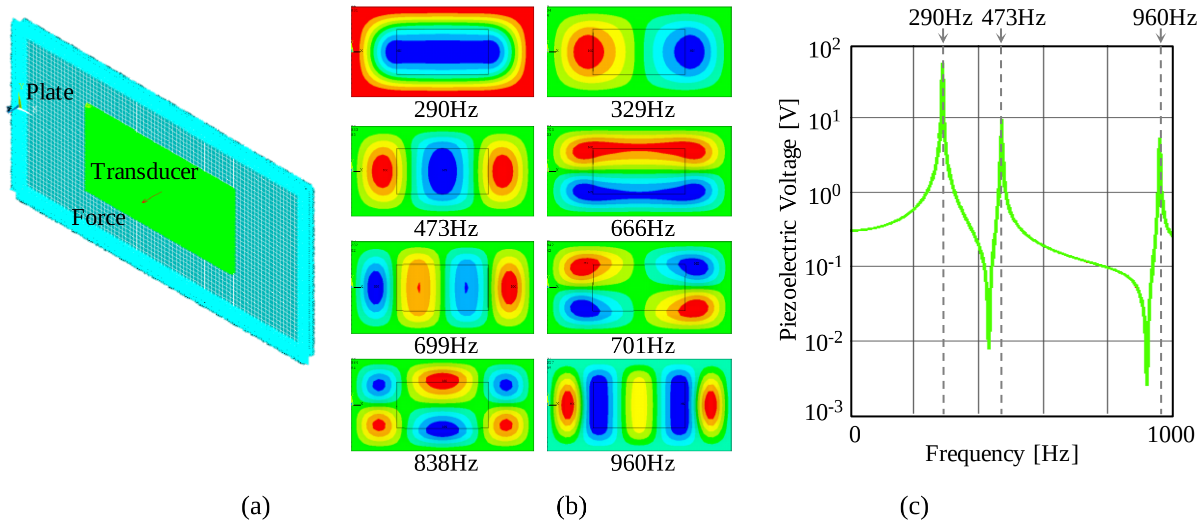


Figure 3. Simulation of a plate (400x200x1.3 mm) containing an open-circuited piezoelectric transducer (200x100x0.5 mm). (b) Vibration modes of the plate. (c) Transducer's voltage (V_P) due to a harmonic unitary force.

Hence, by a proper choice of their location, it is possible to tailor the coupling properties of the transducer and eliminate the contribution of certain modes in the piezoelectric response. In this manner, Ducarne *et al*. (2012) found the optimum transducer location for each mode shape of a cantilever beam.

Moreover, it is possible to design piezoelectric transducers such that they respond to only one vibration mode; this is the concept of modal transducer. Exploring the orthogonality properties of the eigenfunctions, Lee and Moon (1990) elegantly designed modal transducers for beam-like structures by tailoring the width of the electrode according the curvature of the mode of interest, as shown in Fig. 4(a). Similarly, Vasques (2012) shaped piezoelectric modal transducers in beam structures to improve the damping efficiency of a single-mode resonant shunt circuit. Unfortunately, as mentioned by Donoso and Sigmund (2016), this approach cannot be readily extended to two-dimensional structures as illustrated in Fig. 4(b), where the modal transducer is shown to be sensitive to various vibration modes (Jian and Friswell, 2007). This stimulated the research towards the use of topological optimization using arrays of discrete transducers (Pagani and Trindade, 2009) or continuous distributed transducers (Ruiz *et al*., 2016) (Fig 4 (e)). It is important to notice that discrete transducer arrays suffer from spatial aliasing when the wavelengths of structural modes are comparable to the spacing between transducers (Preumont *et al*., 2003).

Kim *et al.* (2001) distributed PVDF transducers in a regular mesh and optimized the electrode pattern, the lamination angle, and the relative poling direction of each segment using genetic algorithm. Sun *et al.* (2002) proposed modal transducers by modulating the thickness distribution of the piezoelectric layer. As illustrated in Fig. 14(c), Casadei *et al.* (2012) used an array of transducers for vibration control of a plate using resonant and negative capacitance shunts. Similarly, Tateo *et al*. (2014) controlled the vibrations of a flexible plate using an array of negative capacitance shunted piezoelectric patches. Preumont *et al*. (2003) introduced an array of variable size transducers (Fig. 14 (d)). Pulskamp *et al.* (2012) developed electrode-shaping techniques to detect modes in plates, discs, rings, and beams. Moreover, several researchers have successfully applied topology optimization techniques to the design of piezoelectric smart structures (Silva and Kikuchi, 1999; Zhang and Kang, 2014).

