24th COBEM - 2017

ABCM Associação Brasileira de Engenharia e Ciências Mecânicas

24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-1082
FRACTIONAL ORDER CALCULUS APPLIED TO GENERALIZE THE ROTT'S LINEAR THERMOACOUSTICS: AN INVESTIGATION TOWARD ENERGY REGENERATION IN AUTOMOBILES

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Abstract. *Exhaust gas from internal combustion engines in automobiles usually contains a large amount of available energy that is simply lost to the atmosphere. A thermoacoustic engine could recover part of this energy by converting it into acoustic waves, which in turn would be converted into electric power to supply the automobile's battery. This process enables the removal of some of the load on the engine crankshaft, which results in energy regeneration. The acoustic field inside the device is commonly described by the Nikolaus Rott's linear thermoacoustics wave equation. We propose in this paper a step toward the generalization of this model under the regard of fractional order calculus, aiming to fit it with experimental acoustic data. The purpose of the resulting model is to be later on applied to investigate the energy regeneration.*

Keywords: *fractional calculus, thermoacoustics, energy regeneration, internal combustion engine.*

1. INTRODUCTION

The gases expelled by internal combustion engines contain available thermal energy that is lost once released to the atmosphere (Al-Najem and Diab, 1992). This free energy could be harnessed by a coupled device and utilized to supply an automobile battery, replacing at least part of the electric power currently provided by the engine crankshaft via the alternator. The resulting energy regeneration may increase the engine performance depending on how efficiently it is converted (Bradfield, 2008). However, even though the amount of wasted energy is important, the usual low thermal potential in these gases limits its availability (Blok, 2010). Furthermore, the device coupled to the exhaust system would have to be capable of operating not constrained by phase shifting in working fluid (Blok, 2010). One suitable device for this job is a thermoacoustic (TA) engine, which is interesting mainly for three reasons: it is more reliable due to the absence of moving parts, its working fluid remains gaseous, and it can operate under low-temperature differences. Despite the fact that TA engines perform up to now less efficiently than normal thermal engines, with regard to this application all energy produced by the device originates from a potential that would be unconditionally lost (Blok, 2010). Although this application for TA engine has already been investigated (Gardner and Howard, 2009), as far as we know, a fractional calculus (FC) approach has never been employed for this purpose.

The TA engine is essentially composed by a core with a porous material, where the thermoacoustic effect takes place, a resonator, and a linear alternator - responsible for converting the generated acoustic power into electric energy. The thermoacoustic effect consists in an interaction between fluid and solid surface where a conversion between thermal and acoustic energy occurs; depending on its direction, the conversion can result in either refrigeration or acoustic field generation (Bannwart and Arruda, 2009). In the case of interest, the conversion toward acoustic energy is triggered when a sufficiently high gradient of temperature is imposed along the solid surface (Swift, 1997). The amount of energy conversion is proportional to the available surface for interaction, favored in porous materials due to its vast internal walls network. Depending on the waveguide configuration of the resonator, the acoustic field can result in stationary or progressive waves, which lead the acoustic particle behavior to approach to the Brayton or Stirling cycle, respectively. In this study, the gradient of temperature is expected to be supplied by the exhaust gas from the internal combustion engine, with the waste heat being rejected to the atmosphere by means of proper heat transfer systems. Preliminary simulations indicate the suitability of the TA engine as a possible energy regenerator in an automobile. We evaluated, for instance, a simplified system where a thermoacoustic engine equivalent to the one proposed by Blok (2010) utilizes the exhaust gas from a 243 kW turbo-diesel combustion engine as a heat source. The results obtained with the same parameter conditions as the ones used by Al-Najem and Diab (1992) showed that the energy harnessed by the TA engine could improve the efficiency of the system up to 39.4%, therefore justifying this investigation.

The main concern of this investigation is on using the concepts of FC to generalize the linear thermoacoustics wave equation aiming to build a better model for TA engines. FC is the generalization of the traditional calculus to a broader scope in which derivatives and integrals can assume non-integer and even complex orders (Oldham and Spanier, 1974). Several mathematicians and physicists contributed to distinct definitions and approaches on the subject (Machado *et al.*, 2011). Those distinct perspectives allow researchers to choose the definition that best fits the physical phenomenon under study (David *et al.*, 2011). In this work, we chose to apply and evaluate the Caputo fractional derivative, based on the Riemann-Liouville definition, because its convergence requirements allow the differentiation of several functions (Herrmann, 2011), what comprises our problem.

