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ON SIMULATIONS AND AN IMPLEMENTATION OF CHAOTIC AND HYPERCHAOTIC MOTIONS OF LORENZ MODELS

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Abstract. *This work is comprised by simulations and experiment of four dimensions generalized Lorenz system with hyperchaos behavior. The system studied was analyzed concerned to Lyapunov exponents' spectrum and, for the designed parameters, the implementation was supposed to have hyperchaos characteristics, what we indeed inferred from qualitative comparison of time series waveforms and phase planes with simulations. Furthermore, we implemented a simplified proportional feedback control in just one dimension of the system, achieving the expected result of following continuous and quasi-sinusoidal references.*

Keywords: *Otimal Control, Nonlinear, Feedback Control*

1. INTRODUCTION

Chaos and hyperchaos study has applications in aeronautics, communications, chemical process, even in biological systems. One of major interest in the study of chaos is that real systems usually are controlled taking into account simplified hypothesis that cut out nonlinearities, delays, dimensions, thermal errors and effects. The control engineer sometimes, set the control parameters with fast and save calculations and does not explore further possibilities and optimizations. However, nowadays the productivity and economic requirements urge the control designer to extract the best of a process and consider conditions that require other approaches. Frequently, one important consequence is related to system's behavior that, excited and controlled in an amplified state and control ranges, or restricted time responses, may perform oscillations characterized as chaos and hyperchaos.

Our work presents simulations and implementation of a four-dimensional Lorenz system. According to Macek and Strumik (2010), this system under specific parameters has hyperchaos characteristics. Actually, we simulated the system and compared the result against a physical implementation on an electronic circuit. Under qualitative validation of the time responses, we observed similar waveforms and behavior. Furthermore, we implemented a simplified version of proportional feedback control Rafikov and Balthazar (2008), archiving the expected result.

In section 2 we present the experimental procedure to run the simulations and the implementation setup. In section 3 we show the results and present a short discussion on data interpretation.

2. EXPERIMENTAL PROCEDURES

2.1 Background on Lorenz

The generalized Lorenz system is described by Macek and Strumik (2010) as:

$$\begin{cases} \frac{dX}{dt} &= \sigma * (Y - X) - \omega_0 W \\ \frac{dY}{dt} &= -X * Z + R * X - Y \\ \frac{dZ}{dt} &= X * Y - b * Z \\ \frac{dW}{dt} &= \omega_0 * Z - \sigma_m * W \end{cases} \tag{1}$$

We adjusted the signals to analogue to digital converter (ADC) with:

$$x = \frac{X}{e};\ y = \frac{Y}{f};\ z = \frac{Z}{g};\ w = \frac{W}{h} \tag{2}$$

and

$$\begin{cases} \frac{dx}{dt} &= \sigma * (y * \frac{f}{e} - x) - \omega_0 w * \frac{h}{e} \\ \frac{dY}{dt} &= -x * z * \frac{e*g}{f} + R * x * \frac{e}{f} - y \\ \frac{dZ}{dt} &= x * y * \frac{e*f}{g} - b * z \\ \frac{dW}{dt} &= \omega_0 * z * \frac{g}{h} - \sigma_m * w \end{cases} \tag{3}$$

According to Macek and Strumik (2010), with:

$$\sigma = 10.0;\ \omega_0 = 5.95;\ R = 460.0;\ \sigma_m = 0.1;\ b = \frac{8}{3} \tag{4}$$

and

$$e = 90;\ f = 120;\ g = 360;\ h = 90 \tag{5}$$

the system has hyperchaos behavior.

2.2 Simulations

The initial system simulations were made in an integrated modeling and simulation environment based in the JModelica tool and Python script language.

We made parameters variation simulations for Lyapunov exponents' estimation in MATLAB®.

2.3 Implementation

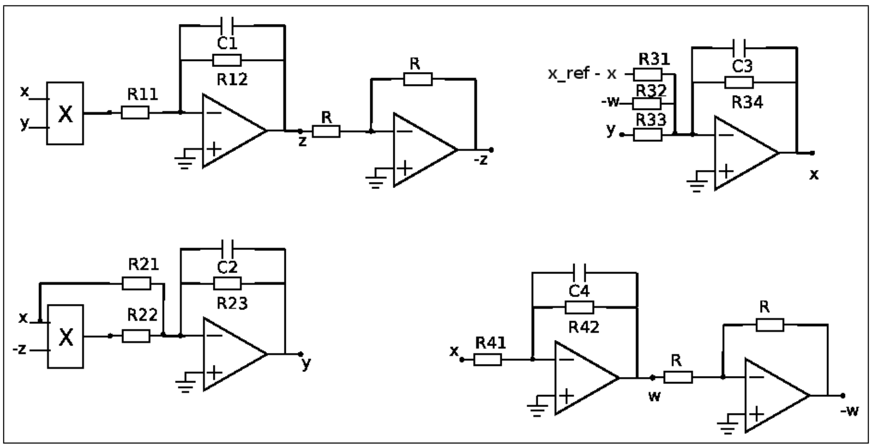


Figure 1. Circuitry Project. Source: The authors.

