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## MODELING AND SIMULATION OF THE THERMAL GENERATOR AND RECTIFIER OF AN ABSORPTION REFRIGERATION PROTOTYPE

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**Abstract.** *Considering the increasing energy demand and the increasingly significant use of refrigeration and HVAC-R equipment on the world, the utilization of absorption refrigerators is an excellent alternative to minimize this problem. In addition, the use of this equipment becomes energy-efficient when compared to the steam compression refrigeration cycles because it is possible to operate with residual heat in the generator as an energy source, even if its performance is lower. Thus, in order to become absorption refrigerators increasingly commercially competitive, both economically and thermodynamically, this study presents a proposal for semi transient mathematical modeling and the computational simulation of the thermal generator and rectifier of a absorption refrigeration prototype by fuel gas (LPG) developed in NPDEAS (Self-Sustainable Energy Research and Development Center) at UFPR. The modeling, which aims to describe mathematically the physical principles of the refrigeration prototype, was based on the application of the mass and energy conservation principles in a transient regime for the control volumes ( $CV_1$ ,  $CV_2$ ,  $CV_3$ ,  $CV_4$  and  $CV_5$ ) that describes the thermal generator and rectifier. Finally, with the developed mathematical model, a computational code was elaborated in Fortran<sup>®</sup> 95 language, which allowed to obtain the numerical and transient thermal and mass responses of the generator and rectifier. The mathematical model considers both components, e.g. thermal generator and rectifier, in a transient regime, which resulted in a system of ten ordinary differential equations, which were integrated at time and using the 4<sup>th</sup>/5<sup>th</sup> order Runge-Kutta method, that are interacting, simultaneously, with the empirical mass flow equations. An investigation of the model sensitivity was conducted, so that a qualitative assessment of how the numerical model results agree with the system expected response could be made, and the results showed that the physically expected trends were captured by the model. As a result, the proposed model can be a useful tool for simulation of single-stage absorption refrigeration systems in operation because presents the temperature and mass fraction distribution over time and that, subsequently, used for testing different conditions of operation and optimization procedures.*

**Key-Words:** *Absorption Refrigeration Prototype, Rectifier, Thermal Generator, Mathematical Model, Computational Simulation.*

### 1. INTRODUCTION

According to the latest National Energy Balance released by the Ministry of Mines and Energy, the industrial sector accounted for approximately 37.6% of Brazil's electricity consumption in 2015, followed by residential (25.1%) and commercial (17.5%). If these three sectors are taken into account together, it is possible to affirm that they consume more than 80% of all electricity consumed in the country. In these sectors, refrigeration systems account for a significant portion of total electricity consumption. Taking into account only the residential and commercial sectors data, it is possible to affirm that approximately 11.8% of the total electricity consumption in Brazil is destined to the various refrigeration systems (Brazil MME, 2016).

Therefore, any technological-scientific measure with the aim of reducing the energy consumption of HVAC-R systems will represent an unequivocal contribution to the search for solutions to the growing world energy demand (Cardoso and Nogueira, 2007). Thus, in the saving and reducing electricity consumption context and taking into account

the high energy consumption of HVAC-R and refrigeration systems, we highlight the refrigeration systems that use alternative sources of energy as, for example, the absorption refrigeration systems that can be fed by residual heat and solar energy, thus dispensing with the use of energy considered traditional and in most cases causing damage to the environment.

In absorption systems, a physicochemical process replaces the mechanical process of the steam compression system by using energy in the form of heat rather than mechanical work (Moran and Shapiro, 2000). The absorption refrigeration ammonia-water cycle has some attractive features when compared to conventional compression cycle, e.g. no use of oil; if there is waste heat available, operating costs drop considerably; little consumption of electric energy and it produces no noise other than pumps (Hudson, 2002). The disadvantages of the absorption refrigeration systems include: higher capital investment is needed, the hardware is more complex and more space required. In Brazil the installed absorption refrigeration capacity is relatively small, despite the potential for it in different economy sectors (Cortez and Muhle, 1994).

Regarding the scientific and technical literature, there are several studies developed in order to mathematically model and computationally simulate the dynamic behavior of absorption refrigeration systems. For example, Vargas *et al.* (2000) developed a dimensionless mathematical model and internally reversible, i.e., when there are no internal irreversibility's or when the same are in the boundary of the system, of an absorption refrigeration unit thermally driven through a fuel burner. Chua, Toh and Ng (2002) developed a stationary model taking into account the irreversibility's of an absorption refrigeration system, as well as a careful thermodynamic analysis applied to the generator, the rectifier and the absorber, since in these components the liquid and the steam are not in thermodynamic equilibrium.

Among dynamic simulation models, recently, Kim and Park (2007) presented a model for a particular type of absorption chiller, therefore not general for all types of single effect absorption refrigeration systems. Vargas *et al.* (2009) proposed a mathematical model, based on empirical and fundamental correlations, in addition to the principles of classical thermodynamics and heat and mass transfer, in order to simulate the behavior, in transient and permanent regimes, of a cogeneration system developed to, simultaneously, generate heating and cooling, under different design and operation conditions. Finally, Martinho *et al.* (2016) proposed a semi-transient dimensionless mathematical model to analyze the dynamic behavior of single-stage absorption refrigeration systems according to several geometric and operation parameters.

