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BICYCLES AND MOTORCYCLES AS AN EDUCATIONAL OBJECT FOR MODELING, ANALYSIS AND SIMULATION OF STABILITY, SPEED, SUSPENSIONS, ATTITUDE AND TRAJECTORY CONTROL

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Abstract. *This article presents linearized models of motorcycles and bicycles built with the main objective of introducing analytical and numerical analysis tools for students. Using the methodology of problem-based learning, the analysis and simulation of control systems were developed by both state variables and transfer functions approaches. The first step is the description and operation of the system's individual components; the mathematical models of their isolated and integrated elements are developed and interpreted and, at last, the simulation and analysis of the open-loop system dynamic behavior is carried out. Once the behaviors that need change are identified, such as oscillations or instability, the appropriate control strategies are employed in order to achieve the desired system performance. Finally, the simulation and analysis of the closed-loop system dynamic behavior are made and improvements in the system performance are verified. All simulations and analysis of the open and closed loop systems are executed with the aid of Simulink / MATLAB.*

Keywords: *Engineering Education, Educational Object for Mechanical and Control and Automation Engineering, Dynamic and Control Systems, Motorcycle and Bicycle Dynamics, Simulation in MATLAB / Simulink*

1. INTRODUCTION

With the popularization and cheapening of mechatronics systems in the last decades, student's interest in Control and Automation or Mechatronics Engineering has arisen, creating a foreseen need of multidisciplinary disciplines to deal with these complex systems, which combine mechanical, electronics and programming studying topics. One of its most complete and analyzed subjects are vehicles in general – aerial, ground and marine – an interdisciplinary field in which theoretical and practical knowledge of modelling, simulation and control of dynamic systems are needed.

In order to further motivate students with practical examples, the Mechatronic System Development Laboratory (*LDSM*, in Portuguese) was created at the Mechanical Engineering Department, a multidisciplinary environment where a series of equipment and components are available for testing, evaluation and experiments. Using technology marketed to model building, the laboratory presents to students a number of scale systems fully instrumented – Speranza Neto (2016).

Regarding specifically the subject of vehicles, the *LDSM* has a greater focus on terrestrial ones. Two-wheeled vehicles, in this context, are particularly interesting due to its unique dynamics: though naturally instable, like an inverted pendulum, they can achieve stability at certain speeds thanks to its unique geometry and gyroscope effect. On a first approach, students may learn about multibody dynamics on a daily system (Escalona, 2010); the bicycle and motorcycle model can be used from undergraduate level to high level courses with less simplifications: Åström (2015) presents models that vary from simple second-order models, to a linear fourth-order model – where the front fork displays a dynamic behavior – until finally arriving at a fourth-order nonlinear model.

Since bicycle and motorcycles are such a familiar system, they become a concrete example of a dynamic system that use a full range of systems-theoretic tools so as to be challenging, but not yet overpowering, to the student (Klein, 1989). Through the modelling of these objects, students also learn another wide variety of concepts such as open-loop stabilization; the role of poles and zeros, especially in the right-half plane; the importance of a good design through parametric influences, as altering mass distribution or damping constants; physical limitations like maximum torque supplied by driver or motor.

Through a multidisciplinary approach, the two-wheeled vehicle is also a good study object, involving the previously cited multibody dynamics, electronics and filters present in the embedded instrumentation and many control techniques to keep the system stable in every desired path. The system allows for continuous learning with increasing difficulty, since the students start with identifying the physical parameters, progress to analyze its geometry and characteristics through computer software, test their own control strategy with the help of computational simulations and, at last, apply the result in the real system (Åström, 2010). Undergraduate students from the first semester can become familiar with simple instrumentation tasks such as collecting data from inertial measurement units via low cost microcontrollers like Arduino, while more advanced classes may apply complex techniques such as modern control or sensors fusion to the same system.

This work presents a linearized dynamic model for analysis: longitudinal and lateral dynamics of the vehicle – including lateral sliding in the tires – which represent the movement of the motorcycle in its main degrees of freedom. The main advantages of the model will be discussed, regarding mainly the students' ability to understand its analysis and simulation results.

Figure 1 presents the variables and parameters of the motorcycle system used in the dynamic models discussed in this paper. The system has four coordinate systems: a global system given by $\{X, Y, Z\}$; a local system $\{x, y, z\}$ originating from the vehicle's center of mass; a local system at the rear wheel's centroid given by $\{x_t, y_t, z_t\}$ and one last system centered at the front wheel geometric center at $\{x_d, y_d, z_d\}$. The motorcycle's roll axis is x_{roll}, the variables h, r_t, r_d, l_r, l_d, l, t e θ_g represent the vehicle's geometric parameters and variables ω_{rol}, ω_{yaw} , ω_{pit} are, respectively, the roll, yaw and pitch angular speeds.