The components values are:

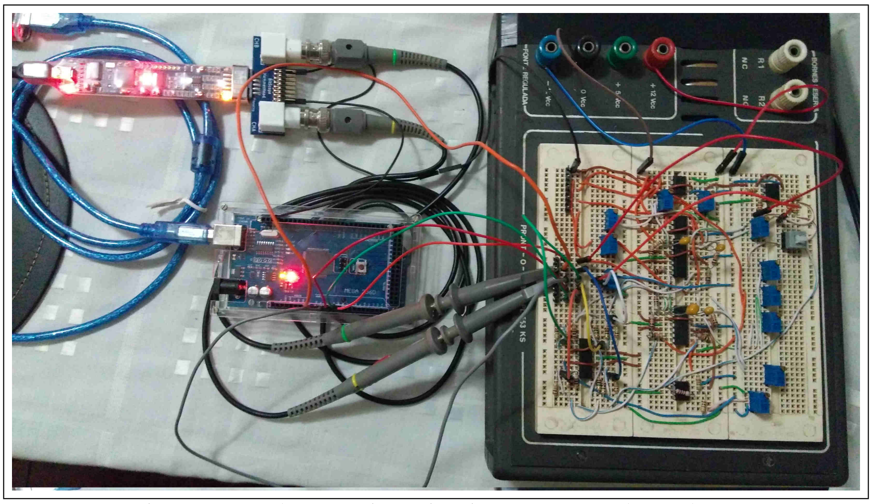


Figure 2. Implementation Circuitry. Source: The authors.

$$\begin{cases} C_1 = C_2 = C_3 = 1.0\mu F; C_4 = 10.0\mu F \\ R_{11} = 3.3K\Omega; R_{12} = 374K\Omega; R = 10K\Omega; \\ R_{21} = 2.9K\Omega; R_{22} = 370\Omega; R_{23} = 1M\Omega; \\ R_{31} = 8.0K\Omega; R_{32} = 168K\Omega; R_{33} = 75K\Omega; R_{34} = 100K\Omega; \\ R_{41} = 16.8K\Omega; R_{42} = 1M\Omega; \end{cases} \quad (6)$$

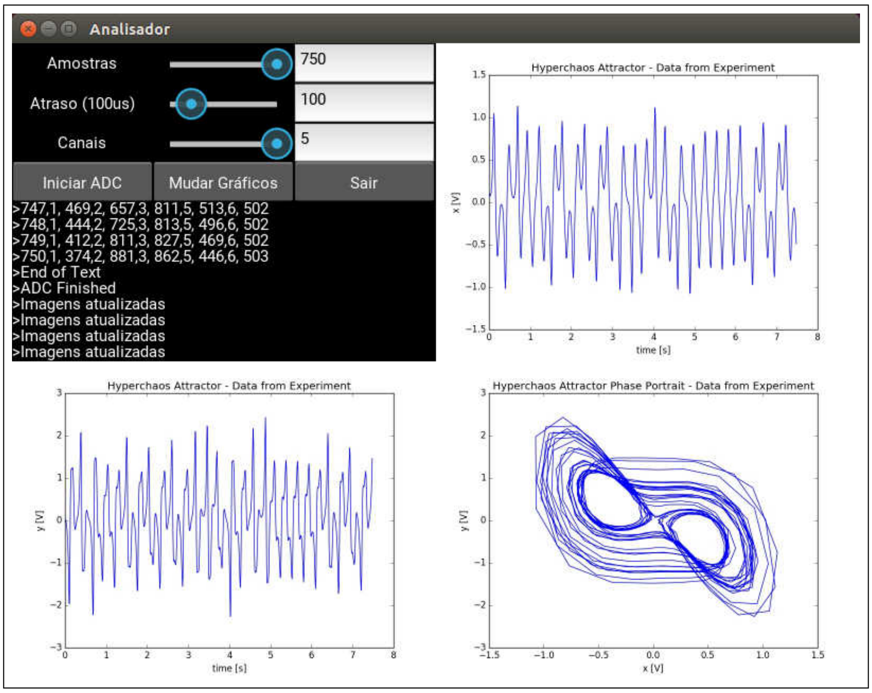
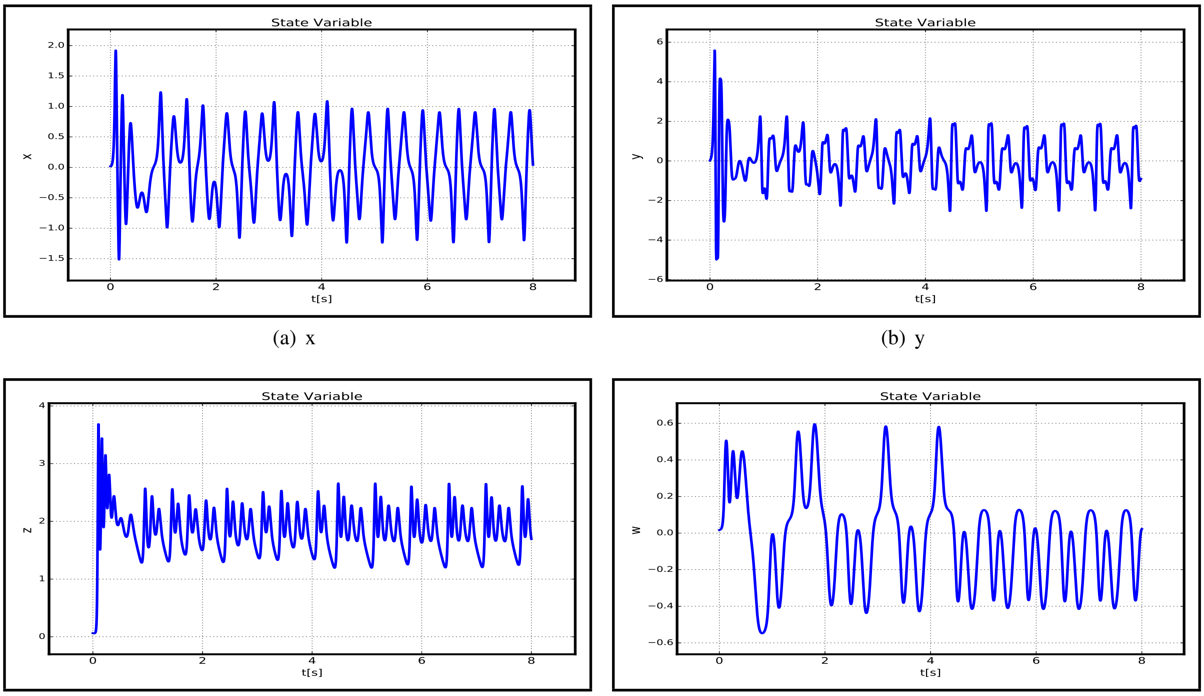


Figure 3. Implementation Circuitry. Source: The authors.