Considering these technical works and scientific studies presented above, it is possible to define the objective of this study as a mathematical modeling and computational simulation in Fortran<sup>®</sup> 95 of the thermal generator and rectifier, basics components of absorption refrigeration systems. The equations that govern the mathematical model will be based in empirical correlations and in the mass and energy conservation laws. Besides it, will be considered the operation in semi-transient regime. The components will be submitted to the division of more than one control volume in order to analyze, individually, the stages of the solution phase equilibrium of an absorption refrigeration system and using ammonia and water as working fluids.

## 2. PHYSICAL PROBLEM

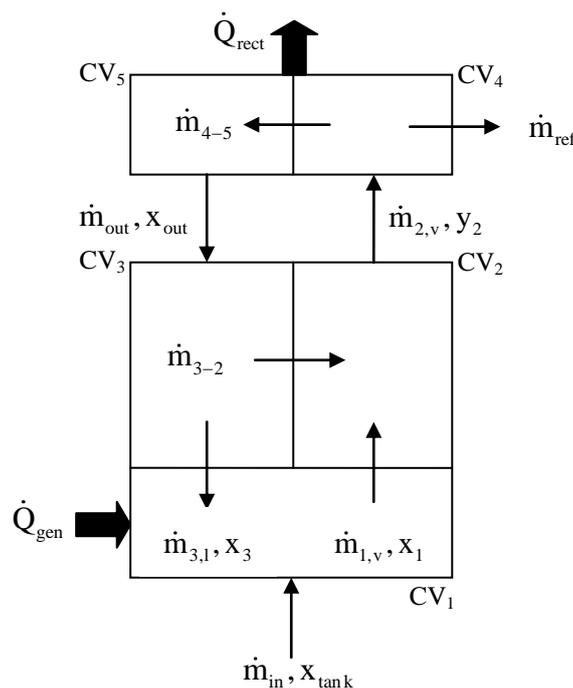


Figure 1. Division of the Generator and Rectifier in Control Volumes. Source: The Authors (2017).

The physical problem constitutes the configuration of the thermal generator of an absorption refrigeration system, divided into three control volumes in order to individually analyze the liquid and steam phases of the ammonia and water solution, as well as the division of the posterior component to the generator, the rectifier, subdivided into two more control volumes, as shown in Figure 1. The CV<sub>1</sub> (Control Volume 1) and CV<sub>3</sub> (Control Volume 3) correspond to the liquid phase, while the CV<sub>2</sub> (Control Volume 2) corresponds to the steam phase of the mixture. The CV<sub>4</sub> (Control Volume 4) and CV<sub>5</sub> (Control Volume 5) represent the rectifier division. In the thermal generator, the rich or strong ammonia solution, preheated in the regenerative heat exchanger, is separated by the heat addition process resulting in two products: a top product, rich in ammonia and a bottom product, low in ammonia (Martinho, 2013).

Considered of fundamental importance for the development of the thermal generator mathematical modeling, the following are presented the laws of mass and energy conservation for open systems already ignored the influence of energies, potential and kinetics, and taking into account that control volume total energy (E<sub>cv</sub>) is present only in the form of internal energy.

a) Mass Conservation Law

$$\frac{dm_{cv}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out} \quad (01)$$

b) Energy Conservation Law

$$\frac{dE_{cv}}{dt} = \frac{d(m \cdot u)_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum \dot{m}_{in} \cdot h_{in} - \sum \dot{m}_{out} \cdot h_{out} \quad (02)$$

where  $\dot{m}_{in}$  e  $\dot{m}_{out}$  represents, respectively, the mass flows that enter and leave the CV,  $\dot{Q}_{cv}$  denotes the energy transfer rate in the heat form in the CV,  $\dot{W}_{cv}$  corresponds to the generation of work in relation to time and  $h_{in}$  and  $h_{out}$  represents, respectively, the specific enthalpies at the input and output of the CV.

### 3. MATHEMATICAL MODELING

The mathematical model developed in this work for the thermal generator takes into account the principles of mass (Eq. 01) and energy (Eq. 02) in a semi-transient regime for a control volume (CV), as well as the division of this component into three control volumes (CV<sub>1</sub>, CV<sub>2</sub> and CV<sub>3</sub>) so that it is possible to individually analyze the stages of liquid and steam in the phase equilibrium of the solution of ammonia and water. In addition, two control volumes were considered for the rectifier: CV<sub>4</sub> and CV<sub>5</sub>.