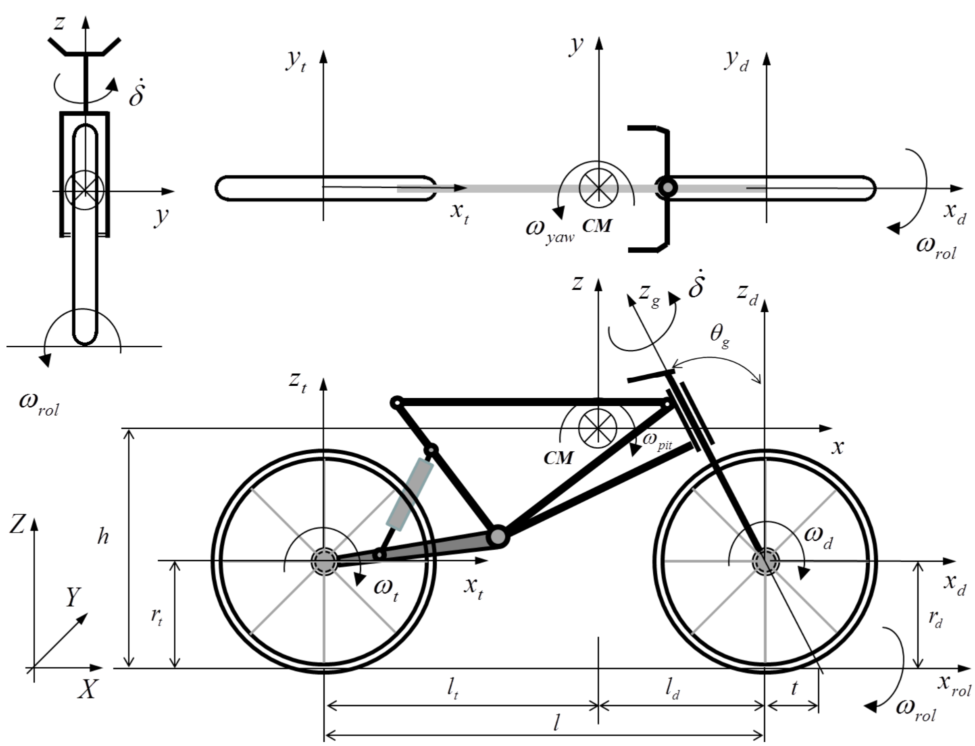


Figure 1: Motorcycle's references, angular velocities and geometric parameters.

2. LONGITUDINAL AND LATERAL DYNAMICS

The detailed model of the longitudinal and lateral dynamics of the motorcycle is quite complex, since the system presents many degrees of freedom, nonlinearities and the dynamic equations are dependent on its very specific geometry. However, it is possible to represent the most relevant aspects of the motorcycle dynamic behavior with a more simplified model, based on some hypotheses: constant longitudinal velocity, gyroscopic effect and trail influence on the direction are neglected, among others (Sharp ,2009).

The linearized open-loop model considers the motorcycle composed of two structures, coupled by a rotation joint on the handlebars, with each frame supporting a free rotation wheel. The conductor is assumed to be an extension of the vehicle, displacing the center of mass of the system on the z-axis, and acts through the steering angle (δ) and the torque generated around the x-axis by the angular displacement of its body (T_x). Lateral sliding and tire deformation are also considered through lateral stiffness and camber coefficients.

The dynamic model was obtained from the sum of forces acting on the global Y axis, including the lateral forces arising due to the lateral slip, and the sum of moments in relation to the X (roll) and Z (yaw) axes, as detailed in Eq. (1) to Eq. (3). The variables not previously described in Tab. 1 are the center of mass position on the global coordinate (Y), the yaw angle on the global coordinate (ψ), the roll angle on the local coordinate (φ), the steering angle (δ) and the roll torque generated by the driver's body angular displacement (T_x); all variables are in the S.I. units.

Equation (4) presents the lateral sliding on the rear and front tires, as described by Sharp (2009), where the lateral forces are described as linear responses to sideslip and camber angles, due to small angles hypothesis; the rear wheel camber angle is simply φ while the sideslip angle is different to each wheel – the rear wheel is related to the local displacement and yaw angle, the front wheel also accounts for the steer angle applied in the handlebar (Sharp, 1971). Finally, Eq. (5) explicit the relation between the local displacement and the global position, considering lateral sideslip.

$$\sum M_x = (J_x + mh^2)\ddot{\varphi} - mh\ddot{y} = T_x + mgh\varphi \tag{1}$$

$$\sum F_y = m\ddot{y} = F_d + F_t \tag{2}$$

$$\sum M_z = J_z\ddot{\psi} = l_d F_d + l_t F_t \tag{3}$$

$$\begin{aligned} F_d &= -C_d\alpha_d + C_d^c\varphi\,, \quad \alpha_d = \left(\dot{y} + l_d\dot{\psi}\right)/v_x - \delta \\ F_t &= -C_t\alpha_t + C_t^c\varphi\,, \quad \alpha_t = \left(\dot{y} - l_t\dot{\psi}\right)/v_x \end{aligned} \tag{4}$$

$$\begin{aligned} \dot{y} &= \dot{Y} + v_x\psi \\ \ddot{y} &= \ddot{Y} + v_x\dot{\psi} \end{aligned} \tag{5}$$

Equation (6) brings the final result of the longitudinal and lateral linearized model.