3. RESULTS AND DISCUSSION

3.1 Simulations Results

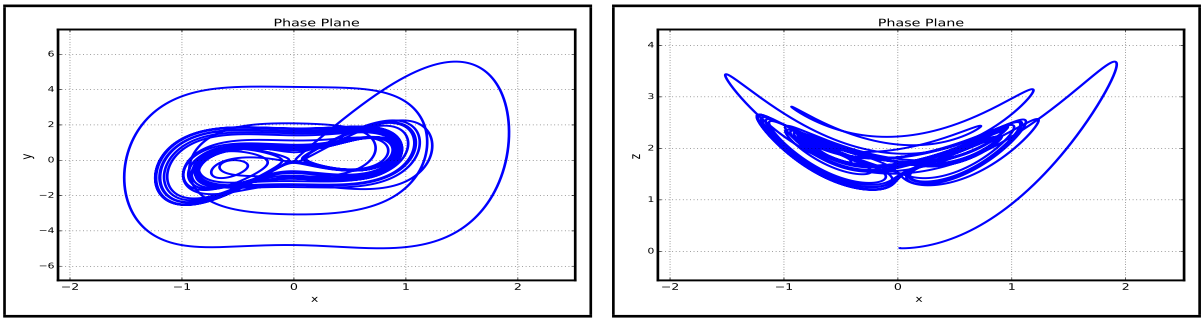
The initial condition was:



(a) x (b) y

(c) z (d) w

Figure 4. System Simulation. Source: the authors.



(a) xy (b) xz

Figure 5. System Simulation. Phase Planes. Source: the authors.

$$x_0 = \frac{1.0}{e};\ y_0 = \frac{1.0}{f};\ z_0 = \frac{15.0}{g};\ w_0 = \frac{1.0}{h} \tag{7}$$

3.1.1 The Uncontrolled System

The results are presented in figures (4) to (6):

3.1.2 Lyapunov Exponents' (LE) Estimative

The simulation results for first and second LE are shown in (7) and (8):

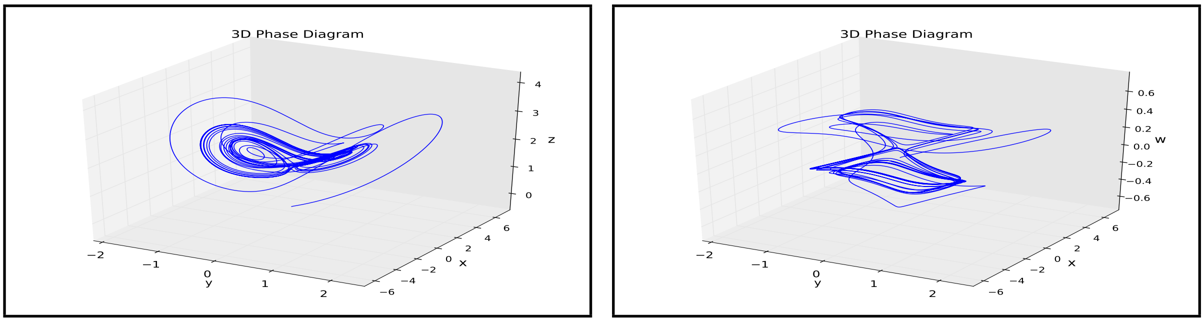
3.1.3 The Controlled System

The results are presented in figures (9) to (12):

We must clarify a divergence from Rafikov and Balthazar (2008). We used only one control component restricted to the proportional feedback error just for 'x' state variable.

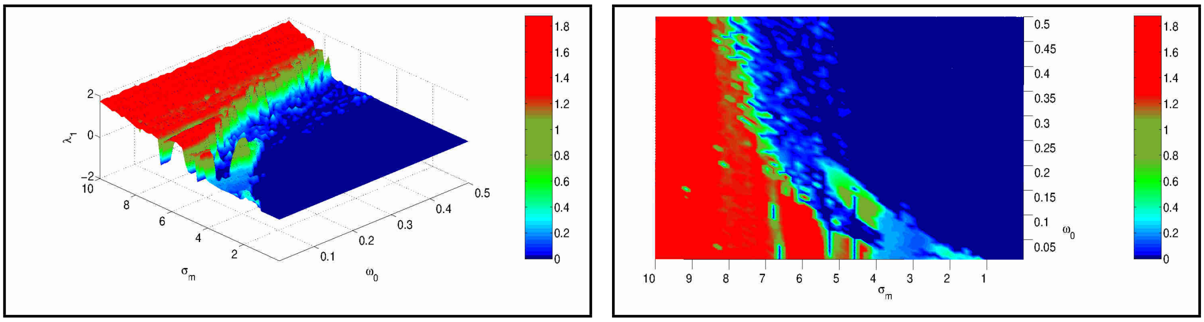
All the matrix calculations of linear feedback control Rafikov and Balthazar (2008) were performed with final control:

$$u_1 = -124.4 * (x - x_{ref}) \tag{8}$$



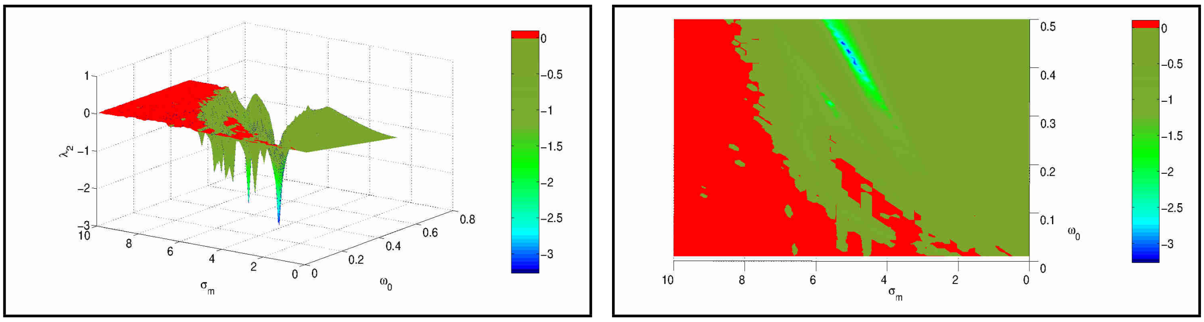
(a) xyz (b) xyw

Figure 6. System Simulation. 3D Phase Planes. Source: the authors.



(a) 3D (b) 2D

Figure 7. First LE. Source: the authors.



(a) 3D (b) 2D

Figure 8. Second LE. Source: the authors.

This control law was not designed to result in hard trajectory following with zero error, instead, our goal was to stabilize the system far from hyperchaos behavior.

3.2 Implementation Results

3.2.1 The Uncontrolled System

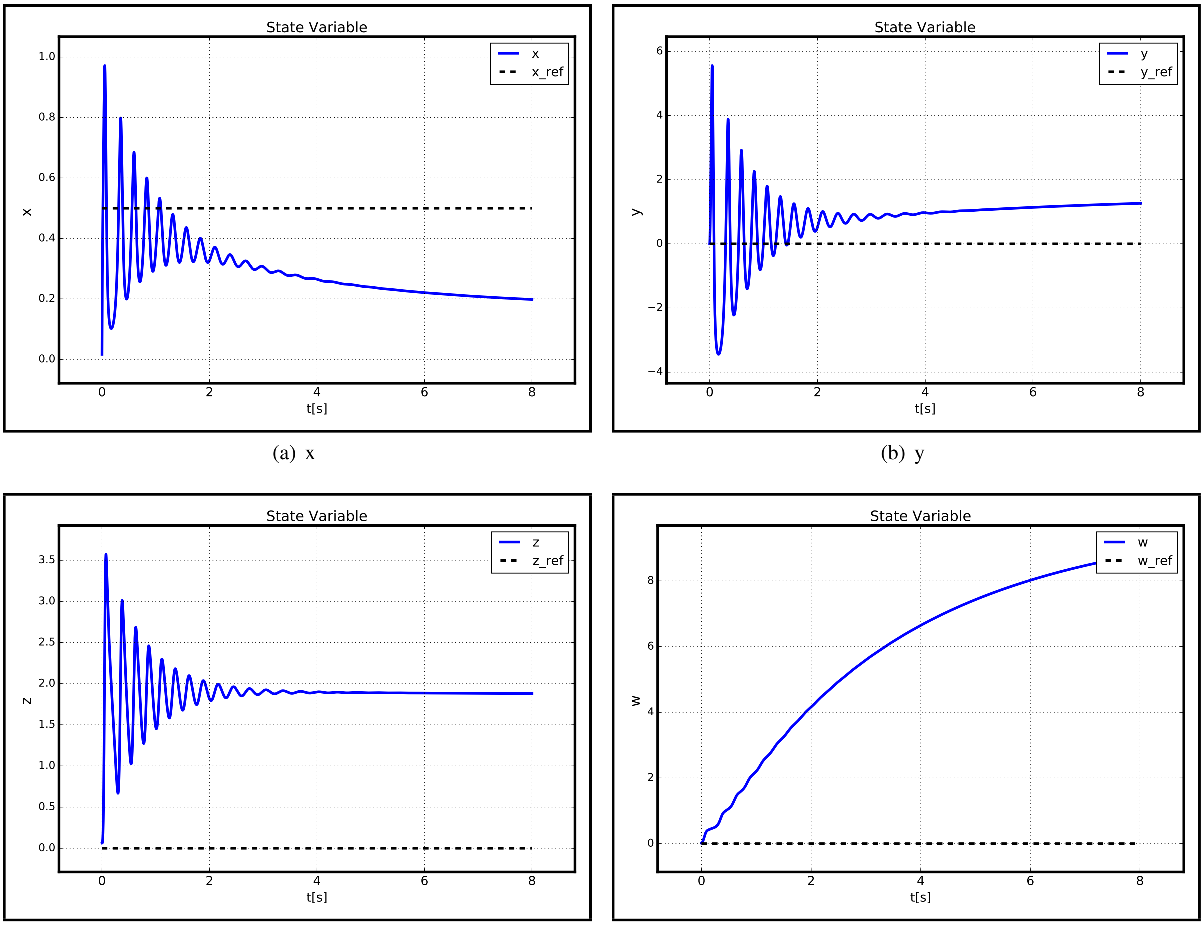
The results are presented in figures (13) to (15):

3.2.2 The Controlled System

The results are presented in figures (16) to (18):
The experience with a quasi-sinusoidal reference is shown in figure (19):