#### 3.1 Mass Flows

With the application of the principle of mass conservation to CV<sub>1</sub>, it is observed that the mass flow of ammonia in the steam phase that leaves this control volume,  $\dot{m}_{1,v}$ , can be described by:

$$\dot{m}_{1,v} = kLa_{NH_3}^{(1)} (x_1 - x_{1,NH_3}^*) + kLa_{H_2O}^{(1)} (1 - x_1 - x_{1,H_2O}^*) \quad (03)$$

where the terms  $x_1$  corresponds to the ammonia fraction in the steam phase that accompanies the mass flow  $\dot{m}_{1,v}$ ,  $kLa_{NH_3}^{(1)}$  and  $kLa_{H_2O}^{(1)}$  represents, respectively, the mass transfer constants for ammonia and water and can be calculated in the following sentence based in the Arrhenius equation:

$$kLa_{NH_3}^{(1)} = kLa_{H_2O}^{(1)} = k_0 \cdot e^{\frac{-E_a}{T_1+273}} \quad (04)$$

and  $x_{1,NH_3}^*$  and  $x_{1,H_2O}^*$  denotes, respectively, a relation for ammonia and water calculated by:

$$x_{1,NH_3}^* = H_{NH_3,1-2} \cdot y_2 \quad (05)$$

$$x_{1,H_2O}^* = H_{H_2O,1-2} \cdot (1 - y_2) \quad (06)$$

where  $y_2$  represents the mass fraction of ammonia in the steam phase present in  $CV_2$ . In addition, the terms  $H_{NH_3,1-2}$  and  $H_{H_2O,1-2}$  correspond, respectively, to the proportionality constants of Henry's law for ammonia and water between  $CV_1$  and  $CV_2$  and can be described by:

$$H_{NH_3,1-2} = h_{0,NH_3} \cdot e^{-c_{NH_3} \cdot T_1} \quad (07)$$

$$H_{H_2O,1-2} = h_{0,H_2O} \cdot e^{-c_{H_2O} \cdot T_1} \quad (08)$$

where the terms  $h_{0,NH_3}$ ,  $h_{0,H_2O}$ ,  $k_0$ ,  $E_a$ ,  $c_{NH_3}$  and  $c_{H_2O}$  are numerical constants presented in Table 1. In addition, it is important to note that  $T_1$  refers to the temperature of the  $CV_1$ .

Table 1. Numerical Constants used in the Simulation of the Mathematical Model.

Variables	Numerical Value	Units
$c_{NH_3}$	0.032	[-]
$c_{H_2O}$	0.014	[-]
$h_{0,NH_3}$	0.551	[-]
$h_{0,H_2O}$	0.919	[-]
$k_0$	$3.6738 \cdot 10^{15}$	[h <sup>-1</sup> ]
$E_a$	14200.62	[J·mol <sup>-1</sup> ]
$c_{p,liq}$	5.03	[kJ·kg <sup>-1</sup> ·K <sup>-1</sup> ]
$c_{p,vap}$	1.50	[kJ·kg <sup>-1</sup> ·K <sup>-1</sup> ]
$x_{\text{tank}}$	0.50	[-]
$T_w$	20	[°C]
$T_g$	150	[°C]
$m_1$	10.0	[kg]
$m_2$	0.10	[kg]
$m_3$	1.00	[kg]
$m_4$	0.50	[kg]
$m_5$	0.50	[kg]
$(UA)_{\text{gen}}$	0.070	[W·K <sup>-1</sup> ]
$(UA)_{\text{rect}}$	0.005	[W·K <sup>-1</sup> ]

Source: The Authors (2017).

The mass flow transferred between  $CV_2$  (Control Volume 2) and  $CV_3$  (Control Volume 3),  $\dot{m}_{3-2}$ , can be described through Eq. (09):

$$\dot{m}_{3-2} = kLa_{NH_3}^{(2)} (x_3 - x_{3,NH_3}^*) + kLa_{H_2O}^{(2)} (1 - x_3 - x_{3,H_2O}^*) \quad (09)$$

where  $x_3$  corresponds to the mass fraction of ammonia in the liquid phase that is present in  $CV_3$ . In addition, the terms  $x_{3,NH_3}^*$  and  $x_{3,H_2O}^*$  are defined through:

$$x_{3,NH_3}^* = H_{NH_3,3-2} \cdot y_2 = \left( h_{0,NH_3} \cdot e^{-c_{NH_3} \cdot T_3} \right) \cdot y_2 \quad (10)$$

$$x_{3,H_2O}^* = H_{H_2O,1-2} \cdot (1 - y_2) = \left( h_{0,H_2O} \cdot e^{-c_{H_2O} \cdot T_3} \right) \cdot (1 - y_2) \quad (11)$$

where  $T_3$  refers to the temperature of the  $CV_3$  and  $kLa_{NH_3}^{(2)}$  and  $kLa_{H_2O}^{(2)}$  can be defined through Eq. (12) with  $k_0$  and  $E_a$  obtained from Table 1.

$$kLa_{NH_3}^{(2)} = kLa_{H_2O}^{(2)} = k_0 \cdot e^{\frac{-E_a}{T_3+273}} \quad (12)$$

Similarly to showed in Eq. (03) and (09), to calculate the amount of mass per unit of time that is transferred between the  $CV_4$  to the  $CV_5$  ( $\dot{m}_{4-5}$ ), i.e., between the control volumes previously defined for the rectifier component, it is used the Eq. (13).