$$\begin{aligned} &\overbrace{\begin{bmatrix} m & 0 & 0 \\ 0 & J_z & 0 \\ 0 & 0 & J_x + mh^2 \end{bmatrix}}^{M} \overbrace{\begin{bmatrix} \ddot{Y} \\ \ddot{\psi} \\ \ddot{\varphi} \end{bmatrix}}^{\ddot{X}} + \overbrace{\frac{1}{v_x}\begin{bmatrix} C_d + C_t & mv_x^2 + C_d l_d - C_t l_t & 0 \\ C_d l_d - C_t l_t & C_d l_d^2 + C_t l_t^2 & 0 \\ h(C_d + C_t) & h(m\dot{x}^2 + C_d l_d - C_t l_t) & 0 \end{bmatrix}}^{C} \overbrace{\begin{bmatrix} \dot{Y} \\ \dot{\psi} \\ \dot{\varphi} \end{bmatrix}}^{\dot{X}} + \\ &\overbrace{\begin{bmatrix} 0 & C_d + C_t & -(C_d^c + C_t^c) \\ 0 & C_d l_d - C_t l_t & -(C_d^c l_d - C_t^c l_t) \\ 0 & h(C_d + C_t) & -h(mg + C_d^c + C_t^c) \end{bmatrix}}^{K} \overbrace{\begin{bmatrix} Y \\ \psi \\ \varphi \end{bmatrix}}^{X} = \overbrace{\begin{bmatrix} C_d & 0 \\ C_d l_d & 0 \\ C_d h & 1 \end{bmatrix}}^{E} \overbrace{\begin{bmatrix} \delta \\ T_x \end{bmatrix}}^{U} \end{aligned} \tag{6}$$

The longitudinal speed v_x is considered constant and it is included in the model as a parameter of the damping matrix. The state space model with lagrangean variables is obtained from the algebraic manipulation of the motion equations, as shown in Eq. (7).

$$\begin{aligned} &\boldsymbol{M\ddot{X} + C\dot{X} + KX = EU} \\ &\begin{bmatrix} \dot{\boldsymbol{X}} \\ \ddot{\boldsymbol{X}} \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{0_{3x3}} & \boldsymbol{I_{3x3}} \\ \boldsymbol{-M^{-1}K} & \boldsymbol{-M^{-1}C} \end{bmatrix}}_{state\ space\ matrix} \begin{bmatrix} \boldsymbol{X} \\ \dot{\boldsymbol{X}} \end{bmatrix} + \underbrace{\begin{bmatrix} \boldsymbol{0_{3x2}} \\ \boldsymbol{M^{-1}E} \end{bmatrix}}_{input\ matrix} \begin{bmatrix} \delta \\ T_x \end{bmatrix} \end{aligned} \tag{7}$$

The model presented in Eqs. (6) and (7) has some very useful characteristics for the teaching of analysis and control of dynamic systems. First, the open loop system is highly unstable, which requires the application of a closed loop control for stabilization. The instabilities of the model are intuitive and easy to demonstrate, confirmed by the existence of positive and null eigenvalues in the state matrix. In addition, the study of the bicycle / motorcycle system can awaken in the student the interest in validating the results obtained via software on a real bicycle, equipment that most people have easy access.

Another important aspect of the model presented here is the coupling between lateral and longitudinal dynamics of the vehicle, including sideslip and camber effects. This way it is possible to explore, in addition to stability control, also the control of the vehicle's trajectory.

2.1. Open loop model analysis

The first model analysis that can be done by students is the characterization of the open loop system's dynamic behavior through the state matrix's eigenvalues. Understanding this concept is very important, since it defines the unforced system response. The motorcycle's parameters that will be used for the analysis are presented in Tab. 1.

Table 1: Parameters of a typical motorcycle.

Parameter	Symbol	Unit	Value
Tire lateral stiffness coefficients front and rear	C_d and C_t	N/rad	11000 and 1000
Tire camber stiffness coefficients front and rear	C_d^c and C_t^c	N/rad	15000 and 1500
Gravity acceleration	g	m/s^2	9.81
Longitudinal constant speed	v_x	m/s	20
Height of vehicle and driver's center of mass	h	m	1.3
Moment of inertia of vehicle and driver mass in relation to the vehicle center of mass in x axis	J_x	kgm^2	10
Moment of inertia of vehicle and driver mass in relation to the vehicle center of mass in z axis	J_z	kgm^2	5
Distance from front axis to center of mass	l_d	m	0.5
Distance from rear axis to center of mass	l_t	m	0.6
Vehicle and driver mass	m	kg	300
Driver mass	m_c	kg	70

Using this parameters, state matrix's eigenvalue are $[0.00 \quad 0.00 \quad 0.052 \quad 5.29 \pm j20.26 \quad -43.75]$. The motorcycle's instability on the roll axis is represented by the positive real eigenvalue, which results in a capsize mode, an exponential fall on uncontrolled systems without oscillation (Cossalter, 2006); the null eigenvalues are associated to the free-body movement on the lateral displacement. The conjugated complex pair represents an oscillating unstable mode which appears when there is lateral tire slippage and it is associated to the lateral displacement (Y) and the yaw angle (ψ). For the parameters on Tab. 1, this mode presents a natural frequency of 20.94 rad/s; this value is strongly attached to the tire lateral stiffness coefficient, which affects the lateral slippage force intensity, as shown in Eq. (4).

Since the longitudinal velocity is considered a model parameter, it is essential that students understand how the speed variation affects the state space matrix eigenvalues and, consequently, the system's stability. The root locus approach is an excellent tool to visualize the poles changing with the speed, as portrayed in Fig. 2: the '×' represent the eigenvalues at low speed (5 m/s) and its transition up to the 'o' positions, at high speed (75 m/s).