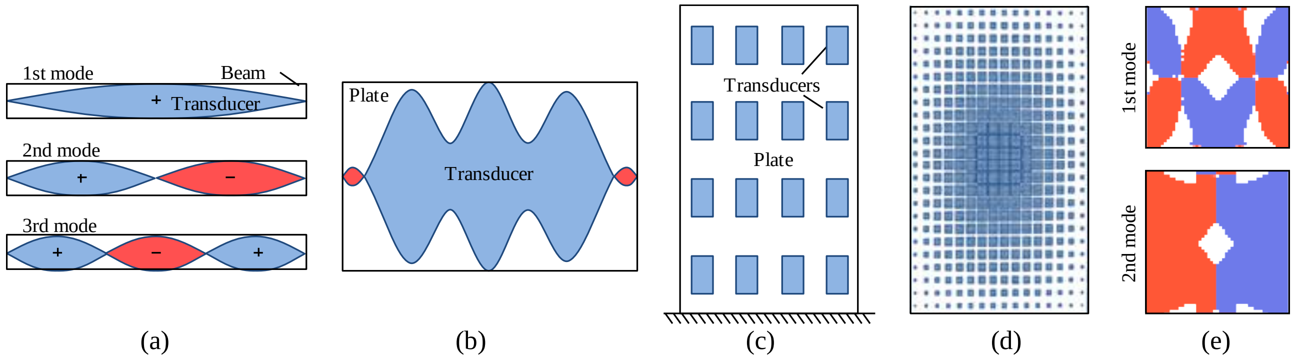


Figure 4. Piezoelectric transducers bonded on the host structure. (a) Modal filters for a beam. (b) Modal filter for a plate (c) Array of transducers. (d) Array of variable size transducers. (e) Topological optimization of modal filters.

4. MODELING OF STRUCTURES CONTAINING PIEZOELECTRIC SHUNTED TRANSDUCERS

In this section, the concept of piezoelectric shunt damping is applied to continuous systems. It is possible, for simple structural elements (rods, beams and plates), to derive analytical models accounting for the electromechanical coupling and the presence of shunt circuits (Preumont, 2006). Alternatively, semi-analytical models, such as the Assumed Modes, or Rayleigh-Ritz method, can be used with advantages in a number of cases (Craig Jr and Kurdila, 2006). However, for more complex structures, including those of industrial interest, finite element modeling is considered as the most efficient modeling strategy, which has been adopted in many research studies (Benjeddou, 2000; De Marqui Jr *et al*., 2009). In addition, many commercial FE codes currently provide all the necessary means for building high fidelity models of structures containing shunted piezoelectric transducers.

The remainder of this Section aims at providing the understanding of the theoretical foundations behind FE modeling of such structures. For an elastic structure having an arbitrary number of piezoelectric elements, either bonded to its surface or embedded in its volume, connected to shunt circuits whose individual electrical impedances are indicated by Z_i as depicted in Fig. 1(d), under the assumption that electromagnetic effects can be neglected, the Extended Hamilton's Principle is expressed under the form (Hagood and Chung, 1990):

$$\int_{t_1}^{t_2}\left[\delta\left(T-U+U_e\right)+\delta W_{ext}\right]dt=0 \tag{1}$$

where

$$T=\int_{V_S}\frac{1}{2}\rho_s\dot{\mathbf{u}}^T\dot{\mathbf{u}}\,dV_s+\int_{Vp}\frac{1}{2}\rho_p\dot{\mathbf{u}}^T\dot{\mathbf{u}}\,dV_p\,,\;U=\int_{V_S}\frac{1}{2}\mathbf{S}^T\mathbf{T}\,dV_s+\int_{V_p}\frac{1}{2}\mathbf{S}^T\mathbf{T}\,dV_p\,,\;U_e=\int_{Vp}\frac{1}{2}\mathbf{E}^T\mathbf{D}\,dV_p \tag{2}$$