$$\dot{m}_{4-5} = kLa_{NH_3}^{(3)} (x_5 - x_{5,NH_3}^*) + kLa_{H_2O}^{(3)} (1 - x_5 - x_{5,H_2O}^*) \quad (13)$$

where  $x_5$  denotes the mass fraction of ammonia in the liquid phase present in the  $CV_5$ . The term  $T_5$  refers to the temperature of the  $CV_5$  and  $x_{5,NH_3}^*$  and  $x_{5,H_2O}^*$  can be written by the following correlations, calculated by:

$$x_{5,NH_3}^* = H_{NH_3,4-5} \cdot y_4 = \left( h_{0,NH_3} \cdot e^{-c_{NH_3} \cdot T_5} \right) \cdot y_4 \quad (14)$$

$$x_{5,H_2O}^* = H_{H_2O,4-5} \cdot (1 - y_4) = \left( h_{0,H_2O} \cdot e^{-c_{H_2O} \cdot T_5} \right) \cdot (1 - y_4) \quad (15)$$

and the volumetric mass transfer coefficients,  $kLa_{NH_3}^{(3)}$  and  $kLa_{H_2O}^{(3)}$ , is obtained through:

$$kLa_{NH_3}^{(3)} = kLa_{H_2O}^{(3)} = k_0 \cdot e^{\frac{-E_a}{T_5+273}} \quad (16)$$

After this, when applying the principle of mass conservation in a steady state and admitting that there is no mass accumulation in the  $CV$ 's, it is possible to define the mass flows  $\dot{m}_{2,v}$ ,  $\dot{m}_{ref}$ ,  $\dot{m}_{out}$ ,  $\dot{m}_{3,l}$  and  $\dot{m}_{in}$  through Eq. (17) to (21).

$$\dot{m}_{2,v} = \dot{m}_{1,v} - \dot{m}_{3-2} \quad (17)$$

$$\dot{m}_{ref} = \dot{m}_{2,v} - \dot{m}_{4-5} \quad (18)$$

$$\dot{m}_{out} = \dot{m}_{4-5} \quad (19)$$

$$\dot{m}_{3,l} = \dot{m}_{out} + \dot{m}_{3-2} \quad (20)$$

$$\dot{m}_{in} = \dot{m}_{1,v} - \dot{m}_{3,l} \quad (21)$$

where  $\dot{m}_{2,v}$  is the ammonia steam mass flow that leaves the  $CV_2$  and arrives the  $CV_4$ ,  $\dot{m}_{ref}$  is the mass flow of pure ammonia leaving the rectifier through the  $CV_4$ ,  $\dot{m}_{out}$  denotes the mass flow of liquid that arrives in the thermal generator through the process of condensation that occurs in the rectifier, that is, from  $CV_5$  to  $CV_3$ ,  $\dot{m}_{3,l}$  represents the mass flow of ammonia in the liquid phase transferred from  $CV_3$  to  $CV_1$  and, finally,  $\dot{m}_{in}$  corresponds to the mass flow of liquid entering  $CV_1$ .

### 3.2 Ammonia Mass Fraction on Liquid (x) and Steam (y) Phase

When performing a mass balance for each of the control volumes present in the thermal generator and rectifier, it is possible to write the equations of the ammonia mass fractions that are present in the  $CV_1$  to  $CV_5$ . Thus, it is defined that the mass fraction in the liquid phase with respect to time and present in  $CV_1$ ,  $x_1$ , can be written by:

$$\frac{dx_1}{dt} = \frac{\dot{m}_{in} \cdot x_{\text{tank}} + \dot{m}_{3,1} \cdot x_3 - \dot{m}_{1,v,NH_3}}{m_1} \quad (22)$$

where  $x_{\text{tank}}$  and  $m_1$  are, respectively, the mass fraction of liquid ammonia that accompanies the flow rate  $\dot{m}_{in}$  and the mass of the  $CV_1$ . The term  $\dot{m}_{1,v,NH_3}$  represents the mass flow of pure steam of ammonia that leaves the  $CV_1$  and can be calculated by:

$$\dot{m}_{1,v,NH_3} = kLa_{NH_3}^{(1)} (x_1 - x_{1,NH_3}^*) \quad (23)$$

where  $kLa_{NH_3}^{(1)}$  and  $x_{1,NH_3}^*$  can be calculated, respectively, from Eq. (04) and Eq. (05).

In a manner analogous to the previous development, the mass fraction of ammonia present in the steam form inside  $CV_2$ ,  $y_2$ , can be described by Eq. (24).

$$\frac{dy_2}{dt} = \frac{\dot{m}_{1,v,NH_3} - \dot{m}_{3-2,NH_3} - \dot{m}_{2,v,NH_3}}{m_2} \quad (24)$$

where  $m_2$  corresponds to the mass of the control volume  $CV_2$ . The terms  $\dot{m}_{3-2,NH_3}$  and  $\dot{m}_{2,v,NH_3}$  can be calculated through:

$$\dot{m}_{3-2,NH_3} = kLa_{NH_3}^{(2)} (x_3 - x_{3,NH_3}^*) \quad (25)$$

$$\dot{m}_{2,v,NH_3} = kLa_{NH_3}^{(1)} (x_1 - x_{1,NH_3}^*) - kLa_{NH_3}^{(2)} (x_3 - x_{3,NH_3}^*) \quad (26)$$

Besides it, the mass fraction of ammonia in the liquid phase present in  $CV_3$ , represented by  $x_3$ , is defined and can be calculated by:

$$\frac{dx_3}{dt} = \frac{\dot{m}_{3-2,NH_3} + \dot{m}_{out} \cdot x_5 - \dot{m}_{3,1} \cdot x_3}{m_3} \quad (27)$$

where  $m_3$  corresponds to the mass of the control volume  $CV_3$ .