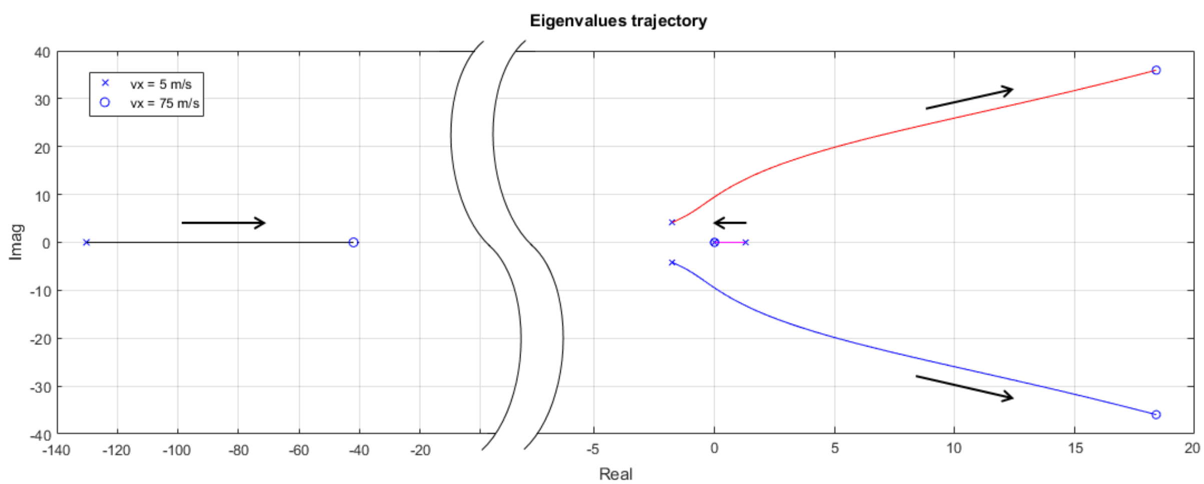


Figure 2: Open loop model's eigenvalues map.

The students should be able to notice that, as the speed increases, the positive real pole becomes closer to zero, representing higher stability; therefore, the speed increase improves the vertical stability (or capsize tendency), as expected from empirical knowledge. Nevertheless, this eigenvalue never cross the imaginary axis, that is, it does not go to the negative half-plane and is instable to all velocities.

Notice that the speed has no effect on the null eigenvalues; the conjugated complex eigenvalues, on the other hand, move further on the right half-plane with the velocity increase. The students must correlate this phenomenon to the increased sensitivity of the vehicle to lateral slip as the motorcycle achieves higher speeds. Through graphical analysis, it is possible to determine that the minimum longitudinal velocity that maintains the oscillatory mode at the stable half-plane is 8.6 m/s^2.

The root locus is an important tool to the dynamic system analysis teaching since it represents, visually, the changes in the system's dynamic behavior due to one parameter variation. In this way, students can measure the sensitivity of the model to some physical, geometric or dynamic vehicle's characteristic; in addition, the root locus can be used as a tool to study the influence of each control system gain on the closed loop, making it an essential tool in teaching systems control.

Figure 3 illustrates the open loop system response to non-unitary pulses with 0.5 second duration and amplitude of 3º in the steer angle (δ) and 170 Nm in the driver's torque (T_x), compatible to a 70 kg driver tilting its body by 10º. The open loop system is clearly instable and there is a large difference in the outputs order of magnitude, which leads to the conclusion that the vehicle is more sensitive to a steer angle variation than to a change in the driver's torque. The system response presents oscillatory characteristics, since the conjugate complex eigenvalues have a larger real part and are, therefore, dominant. Finally, by graphical analysis, students can also identify how long it takes for the bicycle or motorcycle to fall down, i.e., the roll angle (φ) achieve 90º - in this example, it happens at 0.69 seconds.

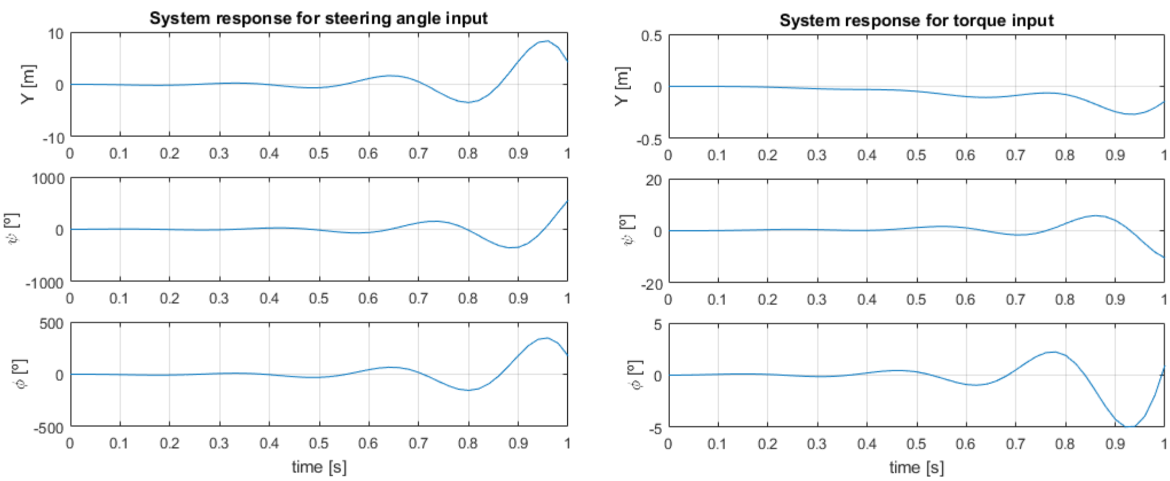


Figure 3: Open loop system response to non-unitary pulses.