$$\delta W_{ext}=\int_{V_s}\delta\mathbf{u}^T\overline{\mathbf{F}}\,dV_s+\int_{V_p}\delta\mathbf{u}^T\overline{\mathbf{F}}\,dV_p+\int_{S_s}\delta\mathbf{u}^T\overline{\mathbf{T}}\,dS_s+\int_{S_p}\delta\mathbf{u}^T\overline{\mathbf{T}}\,dS_p-\int_{S_p}\delta\phi\overline{\sigma}\,dS_p+\sum_i\delta\mathbf{u}_i^T\bar{f}_i-\sum_i\delta\phi_i\bar{q}_i$$

In the equations above, T is the kinetic energy, U is the potential energy, U_e is the electric energy, and δW_{ext} is the virtual work of mechanical and electrical actions. Subscript s and p denote quantities pertaining to the structure and the piezoelectric elements, respectively, and ^{T} indicates matrix transpose. Additionally, **u** is the displacement vector (m), **S** is the strain vector (m/m), **T** is the stress vector (N/m^2), **E** is the electrical field vector (V/m), **D** is the electrical displacement vector (C/m^2), ϕ is the electrical potential, and ρ is the material density (kg/m^3). Integrations are performed either on the structure volume, indicated by V_S, or on the volume of the piezoelectric elements, indicated by V_P. In addition, overbars indicated prescribed quantities: body forces $\bar{\mathbf{F}}$, surface forces $\bar{\mathbf{T}}$, surface electric charges $\bar{\sigma}$, concentrated forces $\bar{f}_i$, concentrated electric charges $\bar{q}_i$.

The constitutive equations of the piezoelectric tranducers are used in the following multidimensional version, expressed as (IEEE, 1988):

$$\begin{aligned} \mathbf{T} &= \mathbf{c}^E \mathbf{S} - \mathbf{e}^T \mathbf{E} \\ \mathbf{D} &= \mathbf{e}\ \mathbf{S} + \boldsymbol{\varepsilon}^S \mathbf{E} \end{aligned} \tag{3}$$

where $\mathbf{c}^E$ is the stiffness matrix under constant electrical field (N/m^2), **e** is the matrix of piezoelectric coefficients (m/V), and $\boldsymbol{\varepsilon}^S$ is the dielectric permittivity matrix under constant strain (C/(V.m)). Assuming that the volumes of the elastic structure and the piezoelectric elements are divided in a given number of elements, each one containing a certain number of nodes, the displacement and electric potential fields are interpolated within each element as follows:

$$\begin{aligned} \mathbf{u}(\mathbf{x},t) &= \boldsymbol{\Psi}_r(\mathbf{x})\ \mathbf{r}^e(t) \\ \phi(\mathbf{x},t) &= \boldsymbol{\Psi}_v(\mathbf{x})\ \mathbf{v}^e(t) \end{aligned} \tag{4}$$

where x is the vector of space coordinates, $\boldsymbol{\Psi}_r(\mathbf{x})$ and $\boldsymbol{\Psi}_v(\mathbf{x})$ are the matrices of interpolation functions for displacement and electric potentials, respectively, and $\mathbf{r}^e(t)$ and $\mathbf{v}^e(t)$ are the vectors formed from values of displacement and electrical potentials at the element nodes. Super-scripts e indicate quantities defined at element level. Strain-displacement and electrical field-electric potential relations are introduced as follows:

$$\begin{aligned} \mathbf{S}(\mathbf{x},t) &= \mathbf{D}_u \mathbf{u}(\mathbf{x},t) = \mathbf{D}_u \boldsymbol{\Psi}_r(\mathbf{x}) \mathbf{r}^e(t) = \mathbf{B}_r(\mathbf{x}) \mathbf{r}^e(t) \\ \mathbf{E}(\mathbf{x},t) &= \mathbf{D}_\phi \phi(\mathbf{x},t) = \mathbf{D}_\phi \boldsymbol{\Psi}_v(\mathbf{x}) \mathbf{v}^e(t) = \mathbf{B}_v(\mathbf{x}) \mathbf{v}^e(t) \end{aligned} \tag{5}$$