The mass fractions of ammonia in the steam and liquid phases and that are presents in the  $CV_4$  and  $CV_5$ , respectively, represented by  $y_4$  and  $x_5$  can be calculated through Eq. (28) and (29).

$$\frac{dy_4}{dt} = \frac{\dot{m}_{2,v,NH_3} - \dot{m}_{ref,NH_3} - \dot{m}_{4-5,NH_3}}{m_4} \quad (28)$$

$$\frac{dx_5}{dt} = \frac{\dot{m}_{4-5,NH_3} + \dot{m}_{4-5} \cdot x_5}{m_5} \quad (29)$$

where  $m_4$  and  $m_5$  represent, respectively, the masses of  $CV_4$  and  $CV_5$ . The both terms  $\dot{m}_{4-5,NH_3}$  and  $\dot{m}_{ref,NH_3}$  can be obtained by the Eq. (30) and (31).

$$\dot{m}_{4-5,NH_3} = kLa_{NH_3}^{(3)} (x_5 - x_{5,NH_3}^*) \quad (30)$$

$$\dot{m}_{ref,NH_3} = kLa_{NH_3}^{(1)} (x_1 - x_{1,NH_3}^*) - kLa_{NH_3}^{(2)} (x_3 - x_{3,NH_3}^*) - kLa_{NH_3}^{(3)} (x_5 - x_{5,NH_3}^*) \quad (31)$$

considering that  $x_{5,NH_3}^*$  can be obtained from Eq. (14). In general, it is important to emphasize that the masses of the  $CV_1$  to  $CV_5$ , that is, from  $m_1$  to  $m_5$ , are accepted as constants and their respective values are shown in Table 1.

### 3.3 Control Volumes Temperatures

In order to be able to model the temperature behavior of each system control volume (from CV<sub>1</sub> to CV<sub>5</sub>), the application of the principle of energy conservation for open systems, as described in Eq. (02), was taken into account. Thus, for CV<sub>1</sub>, it is possible to write the temperature variation of this control volume with respect to time (dT<sub>1</sub>/dt) through Eq. (32).

$$\frac{dT_1}{dt} = \frac{\dot{m}_{in} \cdot h_{liq}(T_{in}) + \dot{m}_{3,1} \cdot h_{liq}(T_3) - \dot{m}_{1,v,NH_3} \cdot h_{v,NH_3}(T_1) + \dot{m}_{1,v,H_2O} \cdot h_{v,H_2O}(T_1) + \dot{Q}_{gen}}{m_1 \cdot c_{p,liq}} \quad (32)$$

where  $h_{liq}(T_{in})$  refers to the specific enthalpy of ammonia in the liquid phase with respect to temperature  $T_{in} = 20^\circ\text{C}$  and defined by the following mathematical relation:

$$h_{liq} = 5.03 \cdot T + 172.02 \quad (33)$$

and the term  $h_{liq}(T_3)$  corresponds to the specific enthalpy of ammonia in the liquid phase as a function of temperature  $T_3$  and can be obtained through the correlation shown in Eq. (33). Besides it, the term  $\dot{m}_{1,v,H_2O}$  is defined as:

$$\dot{m}_{1,v,H_2O} = kLa_{H_2O}^{(1)} (1 - x_1 - x_{1,H_2O}^*) \quad (34)$$

with  $kLa_{H_2O}^{(1)}$  and  $x_{1,H_2O}^*$  calculated, respectively, through Eq. (04) and (06). The terms  $h_{v,NH_3}$  and  $h_{v,H_2O}$  correspond, respectively, to the specific enthalpies of ammonia and water in the steam phase. It is important to note that, in this case, both enthalpies are related to temperature  $T_1$  and can be calculated by:

$$h_{v,NH_3} = -0.0142 \cdot T^2 + 1.3677 \cdot T + 1438.8 \quad (35)$$

$$h_{v,H_2O} = 1.7744 \cdot T_1 + 2502.9 \quad (36)$$

Besides it, the term  $c_{p,liq}$  represents the specific heat at the constant pressure of the liquid solution of ammonia and water, admitted as constant and presented in Table 1. The term  $\dot{Q}_{gen}$  denotes the heat transfer rate transferred from the absorption refrigeration system power supply to the generator, calculated by:

$$\dot{Q}_{gen} = (UA)_{gen} \cdot (T_g - T_1) \quad (37)$$

where  $(UA)_{gen}$  and  $T_g$  represents, respectively, the thermal conductance and the temperature of the exhaust gas from the burning of LPG (Liquefied Petroleum Gas) in combustion chamber. Both terms are constants and can be obtained from Table 1.