As an additional comment, without lateral slip, the tire lateral and camber stiffness coefficients may be disregarded and the dynamic model consists only of the roll angle equation (Eq. (8)), similar to an inverted pendulum.

$$(J_x + mh^2)\ddot{\varphi} = T_x + mgh\varphi \tag{8}$$

2.2. Closed loop model analysis

As previously noted, the motorcycle needs a stability control system to keep upright at any given speed. The control technique chosen for this work is the state feedback, since it helps the understating of vector gain physical interpretation and has only one control loop to all state variables, instead of multiple feedbacks needed to a PID controller, for instance. Moreover, this strategy allows the discussion and analysis of ideal pole placement based on final desired vehicle's behavior. In this paper, two approaches will be discussed: the lateral dynamic stabilization and path tracking.

Both control objectives use the human driver behavior as a bicycle/motorcycle control element through steer angle and roll torque inputs; it is possible to interpret the control system's constants as the driver sensitivity to certain variables during maneuvers and operation conditions. The feasibility of assigning a physical interpretation to control gains is further justification for the use of this model in dynamic control systems teaching; in addition, the development of a stability control system represents an interesting challenge, due to the presence of positive and null eigenvalues in the motorcycle model, making some usual control techniques not efficient to the system.

The first control system is exposed in Fig. 4. It is important to comment that the controllability test, using the *Matlab* commands `ctrb` and `rank` on the open loop model, confirms that this system is controllable for both inputs, together or separately. This is another fundamental control systems concept introduced with this example.

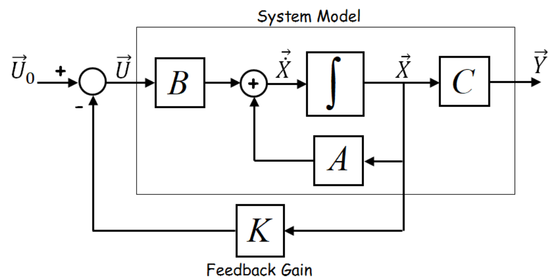


Figure 4: Regulation control loop.

The control input is given by Eq. (9).

$$\boldsymbol{U} = -\boldsymbol{KX} + \boldsymbol{U_0}$$
$$\begin{bmatrix} \delta \\ T_x \end{bmatrix} = -\boldsymbol{KX} + \begin{bmatrix} \delta_0 \\ T_{x_0} \end{bmatrix} \quad (9)$$

where $\boldsymbol{X} = [Y \quad \psi \quad \varphi \quad \dot{Y} \quad \dot{\psi} \quad \dot{\varphi}]^T$, $\boldsymbol{K}$ is the control gain matrix and $\boldsymbol{U_0}$ is an input noise.

The closed loop state space model is given by Eqs. (10) and (11).

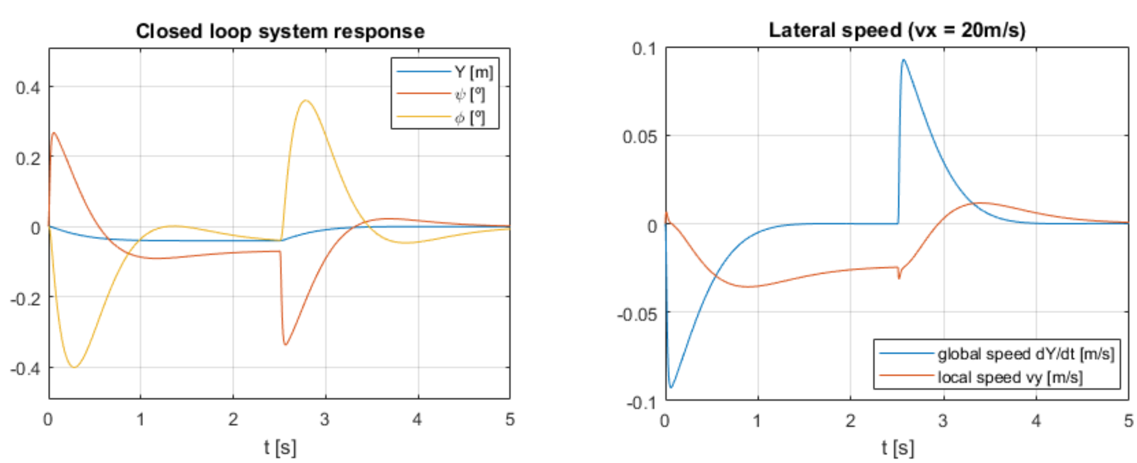
$$\dot{\boldsymbol{X}} = \boldsymbol{AX} + \boldsymbol{BU} = \boldsymbol{AX} + \boldsymbol{B}(-\boldsymbol{KX} + \boldsymbol{U_0}) = (\boldsymbol{A} - \boldsymbol{BK})\boldsymbol{X} + \boldsymbol{BU_0} \quad (10)$$
$$\boldsymbol{Y} = [\boldsymbol{I_{3x3}} \quad \boldsymbol{0_{3x3}}]\boldsymbol{X} + [\boldsymbol{0_{3x2}}]\boldsymbol{U_0} \quad (11)$$

The feedback gains are calculated so that the state space matrix, in closed loop ($A_{CL} = A - BK$), presents a behavior that respects some stability criteria. As an example, it can be proposed to the student that the system reaches the steady state in up to one second, with a damping factor (ξ) smaller than 0.7; these characteristics reflect a fast and almost overdamped system, ideal for a motorcycle. Since the system has six eigenvalues, the student can determine three dominant pole placement and the rest may be significantly higher so that its dynamic will not interfere on the final system response.