where $\mathbf{D}_u$ and $\mathbf{D}_\phi$ are matrices constituted of differential operators. Associating equations (1) to (5), after performing integration by parts of the terms involving $\dot{\mathbf{r}}$ and considering that virtual variations $\delta\mathbf{r}^e(t)$ and $\delta\mathbf{v}^e(t)$ must be arbitrary and independent from each other, one obtains the electromechanical coupled equations of motion at element level:

$$\begin{aligned} \mathbf{M}_{rr}^e \ddot{\mathbf{r}}(t) + \mathbf{K}_{rr}^e \mathbf{r}^e(t) - \mathbf{K}_{rv}^e \mathbf{v}^e(t) &= \bar{\mathbf{f}}^e(t) \\ {\mathbf{K}_{rv}^e}^T \mathbf{r}^e(t) + \mathbf{K}_{vv}^e \mathbf{v}^e(t) &= \bar{\mathbf{q}}^e(t) \end{aligned} \tag{6}$$

where

$$\mathbf{M}_{rr}^e = \int_{V_s} \rho_s \boldsymbol{\Psi}_r^T \boldsymbol{\Psi}_r dV_s + \int_{V_P} \rho_p \boldsymbol{\Psi}_r^T \boldsymbol{\Psi}_r dV_p$$

$$\mathbf{K}_{rr}^e = \int_{V_s} \mathbf{B}_r^T \mathbf{c}_s \mathbf{B}_r dV_s + \int_{V_P} \mathbf{B}_r^T \mathbf{c}^E \mathbf{B}_r dV_p\ ,\ \mathbf{K}_{uv}^e = \int_{V_P} \mathbf{B}_r^T \mathbf{e}^T \mathbf{B}_v dV_p\ ,\ \mathbf{K}_{vv}^e = \int_{V_P} \mathbf{B}_v^T \boldsymbol{\varepsilon}^S \mathbf{B}_v dV_p\ ,$$

$$\bar{\mathbf{f}}^e = \int_{V_S} \boldsymbol{\Psi}_r^T \bar{\mathbf{F}} dV_s + \int_{V_P} \boldsymbol{\Psi}_r^T \bar{\mathbf{F}} dV_p + \int_{S_S} \boldsymbol{\Psi}_r^T \bar{\mathbf{T}} dS_s + \int_{S_P} \boldsymbol{\Psi}_r^T \bar{\mathbf{T}} dS_p + \sum_i \boldsymbol{\Psi}_r(\mathbf{x}_i) \bar{f}_i\ ,\ \bar{\mathbf{q}}^e = \int_{S_P} \boldsymbol{\Psi}_v^T \bar{\boldsymbol{\sigma}} dS_p + \sum_i \boldsymbol{\Psi}_v(\mathbf{x}_i) \bar{q}_i \tag{7}$$

In the equations immediately above, $\mathbf{M}_{rr}^e$ and $\mathbf{K}_{rr}^e$ are the elementary mass and stiffness matrices, respectively, $\mathbf{K}_{rv}^e$ is the electromechanical coupling matrix and $\mathbf{K}_{vv}^e$ is the capacitance matrix. Moreover, $\bar{\mathbf{f}}^e$ and $\bar{\mathbf{q}}^e$ are the element-level vectors of nodal forces and nodal electric charges.

The global electromechanical equations of motion are constructed from element-level equations accounting for the connectivity among neighboring elements. In addition, similar operations must be performed to account for the electrodes that can cover groups of elements, forming equipotential surfaces. Also, mechanical and electrical boundary conditions of Dirichlet type are enforced by imposing prescribed values of displacements and electric potentials at points on the boundaries of the discretized domain. Therefore, the electromechanical equations in global level are obtained in the form (Neubauer and Wallaschek, 2013):