Recently, studies regarding fractional calculus have significantly increased in areas such as signal processing, modeling and control, wave and diffusion problems, and others (Machado *et al.*, 2011). Among these studies, some indicate that, at least for certain applications, fractional models can be more versatile than traditional integer order ones (David *et al.*, 2016a, 2017). They can for instance achieve a better fit to experimental results and cover a wider range of visco-elastic behaviors (David and Katayama, 2013; Di Paola *et al.*, 2011), function as an alternative approach of analysis of continuous variables (Royston *et al.*, 1999), and help design PID controllers with better performance (David *et al.*, 2016b). These results obtained throughout several areas encouraged us to start this work.

At last, seeing the TA engine as a viable energy regenerator, we propose an application of FC as an additional resource to improve the linear TA modeling. Under this regard, the Nikolaus Rott's wave equation (Rott, 1969) will be generalized and used to model optimized TA engines, which will be simulated and confronted with their traditional counterparts. In subsequent works, the fractional orders of the generalized model will be adjusted according to experimental data from previous experiments (Bannwart, 2014; Bannwart *et al.*, 2013), having its accuracy evaluated. Finally, the final and most accurate generalized equation will be used to model a TA engine regenerating energy into an automobile system.

2. PROBLEM MODELING

The Rott's linear equation for the propagation of acoustic pressure $p(x)$ is the basic model for the acoustic field in TA systems. It is usually expressed as:

$$\frac{a^2\rho_m}{\omega^2}\frac{d}{dx}(\frac{1-f_\nu}{\rho_m}\frac{dp}{dx}) - \frac{a^2}{\omega^2}\frac{f_\kappa - f_\nu}{1-\sigma}\frac{1}{T_m}\frac{dT_m}{dx}\frac{dp}{dx} + [1+(\gamma-1)f_\kappa]p = 0. \tag{1}$$

In Eq. (1), p_m and ρ_m are the static pressure and density of the working gaseous fluid, f_ν and f_κ are functions of viscous and thermal losses due to the interaction between gas and solid surface, T_m is the average transversal temperature, σ is the Prandtl Number, ω is the first mode resonance frequency, a is the sound velocity, and γ is the polytropic coefficient.

One way of representing Eq. (1) in a generalized form is using operators of fractional or arbitrary order α:

$$\frac{a^2\rho_m}{\omega^2}{}_RD_x^\alpha(\frac{1-f_\nu}{\rho_m}{}_RD_x^\alpha p) - \frac{a^2}{\omega^2}\frac{f_\kappa - f_\nu}{1-\sigma}\frac{1}{T_m}\frac{dT_m}{dx}{}_RD_x^\alpha p + [1+(\gamma-1)f_\kappa]p = 0. \tag{2}$$

The operator ${}_RD_x^\alpha$ represents a Caputo fractional derivative of order α and may be written in the derivative or integral form for $x > R$, as shown in Eq. (3). Euler's Gamma Function is represented by Γ and ξ is an auxiliary variable.

$${}_RD_x^\alpha f(x) = {}_RI_x^{1-\alpha}\frac{d}{dx}f(x) = \frac{1}{\Gamma(1-\alpha)}\int_0^x (x-\epsilon)^{-\alpha} f'(\xi)d\xi. \tag{3}$$

Even though this operator is commonly known as a fractional derivative operator, it may be more accurately called as an operator of a derivative of arbitrary order (as some authors do), since if $\alpha = 1$ the generalized Eq. (2) returns its traditional form equivalent to Eq. (1).

Before applying any method to directly try deriving fractional solutions of Eq. (1), simpler models are investigated for verification of the generalization proceedings. Three cases are put forward, all departing from linear integer order homogeneous differential equations. In the first case, as an initial approach, the lossless diffusion-wave equation is analyzed in its simplest form. In the second case, we model the lossy wave propagation in the Helmholtz form with zero average temperature gradient, or uniform temperature along the solid surface. In the third case, the only difference with the previous one is the addition of a dissipative term of first order (the simplest possible) disregarding physical meaning. Analysis comparing integer order and fractional models are made for all cases.