The next step is to calculate the control gain matrix (K) in order to achieve the desired closed loop eigenvalues; in this work, the chosen poles are -2; $-3 \pm j2$; -100; -110; -115. Through the `place` command, on *Matlab*, the result is:

$$\boldsymbol{K} = \begin{bmatrix} -1.39 & 1.57 & 2.14 & -0.46 & 0.18 & 0.01 \\ -32087.6 & 833972.4 & 375731.5 & 1949.5 & -1992.7 & 60220.0 \end{bmatrix}$$

An interesting analysis on this result is that the feedback gains associated to the roll torque (T_x) are much higher than the ones associated to the steer angle (δ); this is consistent due to the order of magnitude difference between both inputs. The students should also note that both the control torque as the steer angle adjust with more sensitivity to the yaw and roll angles variations. At last, Figure 5 brings the closed loop simulation response to a δ_0 disturbance as a pulse input with 3º amplitude and 2.5 seconds duration.



(a) (b)
Figure 5: Closed loop regulation system response.

Figure 5(a) shows that all variables reach the steady state at approximately 2.5 seconds, result compatible with the closed loop eigenvalues; both roll and yaw angles have small amplitude variation (less than 0.15º), imperceptible to the pilot, and the lateral displacement caused by the disturbance is also minor, reaching its peak at 40 mm. All variables stabilize at zero, indicating that the motorcycle was successfully brought back into its stable position and the chosen control gains led to a satisfactory response.

Additionally, Fig. 5(b) brings the lateral velocity graphs in the global reference (dY/dt) and local reference (v_y), whose relation was previously stablished in Eq. (5): the difference between both speeds indicates tire slip during movement, even if of small magnitude. Figure 6 shows how the control signals, $\delta(t)$ and $T_x(t)$, varied after the disturbance excited the system.

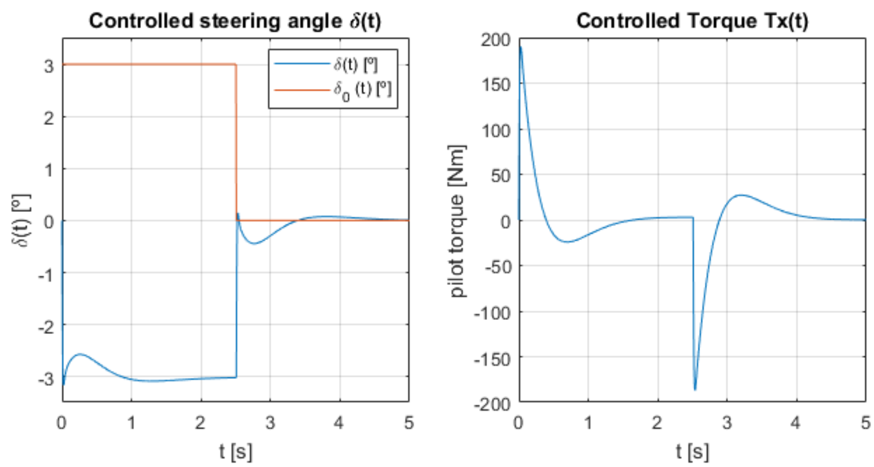


Figure 6: Control inputs to regulation system.

Analyzing the graphs, it is noticeable that the steer angle is quickly corrected to prevent the motorcycle from falling and, in addition the pilot inserts a torque in the opposite direction of the roll angle to bring the vehicle back to its vertical position, proving the effectiveness of the proposed control system.

Another important topic in control teaching that can be presented using this closed-loop system is frequency response analysis. Suppose the pilot is constantly moving the handlebar from one side to the other producing a sine-wave steer angle input, students can learn to evaluate how this motion affects the behavior of the motorcycle for different oscillation frequencies using Bode diagrams and simulations. Figure 7 (a) presents de Bode diagrams for the transfer functions $Y(s)/\delta(s)$, $\psi(s)/\delta(s)$ and $\varphi(s)/\delta(s)$.

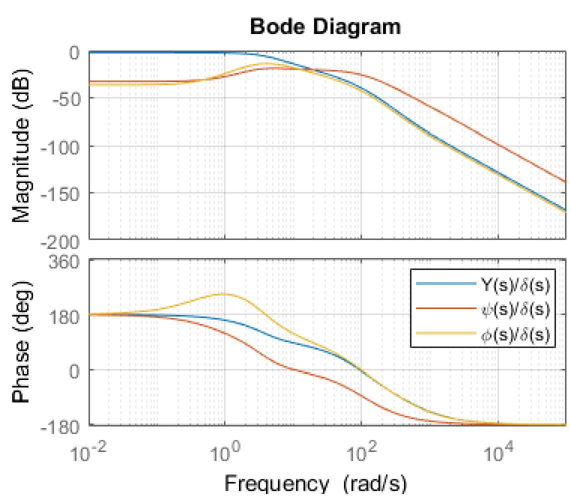
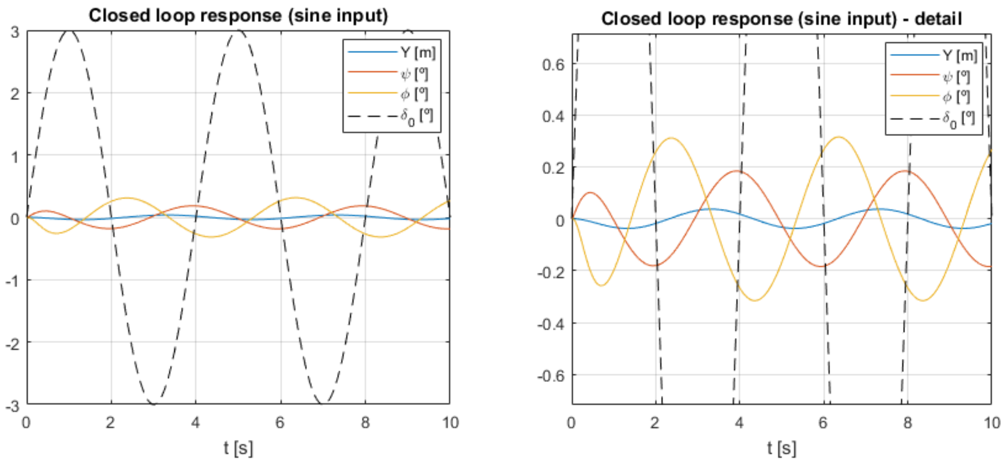