The temperature variation of the CV<sub>2</sub> with respect to time,  $dT_2/dt$ , can be expressed through:

$$\frac{dT_2}{dt} = \frac{A - B - C}{m_2 \cdot c_{p,vap}} \quad (38)$$

where:

$$A = \dot{m}_{1,v,NH_3} \cdot h_{v,NH_3}(T_1) + \dot{m}_{1,v,H_2O} \cdot h_{v,H_2O}(T_1) \quad (39)$$

$$B = \dot{m}_{3-2,NH_3} \cdot h_{v,NH_3}(T_2) + \dot{m}_{3-2,H_2O} \cdot h_{v,H_2O}(T_2) \quad (40)$$

$$C = \dot{m}_{2,v,NH_3} \cdot h_{v,NH_3}(T_2) + \dot{m}_{2,v,H_2O} \cdot h_{v,H_2O}(T_2) \quad (41)$$

where the terms  $h_{v,NH_3}(T_1)$ ,  $h_{v,H_2O}(T_1)$ ,  $h_{v,NH_3}(T_2)$  and  $h_{v,H_2O}(T_2)$  can be calculated through Eq. (35) and (36). In addition, the value of  $c_{p,vap}$  is shown in Table 1 and  $\dot{m}_{3-2,H_2O}$  can be calculated by:

$$\dot{m}_{3-2,H_2O} = kLa_{H_2O}^{(2)} (1 - x_3 - x_{3,H_2O}^*) \quad (42)$$

with  $x_{3,H_2O}^*$  obtained from Eq. (11).

Similar to the previous development, the temperature distributions over time with respect to  $CV_3$  ( $dT_3/dt$ ),  $CV_4$  ( $dT_4/dt$ ) and  $CV_5$  ( $dT_5/dt$ ) are presented, respectively, through Eq. (43), (44) and (45).

$$\frac{dT_3}{dt} = \frac{\dot{m}_{3-2,NH_3} \cdot h_{v,NH_3}(T_2) + \dot{m}_{3-2,H_2O} \cdot h_{v,H_2O}(T_2) + \dot{m}_{out} \cdot h_{liq}(T_5) - \dot{m}_{3,l} \cdot h_{liq}(T_3)}{m_3 \cdot c_{p,liq}} \quad (43)$$

$$\frac{dT_4}{dt} = \frac{D - E - F - \frac{(UA)_{rect} \cdot (T_4 - T_w)}{2}}{m_4 \cdot c_{p,vap}} \quad (44)$$

$$\frac{dT_5}{dt} = \frac{\dot{m}_{4-5,NH_3} \cdot h_{v,NH_3}(T_4) + \dot{m}_{4-5,H_2O} \cdot h_{v,H_2O}(T_4) - \dot{m}_{out} \cdot h_{liq}(T_5) - \frac{(UA)_{rect} \cdot (T_4 - T_w)}{2}}{m_5 \cdot c_{p,liq}} \quad (45)$$

where:

$$D = \dot{m}_{2,v,NH_3} \cdot h_{v,NH_3}(T_2) + \dot{m}_{2,v,H_2O} \cdot h_{v,H_2O}(T_2) \quad (46)$$

$$E = \dot{m}_{ref,NH_3} \cdot h_{v,NH_3}(T_4) + \dot{m}_{ref,H_2O} \cdot h_{v,H_2O}(T_4) \quad (47)$$

$$F = \dot{m}_{4-5,NH_3} \cdot h_{v,NH_3}(T_4) + \dot{m}_{4-5,H_2O} \cdot h_{v,H_2O}(T_4) \quad (48)$$

where  $(UA)_{rect}$  represents the thermal conductance in rectifier and  $\dot{m}_{4-5,H_2O}$  can be calculated through Eq. (49) and  $x_{5,H_2O}^*$  obtained from Eq. (15).

$$\dot{m}_{4-5,H_2O} = kLa_{H_2O}^{(3)} (1 - x_5 - x_{5,H_2O}^*) \quad (49)$$

#### 4. NUMERICAL RESULTS AND DISCUSSIONS

The main results presented in this study are the temperature profiles of the ammonia and water solution and the behavior of the ammonia mass fractions in the liquid and steam phases for the control volumes of the thermal generator and rectifier of an absorption refrigeration system in transient regime.

The numerical results were produced with a code written in Fortran<sup>®</sup>-based mathematical model described in Section 3 and the system of ordinary differential and algebraic equations was numerically integrated in time using the 4<sup>th</sup>-5<sup>th</sup> order Runge-Kutta method with some initial conditions, as showed in Table 2. It is important to emphasize that the mass fraction of the solution in steam phase ( $y$ ) was considered constant during all simulation and stipulated  $y_2 = 0,80$  (or 80%) and  $y_4 = 0,99$  (or 99%).

Table 2. Initials Conditions used in the Simulation of the Mathematical Model.

Variables	CV <sub>1</sub>	CV <sub>2</sub>	CV <sub>3</sub>	CV <sub>4</sub>	CV <sub>5</sub>
Initial Temperatures [°C]	20	20	20	20	20
Initial Mass Fraction [ - ]	0,50	0,80	0,50	0,99	0,50

Source: The Authors (2017).