2.1 Case 1: The lossless diffusion-wave equation

The first case consists in a generalization of the well known partial differential equation that describes the lossless wave propagation in space and time. The order of the derivative regarding space is kept integer while the derivative of pressure in time assumes an arbitrary fractional order between 1 (fully dissipative) and 2 (fully oscillatory):

$$\frac{\partial^\alpha p}{\partial t^\alpha} = c^2 \frac{\partial^2 p}{\partial x^2}. \tag{4}$$

Similarly to the approach in (Al-Khaled and Momani, 2005), the Adomian's decomposition method (Adomian, 1988) and the Caputo operator are used to obtain the generalized solutions to Eq. (4) for $c = 1$ and $1 \leq \alpha \leq 2$. The solution obtained through recurrence equations is expressed in a series form by:

$$p(x,t) = sin(x)[1 - t - \frac{t^\alpha}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \ldots]. \tag{5}$$

Simulations of Eq. (5) are performed for arbitrary orders of α . The behavior of $p(x,t)$ exhibits characteristics more related to diffusion and oscillation for values of α around 1 and 2, respectively (see the results in section 3.1).

2.2 Case 2: The lossy wave equation with uniform temperature

From Eq. (1), if temperature gradient with position is set to zero:

$$\frac{a^2 \rho_m}{\rho_m \omega^2}(1 - f_\nu)\frac{d^2 p}{dx^2} + [1 + (\gamma - 1) f_\kappa] p = 0. \tag{6}$$

After rearranging and considering the working fluid as an ideal gas, Eq. (6) is written as an 1-D Helmholtz traditional equation:

$$\frac{d^2 p}{dx^2} + k^2 p = 0, \tag{7}$$

$$k = k_0 \sqrt{\frac{1 + (\gamma - 1) f_\kappa}{1 - f_\nu}}, \tag{8}$$

where k is the wave number considering thermal and viscous losses and k_0 is simply ω / a.

The arbitrary boundary conditions

$$p(0) = P_0, \tag{9}$$

and

$$\frac{dp}{dx}|_{x=0} = \mu_0 \tag{10}$$

are used, and the well known solution of Eq. (7) is:

$$p(x) = P_0 Cos(kx) + \frac{\mu_0}{k} Sin(kx). \tag{11}$$

In the works of Trujillo *et al.* (1999), the solution for the arbitrary order form of

$${}_R D_x^{2\alpha} p(x) + k^2 p(x) = 0 \tag{12}$$

can be represented as a power series

$$p(x) = \sum_{n=0}^{\infty} C_n x^{n\alpha}. \tag{13}$$

We used the Caputo approach in order to obtain the recurrence equation to find the values of the coefficients

$$C_{n+2} = -k^2 \frac{\Gamma(n\alpha + 1)}{\Gamma[(n+2)\alpha + 1]} C_n. \tag{14}$$

Using this formulation, the acoustic pressure $p(x)$ can be expressed as

$$p(x) = P_0 \sum_{n=0}^{\infty} \frac{(-1)^n k^{2n}}{\Gamma(2n\alpha + 1)} x^{2n\alpha} + \mu_0 \sum_{n=0}^{\infty} \frac{(-1)^n k^{2n}}{\Gamma[(2n+1)\alpha + 1]} x^{(2n+1)\alpha}. \tag{15}$$

Finally, Eq. (15) is simulated with up to 100 terms using the values presented in Tab. 1.

Table 1. Parameters used for the simulation of cases 2 and 3.
Working fluid: air at 295.15 K and 101900 Pa.

Description	Parameter	Value
Polytropic coefficient	γ	1.402
Sound velocity	a	344.6 m/s
Thermal conductivity	K	0.0263 W/(m K)
Density	ρ_m	1.203 kg/m^3
Specific heat at constant pressure	C_P	1.003 kJ/(kg K)
Viscosity	μ	1.84×10^{-5} Pa
Frequency	f	100 Hz
Wave number (lossless)	k_0	1.8233 m^{-1}
Pore radius$^{(1)}$	R	200 μm
Thermal losses$^{(1)}$	f_κ	$0.7284 + 0.36149$ i
Viscous losses$^{(1)}$	f_ν	$0.6038 + 0.3772$ i
Wave number (with losses)$^{(1)}$	k	$2.5482 + 1.1885$ i m^{-1}
First boundary condition (normalized initial pressure)	P_0	1
Second boundary condition	μ_0	$P_0 k i$

$^{(1)}$Considering a cylindric pore.