3.3 Discussion

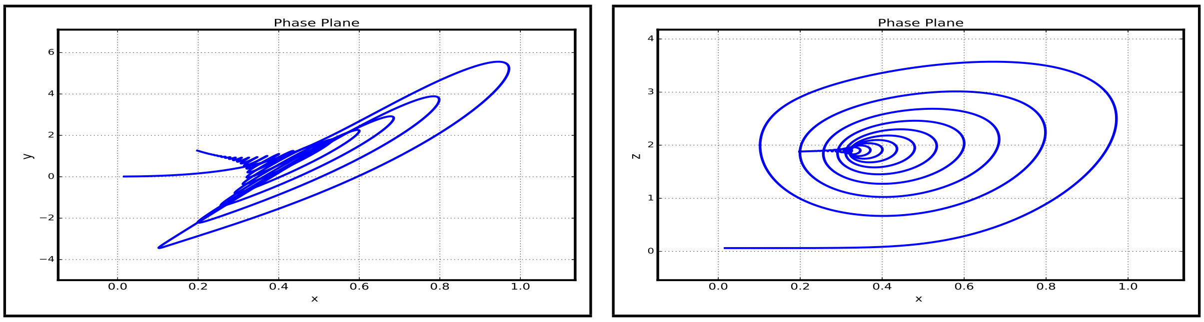
Despite the effort to get numerically verification of hyperchaos on experimental data, we could not estimate LE for the experimental time series as indicated for hyperchaos. However, the waveforms and phase portraits from results were qualitatively similar to simulations. Our understanding is that the experiment importance is the verification that a four dimension system, supposed to be in hyperchaos behavior for the designed parameters, has been taking from oscillations



(a) x (b) y

(c) z (d) w

Figure 9. Simulation of Controlled System. Source: the authors.



(a) xy (b) xz

Figure 10. Simulation of Controlled System. Phase Planes. Source: the authors.

to follow continuous and quasi-sinusoidal references by actuation in only one-dimensional control.

4. CONCLUSIONS

This work presented the results of simulation and implementation of a Lorenz hyperchaotic system. We designed an one-dimensional control that was succeeded on making a variable to follow continuous and quasi-sinusoidal references.

The result suggests an alternative approach on how to interact and control a hyperchaos attractor with just one-dimensional control.

5. ACKNOWLEDGEMENTS

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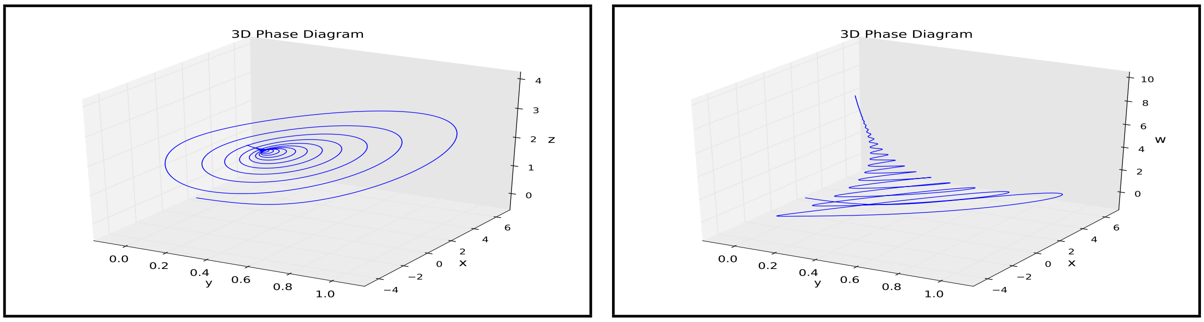

(a) xyz (b) xyw

Figure 11. Simulation of Controlled System. 3D Phase Planes. Source: the authors.

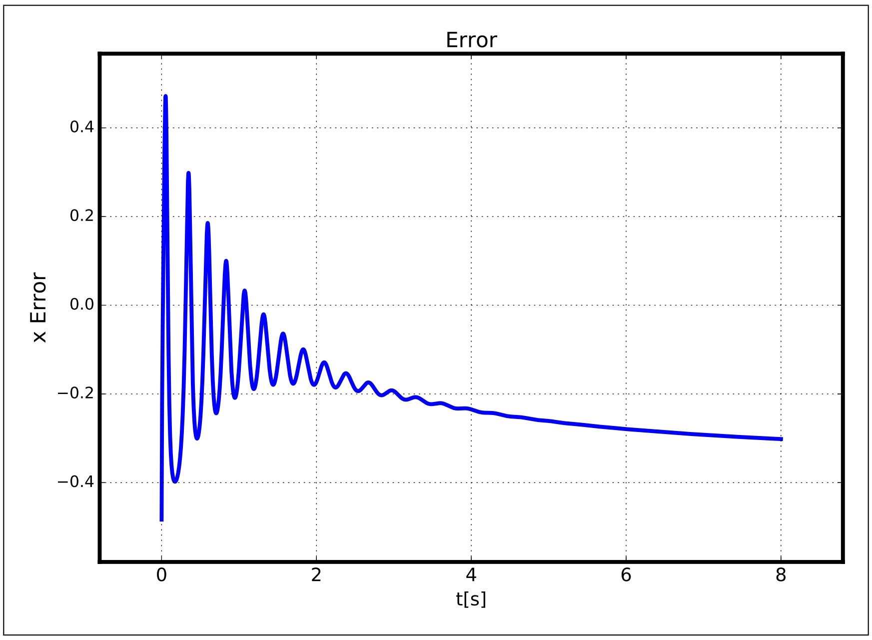

Figure 12. Simulated Error. Source: The authors.

6. REFERENCES

Macek, W.M. and Strumik, M., 2010. “Model for hydromagnetic convection in a magnetized fluid”. *Phys. Rev. E*, Vol. 82, p. 027301. doi:10.1103/PhysRevE.82.027301. URL `http://link.aps.org/doi/10.1103/PhysRevE.82.027301`.