$$\begin{aligned} \mathbf{M}_{rr}\ddot{\mathbf{r}}(t)+\mathbf{K}_{rr}\mathbf{r}(t)-\mathbf{K}_{rv}\mathbf{v}(t) &= \bar{\mathbf{f}}(t) \\ \mathbf{K}_{rv}^{T}\mathbf{r}(t)+\mathbf{K}_{vv}\mathbf{v}(t) &= \bar{\mathbf{q}}(t) \end{aligned} \tag{8}$$

where the intervening matrices and vectors are given as:

$$\mathbf{M}_{rr}=\sum_i \mathbf{R}_i^T \mathbf{M}_{rr}^{e_i}\mathbf{R}_i \ , \ \mathbf{K}_{rr}=\sum_i \mathbf{R}_i^T \mathbf{K}_{rr}^{e_i}\mathbf{R}_i \ , \ \mathbf{K}_{rv}=\left(\sum_i \mathbf{R}_i^T \mathbf{K}_{rv}^{e_i}\mathbf{P}_i\right)\tilde{\mathbf{P}} \ , \ \mathbf{K}_{vv}=\tilde{\mathbf{P}}^T\left(\sum_i \mathbf{R}_i^T \mathbf{K}_{vv}^{e_i}\mathbf{P}_i\right)\tilde{\mathbf{P}} \ ,$$
$$\bar{\mathbf{f}}=\sum_i \mathbf{R}_i^T \bar{\mathbf{f}}^{e_i} \ , \ \bar{\mathbf{q}}=\sum_i \mathbf{P}_i^T \bar{\mathbf{q}}^{e_i} \tag{9}$$

It is seen that the connectivity of the mechanical degrees of freedom and the constrains imposed on the electrical degrees of freedom by the existence of electrodes are enforced by using Boolean matrices $\mathbf{R}_i$, $\mathbf{P}_i$, and $\tilde{\mathbf{P}}$.

At this point, the influence of shunt circuits on the dynamic response predicted by the FE model can be included in the formulation. After converting Eqs. (15) to the frequency domain, accounting for the voltage-to-current relationship $\mathbf{V}(\omega)=-\mathbf{Z}(\omega)\mathbf{I}(\omega)$, and manipulating the resulting equations, the following relation between nodal displacement amplitudes and nodal forces is obtained:

$$\mathbf{r}(\omega)=\mathbf{H}(\omega)\bar{\mathbf{f}}(\omega) \tag{10}$$

where

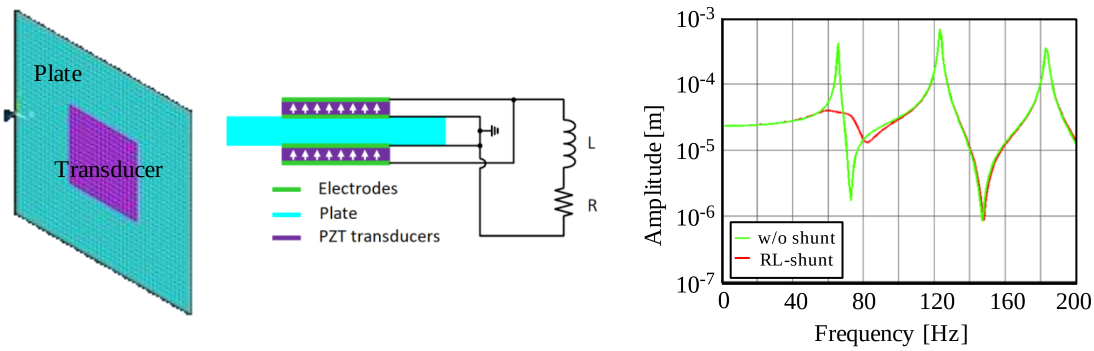
$$\mathbf{H}(\omega)=\left[-\omega^2\mathbf{M}_{rr}+\mathbf{K}_{rr}+j\omega\mathbf{K}_{rv}\left(\mathbf{Z}^{-1}+j\omega\mathbf{K}_{vv}\right)^{-1}\mathbf{K}_{rv}{}^{T}\right]^{-1} \tag{11}$$

is the receptance matrix.

It should be noticed that, in Eq. (11), $\mathbf{Z}(\omega)$ is a diagonal matrix formed by the impedances of the shunt circuits.