Figure 7: Closed loop system Bode diagram.

All three diagrams present an attenuation pattern (negative magnitude gains) and non-minimum phase diagrams, which is a consequence of the presence of zeroes in the right-half-plane. That is another reason why modern control is a good approach for this system. Non-minimum phase systems are usually more difficult to design using classic control approaches, such as root-locus, due to the zero-pole cancellation. A motorcycle maneuver that is a reflection of the "excess of phase" is known as the counter-steering, where the rider turn the handlebar in the counter direction to compensate for the rear sideslip, that is much greater than the front sideslip at higher speeds. Figure 8 (a) shows the variation of the roll and yaw angles and the lateral displacement for a sinusoidal steer angle input of 3º of amplitude and 4 Hz frequency. All outputs are greatly attenuated in relation to the input signal and the oscillation amplitude of the scroll angle is 0.3°, which is a very satisfactory result. In Fig. 8 (b) it is more visible the phase difference between the signals, especially between the steer angle and the lateral displacement. Notice that it takes a few seconds for the vehicle to begin turning after the handlebar was moved.



(a) (b)

Figure 8: System frequency input response.

Once the regulation loop has been tuned, the students can develop a tracking control loop for the motorcycle trajectory control. The input is the desired lateral displacement profile (Y_d) and the output, the real vehicle's displacement (Y); the control signal is proportional to the difference between both displacements. The tracking gain (K_d) is determined in order to keep the signal error null in a steady state condition. Figure 9 illustrates the complete control loop.

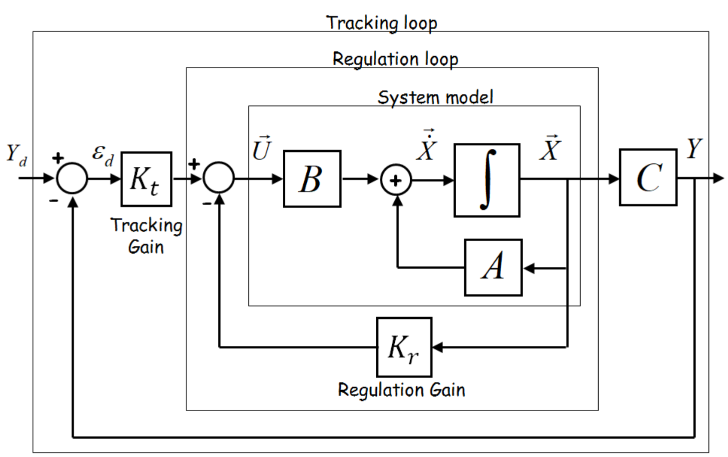


Figure 9: Tracking control loop.

With lateral displacement feedback, the control signal $\boldsymbol{U}$ is given by Eq. (12).

$$\boldsymbol{U} = -\boldsymbol{K}_r\, \boldsymbol{X} + \boldsymbol{K}_t(Y_d - Y) \tag{12}$$

Substituting Eq. (12) into the state space equation, the result is Eq. (13).

$$\dot{\boldsymbol{X}} = \boldsymbol{AX} + \boldsymbol{BU} = \boldsymbol{AX} + \boldsymbol{B}(-\boldsymbol{K}_r\, \boldsymbol{X} + \boldsymbol{K}_t(Y_d - Y)) \tag{13}$$

In order to have the lateral displacement as the output of the system, $\boldsymbol{C} = [1\ 0\ 0\ 0\ 0\ 0]$ and $Y = \boldsymbol{CX}$. After proper algebraic manipulations, Eq. (13) becomes Eq. (14).

$$\dot{\boldsymbol{X}} = (\boldsymbol{A} - \boldsymbol{B}(\boldsymbol{K}_r + \boldsymbol{K}_t\boldsymbol{C}))\boldsymbol{X} + \boldsymbol{BK}_t Y_d \tag{14}$$

Comparing Eq. (14) to Eq. (10), one can affirm that:

$$\boldsymbol{K}_r + \boldsymbol{K}_t\boldsymbol{C} = \boldsymbol{K} \tag{15}$$

The gain vector $(\boldsymbol{K}_t)$ can be determined as to eliminate the steady state error; the lateral displacement is given by considering $\dot{\boldsymbol{X}} = \boldsymbol{0}$ in Eq. (14).