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7. RESPONSIBILITY NOTICE

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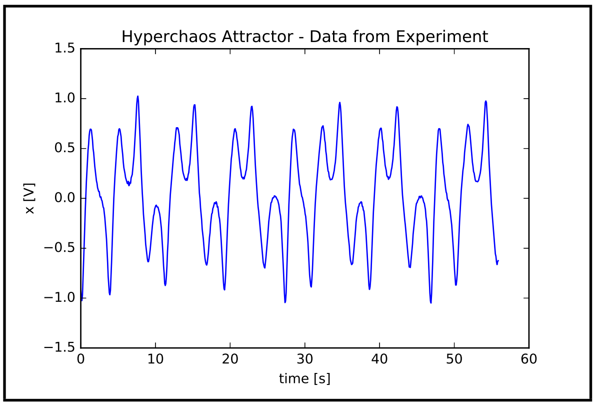
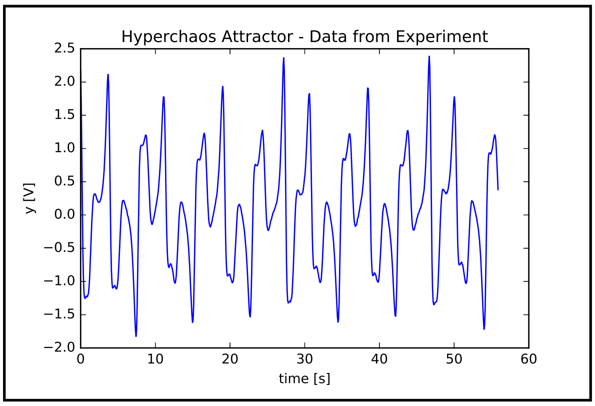
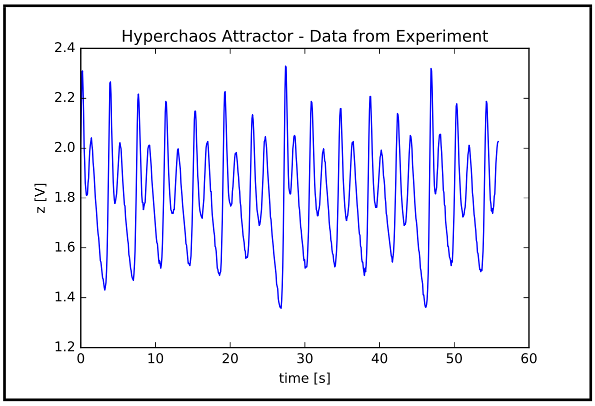


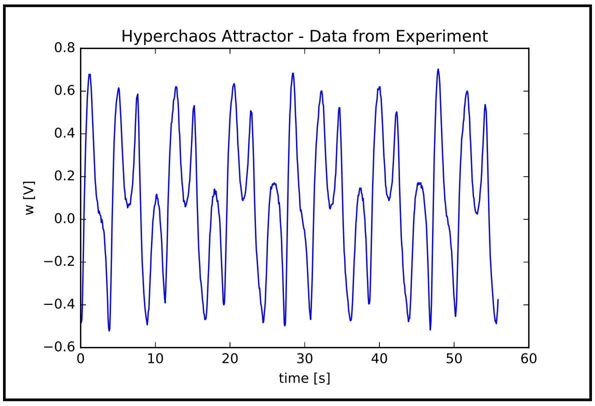
(a) x



(b) y

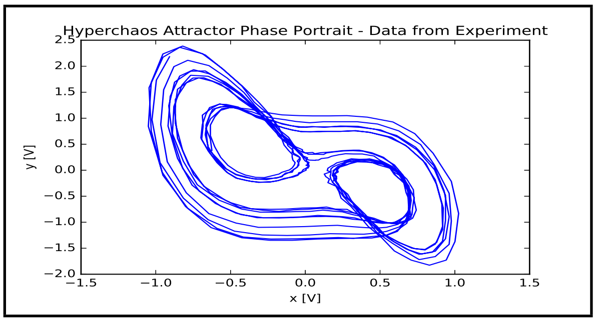


(c) z

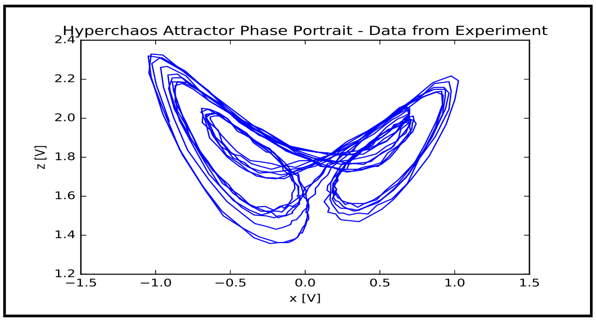


(d) w

Figure 13. Implementation of Uncontrolled System. Source: the authors.

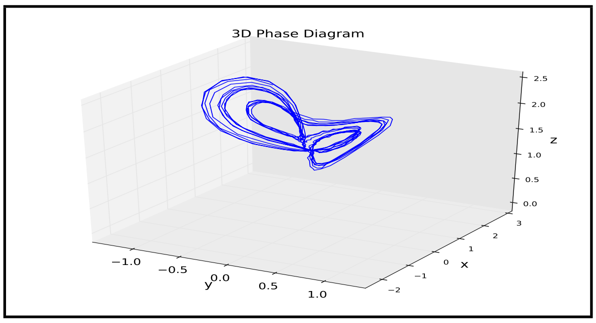


(a) xy

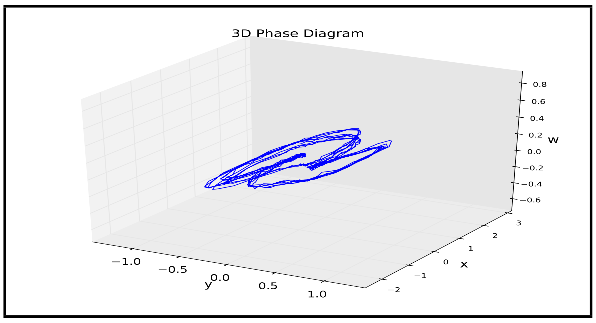


(b) xz

Figure 14. Implementation of Uncontrolled system. Phase Planes. Source: the authors.