Equations (10) and (11) clearly show how the electrical impedances of the shunt circuits influence the dynamics of the system by intervening in the expression of the receptance matrix. In practice, the circuits can be designed to attenuate the vibration levels in such a way to satisfy certain criteria. Additionally, in a manner similar to the single-degree-of-freedom model, the electromechanical coupling coefficients can be computed based on the difference between open- and short-circuited modal frequencies (Neubauer and Wallaschek, 2013).

For illustration, the formulation above has been used for the modeling of a fully clamped aluminum plate (500 x 500 x 1.5 mm) having two opposite surface-bonded ceramic piezoelectric patches (200 x 200 x 0.5 mm) connected to an RL circuit, using the commercial FE modeling software ANSYS®.



(a) (b) (c)

Figure 5. Illustration of the FE model of a fully clamped square plate having two opposite surface-bonded ceramic piezoelectric patches connected to an RL circuit. (a) Geometry and discretization mesh. (b) Detail of the electric connection of the piezoelectric transducers to the circuit. (c) Amplitudes of a typical frequency response function showing the vibration attenuation near the first natural frequency.

The geometry of the model and detail about the electrical connections are shown in Figs. 5(a) and 5(b). The aluminum plate is modeled using finite elements SOLID186 with elastic modulus E = 7.0 x 10^{10}N/m^2, Paisson ratio 0.33, and density 2700kg/m^3. The PZT-5H transducers have compliances s_{21}^E = -4.78x10^{-12}, s_{31}^E = s_{32}^E = -8.54x10^{-12}, s_{33}^E = 20.7x10^{-12} m^2/N, density 7800Kg/m^3, piezoelectric coefficients $d_{13} = d_{23}$ = -2.74x10^{-10}, d_{33} = 5.93x10^{-10}, $d_{42} = d_{51}$ = 7.41x10^{-10}, and are modeled as finite elements SOLID226 connected to circuits CIRCU94. The shunt circuit has been designed to attenuate the vibration amplitudes near the first natural frequency of the plate. Figure 5(c), which depicts the amplitudes of a typical frequency response function, enables to evaluate the effectiveness of the shunt in terms of attenuation of the resonant vibration amplitudes around the first natural frequency.

5. CONCLUSIONS

This paper discussed the use of piezoelectric transducers to convert mechanical vibration energy into electrical energy, which is dissipated in an electrical circuit. The vibration damping performance of shunted piezoelectric transducers can be further improved by exploring more effectively some already available resources, such as: a) the use of evolved piezoelectric transducers, such as Microfiber Composites (MFC) and DuraAct which, being flexible, are light weight and can be attached to curved surfaces; b) the development of miniaturized electronic circuits embedded on the mechanical structures and based on surface mounted devices (SMD); and, specially, c) the use of multiphysics simulation tools. The optimal design in terms of shape and position of the piezoelectric transducers can be achieved using advanced CAE tools, specially finite element codes with multiphysics modeling capabilities, in combination with numerical optimization tools.

6. ACKNOWLEDGEMENTS

The authors acknowledge the support to their research activities provided by the Brazilian Agencies CAPES (PNPD/CAPES - National Postdoctoral Program), CNPq (grants n. 442238/2013-3 and 310633/2013-3), FAPESP (grant n. 2015/20363-6), and the National Institute of Science and Technology of Smart Structures in Engineering (INCT-EIE, grant n. 574001/2008-5). The authors also would like to thank Thiago de Paula Sales and Everton Spuldaro for their help during this research.

7. REFERENCES

Andreaus, U. and Porfiri, M., 2007. “Effect of electrical uncertainties on resonant piezoelectric shunting”, *Journal of Intelligent Material Systems and Structures* Vol. 18. p. 477–485.

Aridogan, U. and Basdogan, I., 2005. “A review of active vibration and noise suppression of plate-like structures with piezoelectric transducers”. *Journal of Intelligent Material Systems and Structures* Vol. 26 (12), p.1455-1476

Benjeddou, A., 2000. “Advances in piezoelectric finite element modeling of adaptive structural elements: a survey”. Computers and Structures Vol. 76 (1) p. 347-363.