Figure 2 shows the distribution of the temperatures  $T_1$  to  $T_5$ , which correspond to the control volumes  $CV_1$  to  $CV_5$ , during a period of computational simulation of 5000 seconds. According to the graph, it is possible to notice that there is a time difference so that the temperatures of  $CV_1$  ( $T_1$ ),  $CV_2$  ( $T_2$ ),  $CV_4$  ( $T_4$ ) and  $CV_5$  ( $T_5$ ) reach the permanent regime. The steady state was reached by the temperatures  $T_1$ ,  $T_2$ ,  $T_4$  and  $T_5$ , respectively, in 500 s, 750 s, 1000 s and 2500 s. In general, as expected, the temperatures of the thermal generator control volumes are higher when compared to temperature of the rectifier control volumes, because the generator is the subsystem that is directly connected to the heat source ( $\dot{Q}_{gen}$ ) of the absorption refrigeration prototype.

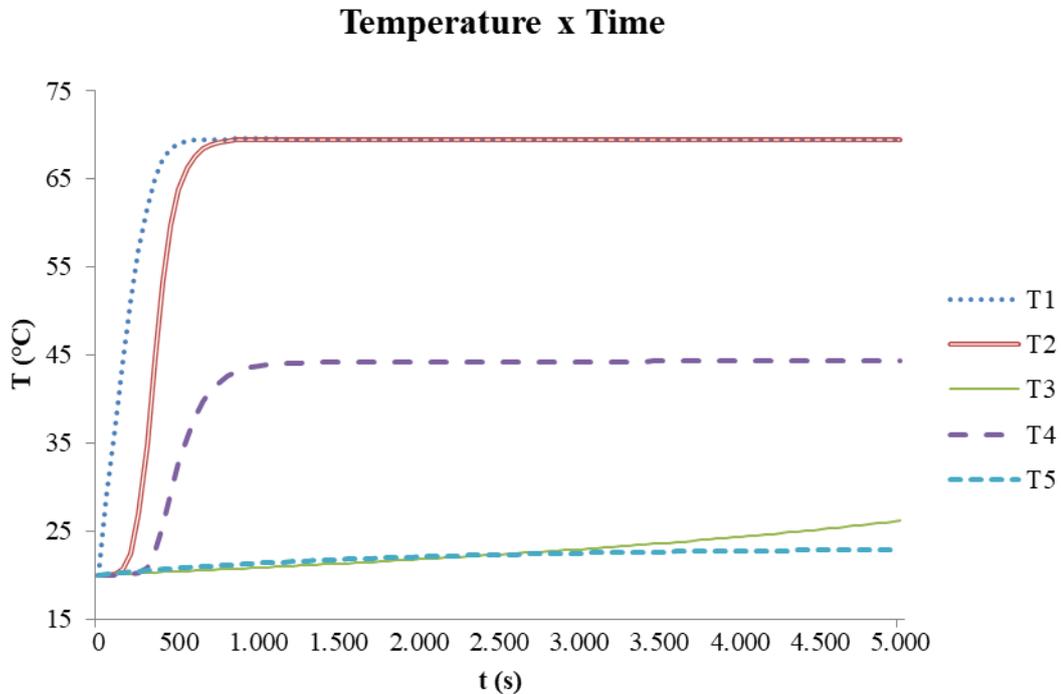


Figure 2. Temperature Distribution in all Control Volumes of the System. Source: The Authors (2017).