(a) xyz



(b) xzw

Figure 15. Implementation of Uncontrolled system. 3D Phase Planes. Source: the authors.

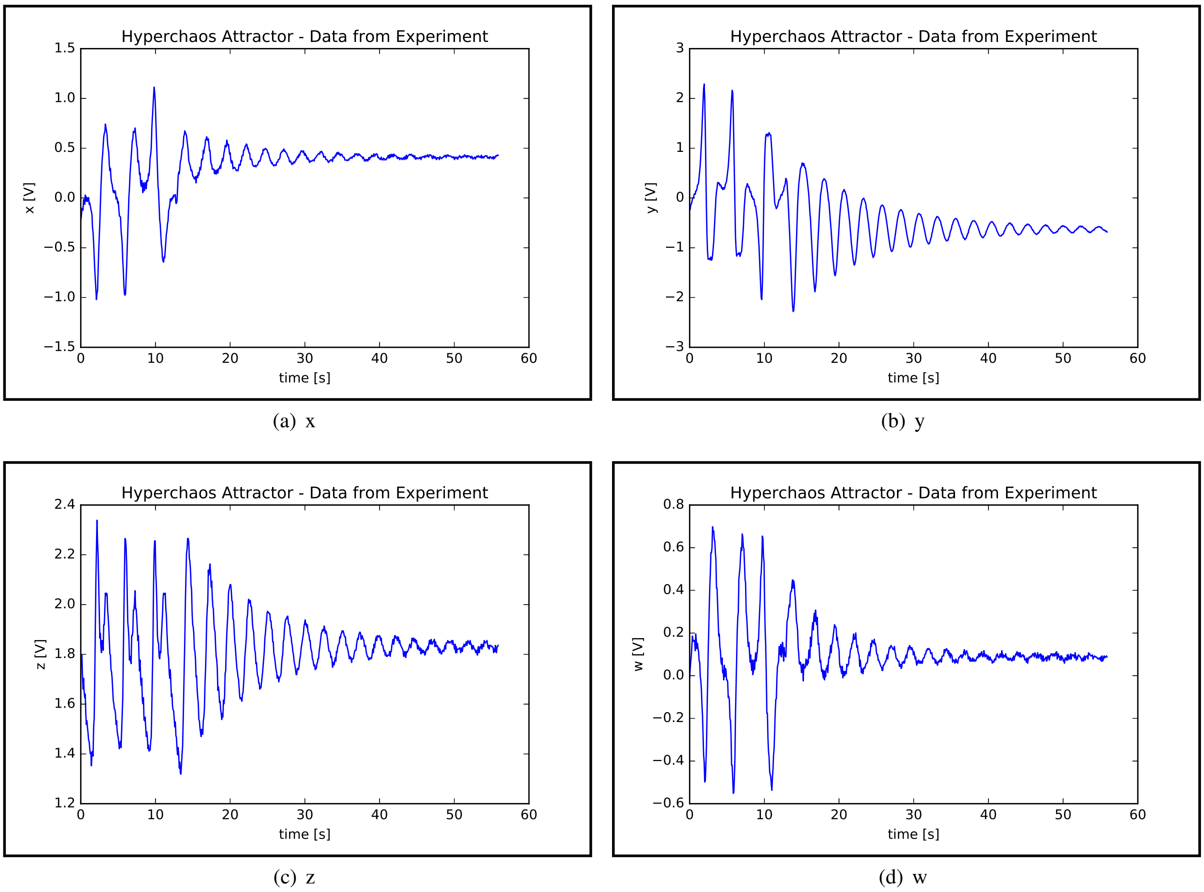


Figure 16. Implementation of Controlled System. Source: the authors.

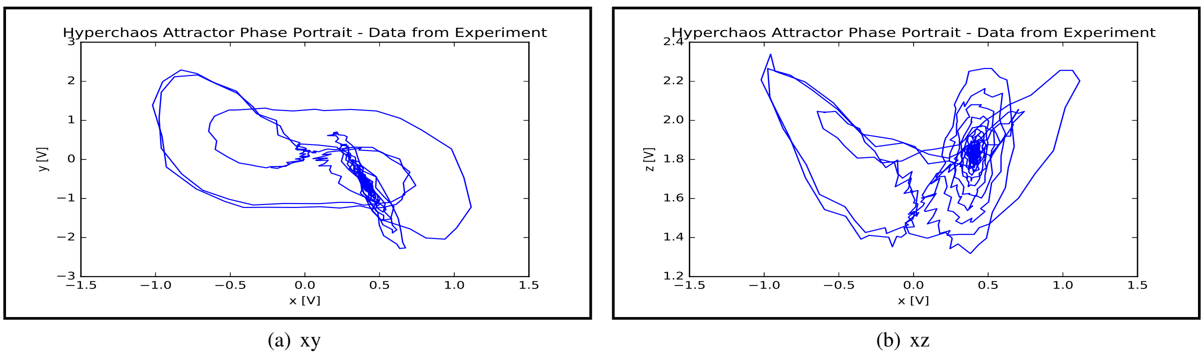


Figure 17. Implementation of Controlled System. Phase Planes. Source: the authors.