Behrens, S., Moheimani, S.O.R. and Fleming, A.J., 2003. “Multiple mode current flowing passive piezoelectric shunt controller.” *Journal of Sound and Vibration.* Vol. 266. p. 929–942.

Casadei, F.. Beck, B.S., Cunefare, K.A. and Ruzzene, M., 2012. “Vibration control of plates through hybrid configurations of periodic piezoelectric shunts.” *Journal of Intelligent Material Systems and Structures*. Vol. 23 (10) p. 1169-1177.

Craig Jr, R.R. and Kurdila, A.J., 2006. *Fundamentals of Structural Dynamics*. 2nd Edition. Springer.

De Marqui Jr, C., Erturk, A. and Inman., D.J., 2009 “An electromechanical finite element model for piezoelectric energy harvester plates.” *Journal of Sound and Vibration* Vol. 327 (1-2) p. 9-25.

Den Hartog, J.P., 1985. *Mechanical Vibrations,* Dover.

Donoso, A. and Sigmund, O., 2016. “Topology optimization of piezo modal transducers with null-polarity phases.” *Structural and Multidisciplinary Optimization*. Vol. 53 (2) p. 193-203.

Ducarne, J., Thomas, O., Deü, J. F., 2012. “Placement and dimension optimization of shunted piezoelectric patches for vibration reduction.” *Journal of Sound and Vibration*. Vol. 331 (14) p. 3286-3303

Erturk, A. and Inman, D.J., 2008. “Issues in mathematical modeling of piezoelectric energy harvesters”. *Smart Materials and Structures*. Vol. 17, 065016.

Forward, R.L., 1979. “Electronic damping of vibrations in optical structures”. *Applied Optics*. Vol. 18. p. 690.

Gripp, J.A.B., Góes, L.C.S., Heuss, O. and Scinocca, F, 2015. “An adaptive piezoelectric vibration absorber enhanced by a negative capacitance applied to a shell structure”. *Smart Materials and Structures*. Vol. 24, 125017.

Hagood, N.W. and Chung, W.H., 1990. “Modelling of piezoelectric actuator dynamics for active structural control.” *Journal of Intelligent Material Systems and Structures*. p. 327-354.

Hagood, N.W. and von Flotow, A.,1991. “Damping of structural vibration with piezoelectric materials and passive electrical networks.” *Journal of Sound and Vibration*. Vol. 146.

IEEE, 1988. *Standard on Piezoelectricity*.

Jian, K. and Friswell, M.I., 2007. “Distributed modal sensors for rectangular plate structures.” *Journal of Intelligent Material Systems and Structures*. Vol. 18 (9) p. 939-948.

Kelley, C.R. and Kauman, J. L., 2017. “Adaptive synchronized switch damping on an inductor: a self-tuning switching law.” *Smart Materials and Structures* Vol. 26 (3).

Kim, J., Hwang, J.-S. and Kim, S. J., 2001. “Design of modal transducers by optimizing spatial distribution of discrete gain weights.”. *AIAA Journal* . Vol. 39 (10). p.1969-1976.

Lee, C. K. and Moon, F. C., 1990. “Modal Sensors/Actuators .” *ASME Journal of Applied Mechanics*. Vol. 57 (2) p. 434-441.

Manzoni, S., Moschini, S., Redaelli, M. and Vanali, M., 2012. “Vibration attenuation by means of piezoelectric transducer shunted to synthetic negative capacitance.” *Journal of Sound and Vibration*. Vol. 331. p. 4644–4657.

Moheimani, S.O.R., 2003. “A survey of recent innovations in vibration damping and control using shunted piezoelectric transducers”. *IEEE Transactions on Control Systems Technology*. Vol. 11 (4), p.482-494.

Neubauer, M., Han, X. and Wallaschek, J., 2012. “On the maximum damping performance of piezoelectric switching techniques.” *Journal of Intelligent Material Systems and Structures*. Vol. 24 (6). p. 717-728.

Neubauer, M. and Wallaschek, J., 2013. “Vibration damping with shunted piezoceramics: Fundamentals and technical applications.” *Mechanical Systems and Signal Processing*. Vol. 36.