2.3 Case 3: The lossy wave equation with uniform temperature and an extra dissipative term

An additional dissipative term is added in Eq. (7) for a comparative analysis, providing

$$\frac{d^2p}{dx^2} + \frac{dp}{dx} + k^2 p = 0, \tag{16}$$

whose arbitrary order form is

$${}_R D_x^{2\alpha} p(x) + {}_R D_x^{\alpha} p(x) + k^2 p(x) = 0. \tag{17}$$

The analytical solution for Eq. (16) is

$$p(x) = \frac{(-P_0 - 2\mu_0 + P_0\sqrt{1-4k^2})e^{(-\frac{1}{2}-\frac{1}{2}\sqrt{1-4k^2})} + (2\mu_0 + P_0 + P_0\sqrt{1-4k^2})e^{(-\frac{1}{2}+\frac{1}{2}\sqrt{1-4k^2})}}{2\sqrt{1-4k^2}}. \tag{18}$$

Having in mind the arbitrary order in Eq. (17), the recurrence formula can be obtained such as

$$C_{n+2} = \frac{-C_{n+1}\Gamma[(n+1)\alpha+1] - k^2 C_n \Gamma(n\alpha+1)}{\Gamma[(n+2)\alpha+1]}. \tag{19}$$

The solution $p(x)$ for Eq. (17) is approximated by:

$$p(x) = P_0[1 - \frac{k^2}{\Gamma(2\alpha+1)}x^{2\alpha} + \frac{k^2}{\Gamma(3\alpha+1)}x^{3\alpha} + \ldots] + \mu_0[\frac{1}{\Gamma(\alpha+1)}x^{\alpha} - \frac{1}{\Gamma(2\alpha+1)}x^{2\alpha} + \frac{1-k^2}{\Gamma(3\alpha+1)}x^{3\alpha} + \ldots] \tag{20}$$

This solution is tested using the values in Tab. 1.

3. SIMULATION RESULTS AND DISCUSSION

3.1 Case 1: The diffusion-wave equation

The solution represented in Eq. (5) is simulated for different values of α. The simulation results shown in Fig. 1 indicate how the fractional order can lead to an additional degree of freedom, as models with lower values of α tend to behave as described by the diffusion equation ($\alpha = 1$), while higher values of α make models approach to the wave equation ($\alpha = 2$). The effect observed in these models is commonly referred to as "Memory function", where a purely diffusive system indicates the absence of memory and an oscillatory one represents full memory (Agrawal, 2002). These results help to validate the view that generalized models as the one proposed in this work may be more adequate to fit with experimental data since they have one more adjustable variable (the arbitrary order α).

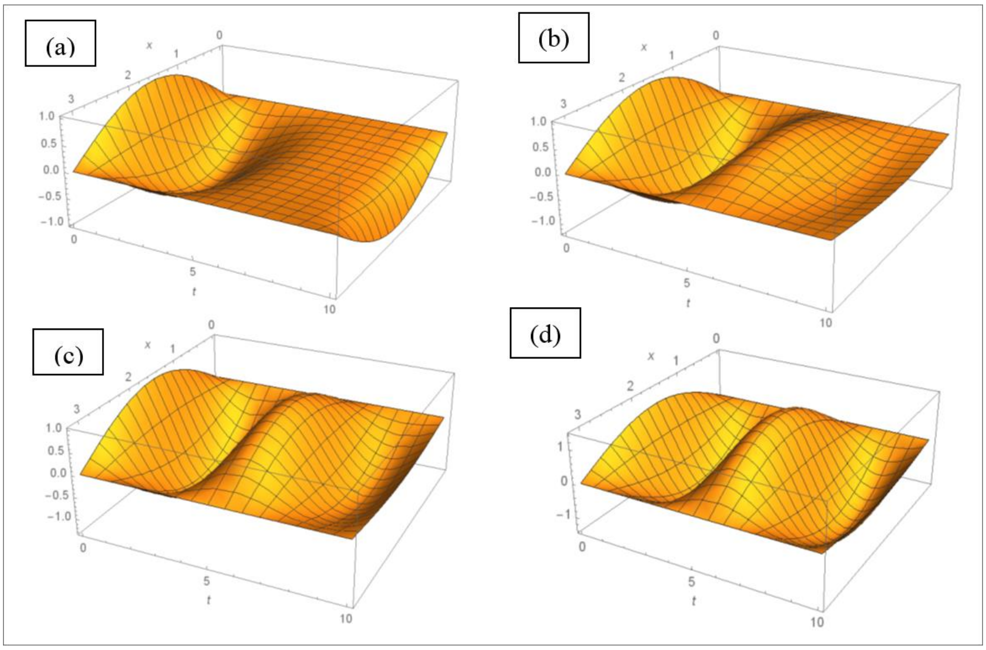

Figure 1. Surface of the approximate solution of the wave $p(x,t)$ using 15 terms for (a) $\alpha = 1.5$; (b) $\alpha = 1.75$; (c) $\alpha = 1.9$; (d) $\alpha = 2$.