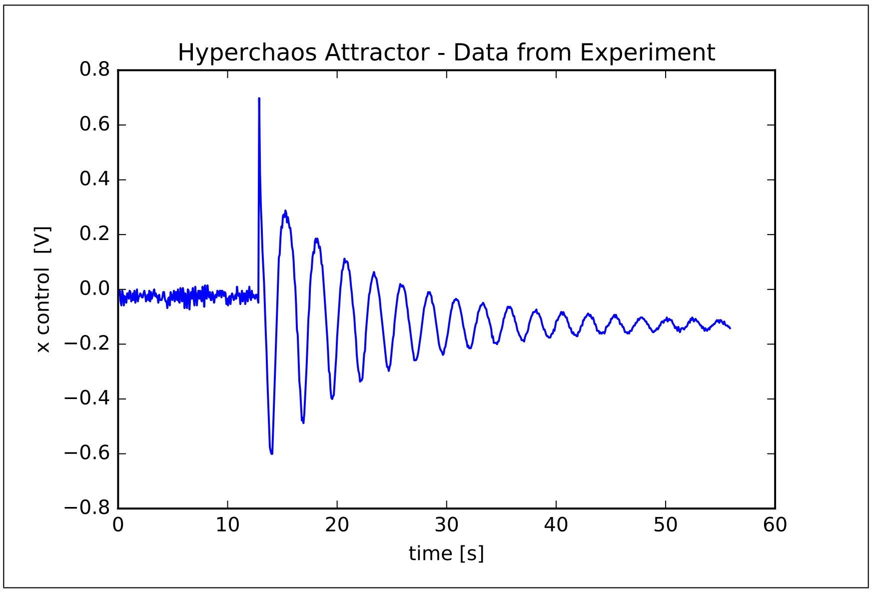
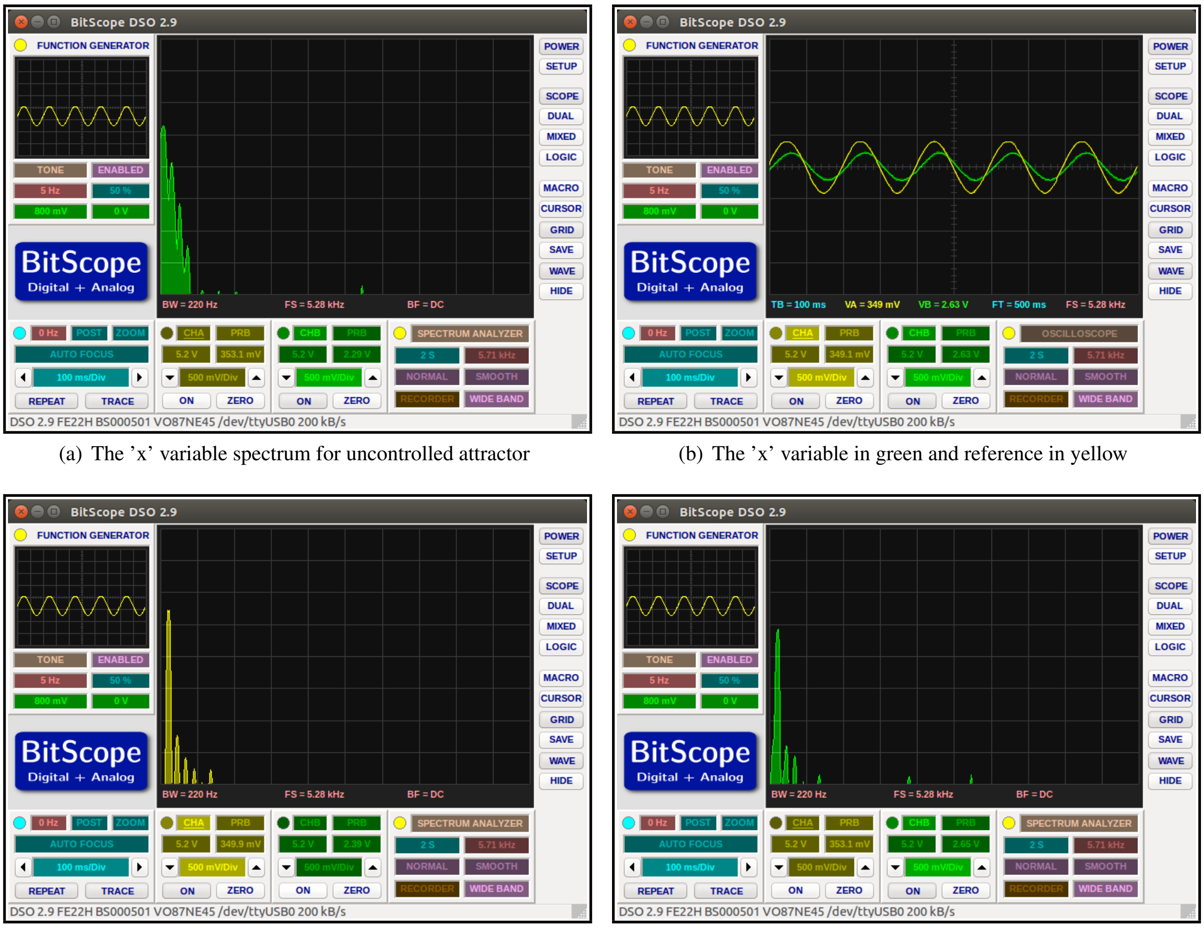


Figure 18. Observed x-error: Control before gain. Source: The authors.



(a) The 'x' variable spectrum for uncontrolled attractor

(b) The 'x' variable in green and reference in yellow

(c) The quasi-sinusoidal reference spectrum

(d) The controlled 'x' variable spectrum

Figure 19. Following a Quasi-Sinusoidal Reference. Source: the authors.