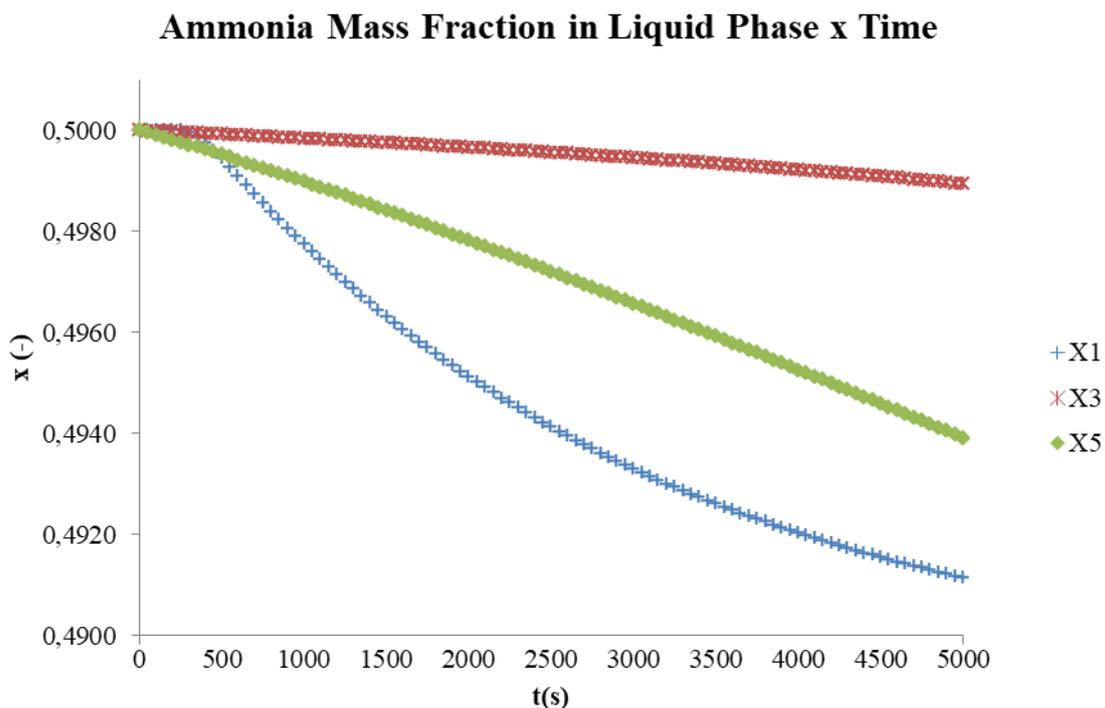


Figure 3. Ammonia Mass Fraction of the Control Volumes in Liquid Phase. Source: The Authors (2017).

Figure 3 shows the evolution of the values of the mass fraction of ammonia with time. Thus, it can be stated that the proposed mathematical model is able to calculate the variation of the mass fraction of ammonia in the liquid phase with time for the generator and the rectifier. As the mass fractions were modeled by transient analysis, the same initial values of "x" were adopted for all control volumes in the liquid phase ( $x = 0.50$ ), as shown in Table 2. In this case, the computational simulations were also 5000 seconds and, as expected through experimental and numerical observations, the mass fractions of ammonia in the liquid phase decrease over time. In the case of  $CV_1$ , the value of "x" reaches  $x_1 = 0.491$ ; in  $CV_3$ ,  $x_3 = 0.498$  and in  $CV_5$ , the  $x_5 = 0.494$ . In order to obtain a better representation of this behavior, the authors suggest that the model be treated in a dimensionless way.

## 5. CONCLUSIONS

This study presents a transient mathematical model and computational simulation, developed through the application of the mass and energy conservation principles, for the thermal generator and the rectifier of an absorption refrigeration system that uses ammonia and water as work fluids. Through the computational simulations of the mathematical model developed in this work, it was possible to obtain the temperatures distribution profile and the behavior of the ammonia mass fraction in liquid phase of the control volumes that compose the system in analysis. In general, the numerical results have been consistent with the experimental data, showing the effectiveness of the model for procedures of simulation, design and optimization of the generator and rectifier of an absorption refrigeration prototype.

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## 7. REFERENCES

- Brasil. MME – Ministério de Minas e Energia. Balanço Energético Nacional 2016: Ano Base 2015, 2016.
- Bruno, J.C. Miquel, J. Castells, S.F., 1999. Modeling of Ammonia Absorption Chillers Integration in Energy Systems of Process Plants. *Applied Thermal Engineering*, v. 19, p. 1297-1328.
- Cardoso, R.B. Nogueira, L.A.H., 2007. Estimativa do Consumo de Energia Elétrica em Refrigeradores no Setor Residencial Brasileiro. *Revista Brasileira de Energia*, v. 13, p. 55-67.
- Chua, H. T. Toh, H.K. Ng K.C., 2002. Thermodynamic Modeling of an Ammonia-Water Absorption Chiller. *International Journal of Refrigeration*, v. 25, p. 896-906.
- Cortez, L. A. B., Muhle, I. N., and Silva, A., 1994, "Refrigeração por Absorção com o par Água-Amônia e seu Potencial no Caso Brasileiro", *Revista ABRAVA*, p. 33-38.
- Hudson, D. W., 2002, "Ammonia Absorption Refrigeration Plant", *The Official Journal of AIRATH*, p.26-30.
- Kim, B. Park, J., 2007. Dynamic Simulation of a Single-Effect Ammonia-Water Absorption Chiller. *International Journal of Refrigeration*, v. 30, p. 535-545.
- Martinho, L.C.S., 2013. Modelagem, Simulação e Otimização de Refrigeradores por Absorção. Tese de Doutorado. Universidade Federal do Paraná, Curitiba.
- Martinho, L.C.S. Vargas, J.V.C. Balmant, W. Ordonez, J.C., 2016. A Single Stage Absorption Refrigeration System Dynamic Mathematical Modeling, Adjustment and Experimental Validation. *International Journal of Refrigeration*, v. 68, p. 130-144.
- Nunes, T.K., 2015. Modelagem, Simulação e Otimização de Sistemas de Refrigeração por Compressão de Vapor. Tese de Doutorado. Universidade Federal do Paraná, Curitiba.
- Vargas, J.V.C. Ordonez, J.C. Dilay, E. Parise, J.A.R., 2009. Modeling, Simulation and Optimization of a Solar Collector Driven Water Heating and Absorption Cooling Plant. *Solar Energy*, v. 83, p. 1232-1244.
- Vargas, J.V.C. Parise, J.A.R. Ledezma, G.A. Bianchi, M.V.A., 2000. Thermodynamic Optimization of Heat-Driven Refrigerators in the Transient Regime. *Heat Transfer Engineering*, v. 25, p. 35-45.

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