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3.2 Case 2: The linear acoustic equation with uniform temperature

The generalized solution for case 2 is simulated both for the fully conservative and lossy cases (using the parameters k_0 and k, respectively). Firstly, Fig. 2 presents the solutions of Eq. (11) and Eq. (15) (when $\alpha = 1$) for both situations. Once again, it is possible to see how generalized forms can return traditional solutions, as both curves of Fig. 2 are overlapping to their traditional solution curves. Another important visual information is that, for the parameters adopted in this simulation, namely working fluid conditions and pore geometry (see Tab. 1), the thermal and viscous losses rapidly change the behavior of the curve within the first centimeters of propagation. The predicted attenuation can be steeper or flatter depending on the thermal and viscous losses, which are function of the pore geometrical parameters and the gas thermophysical properties.

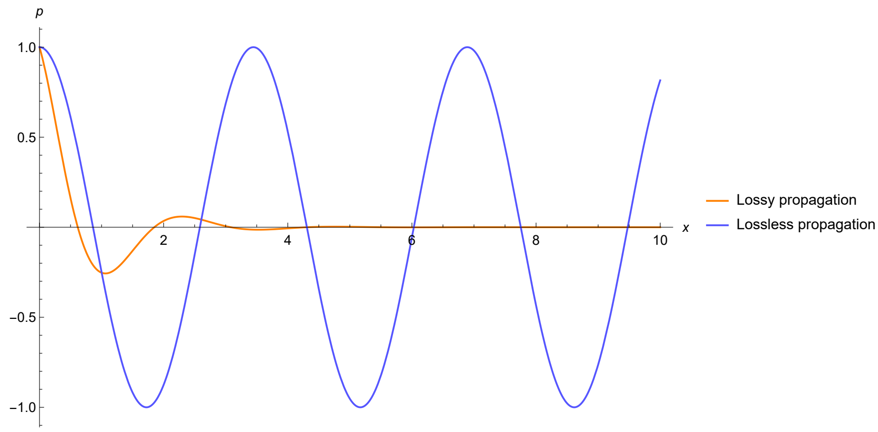

Figure 2. Comparison of lossy and conservative versions of integer order models for $p(x)$ with constant temperature.

Figure 3 shows the curves for $p(x)$ with $k = k_0$. Only the solution with $\alpha = 1$ maintains the conservative aspect of the lossless acoustic wave. All solutions for $\alpha < 1$ have lossy behaviors even though the wave number is purely real, which implies that fractional models can simulate dissipations even when other coefficients indicate that a solution is unattenuated. This characteristic signals that generalized forms may help build more powerful models and, at the very least represent equivalent solutions in different manners. For instance, the wave curve with $\alpha = 0.8$ in Fig. 3 is very close to the behavior of the lossy propagation wave in Fig. 2, which suggests that there is a specific α value capable of turning a generalized solution of a lossless wave into its lossy counterpart (for a specified pore configuration) without changing any parameter.

Finally, Fig. 4 displays the behavior of fractional solutions of propagative waves already lossy in its traditional form. The attenuation is even higher, confirming the additional damping present fractional in terms of fractional order.

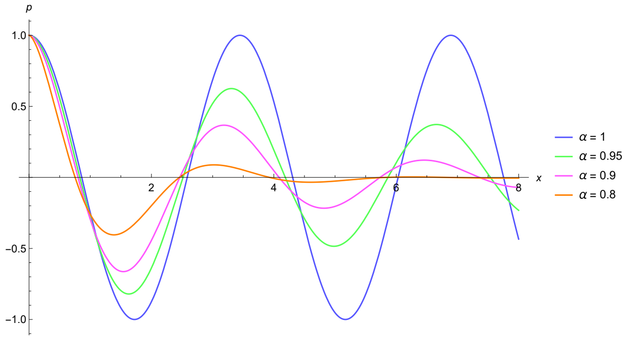

Figure 3. Comparison of fractional models for the conservative version of $p(x)$ with $\alpha = 1$ (traditional model), $\alpha = 0.95$, $\alpha = 0.9$ and $\alpha = 0.8$.