Niederberger, D., 2005. *Smart Damping Materials using Shunt Control*. PhD Thesis. ETH Zurich.

Niederberger, D., Fleming, A., Moheimani, S.O.R. and Morari, M., 2004. “Adaptive multi-mode resonant piezoelectric shunt damping.” *Smart Materials and Structures*. Vol. 13. p. 1025–1035.

Pagani Jr, C. C. and Trindade, M. A., 2009. “Optimization of modal filters based on arrays of piezoelectric sensors.” *Smart Materials and Structures*. Vol. 18 (9) 095046.

Park, C.H. and Park, H.C., 2003. “Multiple-mode structural vibration control using negative capacitive shunt damping”. *KSME International Journal*. Vol. 17. p. 1650–1658.

Park, C.H. and Baz, A., 2005. “Vibration control of beams with negative capacitive shunting of interdigital electrode piezoceramics.” *Journal of Vibration and Control*. Vol.11 (3).

Preumont, A., 2006. *Mechatronics: Dynamics of electromechanical and piezoelectric systems*. Springer.

Preumont, A., François, A., De Man, P. and Piefort, V. , 2003. “Spatial filters in structural control.” *Journal of Sound and Vibration.* Vol. 265 (1) p. 61-79.

Preumont, A. and Seto, K., 2008. *Active Control of Structures,* Wiley.

Pulskamp, J.S., Bedair, S.S., Polcawich, R.G., Smith, G.L., Martin, J., Power, B., and Bhave, S.A., 2012. “Electrode-shaping for the excitation and detection of permitted arbitrary modes in arbitrary geometries in piezoelectric resonators.” *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*.Vol. 59 (5) p. 1043-1060.

Qureshi, E.M., Shen X. and Chen J.J., 2014. “Vibration control laws via shunted piezoelectric transducers: A review”. *International Journal of Aeronautical and Space Sciences.* Vol. 15 (1), p. 1–19.

Ruiz, D., Bellido, J. C., and Donoso, A., 2016. “Design of piezoelectric modal filters by simultaneously optimizing the structure layout and the electrode profile.” Structural and Multidisciplinary Optimization 53 (4) (2016) 715-730.

Sales, T., Rade, D. and de Souza, L., 2013. “Passive vibration control of flexible spacecraft using shunted piezoelectric transducers.” *Aerospace Science and Technology*. Vol. 29 (1) p. 403-412.

Silva, E. and Kikuchi, N., 1999. “Design of piezoelectric transducers using topology optimization”. *Smart Materials and Structures*.Vol. 8 (3) p. 350-364.

Sun, D., Tong, L. and Wang, D., 2002. “Modal actuator/sensor by modulating thickness of piezoelectric layers for smart plates”. *AIAA Journal*. Vol. 40 (8). p. 1676-1679.

Tateo, F., Collet, M., Ouisse, M. and Cunefare, K., 2014. "Design variables for optimizing adaptive metacomposite made of shunted piezoelectric patches distribution." *Journal of Vibration and Control*..

Trindade, M.A. and Benjeddou, A.,2002. "Hybrid active-passive damping treatments using viscoelastic and piezoelectric materials: review and assessment". *Journal of Vibration and Control,* Vol. 8 (6), p.699-745.

Uchino, K. and Ishii, T., 1988. "Mechanical Damper Using Piezoelectric Ceramics." *Journal of the Ceramic Society of Japan*. Vol. 96. P. 863–867.

Vasques, C. M. A., 2012. "Improved passive shunt vibration control of smart piezo-elastic beams using modal piezoelectric transducers with shaped electrodes." *Smart Materials and Structures*. Vol. 21 (12) 125003.

Vér, I.L. and Beranek, L.L., 2005. *Noise and Vibration Control Engineering*. John Wiley & Sons, New Jersey, 2^{nd} ed.

Zhang, X. and Kang, Z., 2014. "Topology optimization of piezoelectric layers in plates with active vibration control." *Journal of Intelligent Material Systems and Structures*. Vol. 25 (6) p. 697-712.

8. RESPONSIBILITY NOTICE

The author(s) is (are) the only responsible for the printed material included in this paper.