$$\begin{aligned} Y &= -\boldsymbol{C}(\boldsymbol{A} - \boldsymbol{B}(\boldsymbol{K}_r + \boldsymbol{K}_t\boldsymbol{C}))^{-1}\boldsymbol{BK}_t Y_d \\ Y &= -\boldsymbol{C}(\boldsymbol{A} - \boldsymbol{BK})^{-1}\boldsymbol{BK}_t Y_d \end{aligned} \tag{16}$$

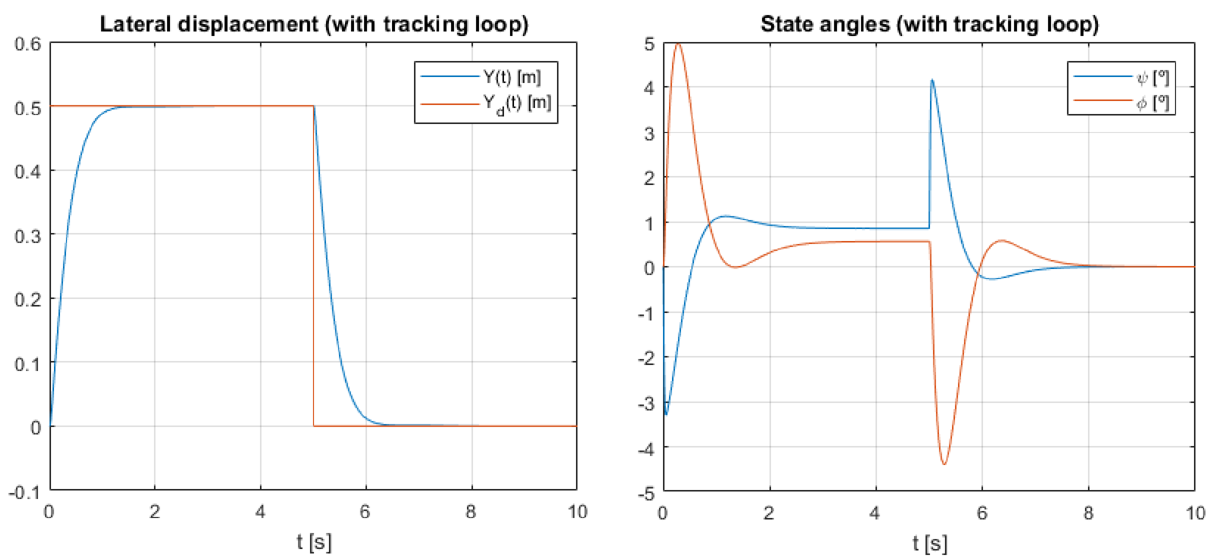
To guarantee that the real lateral displacement is equivalent to the desired value, the vector gain $\boldsymbol{K}_t$ should be calculated through Eq. (17).

$$\boldsymbol{K}_t = -(\boldsymbol{C}(\boldsymbol{A} - \boldsymbol{B}(\boldsymbol{K}_r + \boldsymbol{K}_t\boldsymbol{C}))^{-1}\boldsymbol{B})^{+} \tag{17}$$

where the superscript '+' indicates the pseudo-inverse, since $\boldsymbol{K}_t$ is not a square matrix; in *Matlab*, the `pinv` command can be used.

Once the gain $\boldsymbol{K}_t$ has been determined, the value of $\boldsymbol{K}_r$ is obtained from Eq.(15); to the regulation gain matrix previously considered, $\boldsymbol{K}_t = [-1.29 \quad 3.7 \cdot 10^{-6}]^T$. One interesting observation is that, in order to control the lateral displacement, with this dynamic model and control strategy, only the steer angle input is need to be applied.

By simulating the closed loop system shown in the block diagram of Fig. 9, the following the results shown in Fig. 10 are obtained.



(a) (b)

Figure 10: Regulation and tracking control simulation results.

The desired lateral displacement profile was the double lane change maneuver. Figure 10(a) proves the main objective of keeping the steady state error null was achieved, validating the proposed methodology; moreover, the system response is non oscillatory and reaches the desired value in 1.5 seconds. Figure 10(b) displays the yaw and roll angles throughout the maneuver, both under the small angles hypothesis, i.e., smaller than 10º.

3. CONCLUSIONS

This work has presented a linearized model of motorcycles and bicycles to the longitudinal and lateral dynamics with tire slipping. The main objective of both models is to introduce to students analytical skills such as system modelling, numerical analysis and control strategies through a familiar, yet complex, system.

Using typical motorcycle parameters, mathematical models for individual components were developed, which were then combined to create a more complete state space model. The students had to interpret and analyze the uncontrolled system, allied with simulation tools (*Simulink/Matlab*), in order to identify undesirable behaviors such as oscillations or instability and posteriorly propose appropriate control technique to attenuate or eliminate those characteristics and achieve the desired performance.

Through a problem-based learning approach, all this work was done in a semester, when students began analyzing the open loop lateral dynamics model, which brings the most tangible results, then the model with added tire slipping, before finally reaching the stability and path tracking control system. The same system was used to introduce frequency domain analysis concepts and it was particularly important to discuss non-minimum phase systems, which are not well addressed in typical control system examples. The study of the motorcycle system was well received through periodic and increasingly difficult assignments, which corroborate the good results of this teaching method, as well as the practical implementation of the theory to students.

4. ACKNOWLEDGEMENTS

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