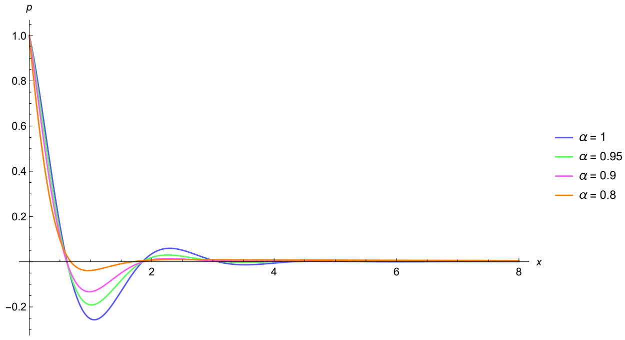

Figure 4. Comparison of fractional models for the lossy version of $p(x)$ with $\alpha = 1$ (traditional model), $\alpha = 0.95$, $\alpha = 0.9$ and $\alpha = 0.8$.

3.3 Case 3: The linear acoustic equation with uniform temperature and an extra dissipative term

The generalized solution for case 3 represented by the series in Eq. (20) is only an approximation of the traditional solution represented by Eq. (18). The accuracy and time for simulation heavily depend on the number of terms of this series, as shown in Fig. 5. The traditional solution is compared to the generalized solution with $\alpha = 1$ for distinct numbers of terms. For the purpose of this work, the number of terms in the solution series does not need to be large, as the comparisons between models will be drawn for the first segments of propagation. We have found good compromise for 30 terms.

Figure 6 shows the behavior of the solutions represented in Eq. (20) for orders of $\alpha < 1$ compared with the analytical solution (equivalent to $\alpha = 1$). Even though all approximations start to diverge for $x > 5$ due to the limited number of terms, the curves for $0 \leq x \leq 4$ should not have any significant errors. In this interval, we can see that regardless of the wave number not taking into account any losses (k_0 instead of k is used for this case), there is a dissipative term in the model that attenuates the solution curve for every case, including $\alpha = 1$. However, the simulation results show that, as the fractional orders α get smaller, the attenuation effect of the curve is increased, confirming the fact that fractional orders contain a complementary description of the attenuation.

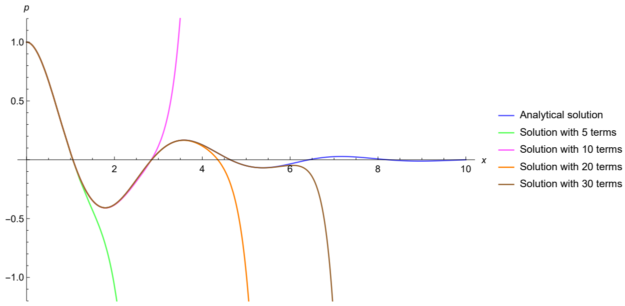

Figure 5. Comparison between analytic and power series integer order solutions for Eq. (16) with $k = k_0$.

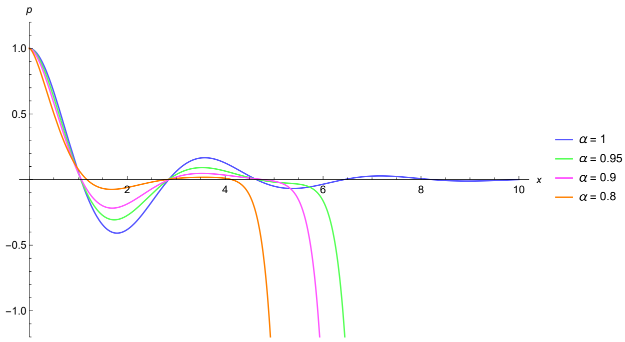

Figure 6. Comparison of fractional models for the solution of Eq. (17) for $k = k_0$ with $\alpha = 1$ (traditional model), $\alpha = 0.95$, $\alpha = 0.9$ and $\alpha = 0.8$.

4. FINAL CONSIDERATIONS

This study addresses a new modeling and numerical simulations for the linear Rott's thermoacoustics wave equation by applying fractional order calculus. The preliminary results are verified correct and pave the sequence of this work, which will be the application of this modeling to investigate energy regeneration of a internal combustion engine by means of a thermoacoustic engine. Experimental acoustic data from the works of Bannwart (2014) and Bannwart *et al.* (2013) will be used to adjust the fractional orders so that it simulates a new TA engine, which will be confronted later on with the predicted performance for the same data for the original integer order consideration. If successful in obtaining a more accurate approximation, which is usually expected for fractional calculus modeling, we will use it to model a TA engine applied to the energy regeneration application.

5. ACKNOWLEDGEMENTS

This work was conducted during a scholarship supported and financed by CAPES - Brazilian Federal Agency for Support and Evaluation of Graduate Education within the Ministry of Education of Brazil.

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