



24th COBEM - 2017



24th ABCM International Congress of Mechanical Engineering  
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-2738

## HVAC DUCT SYSTEM MODELING THROUGH THE COMPONENT MODE SYNTHESIS FOR STRUCTURAL VIBRATION ANALYSIS

**Jose Manuel Bautista Ordoñez**

Universidade de Brasília, Faculdade UnB Gama, Brasília, Brasil.

[josebautistaordonez@gmail.com](mailto:josebautistaordonez@gmail.com)

**Maria Alzira de Araújo Nunes**

Universidade de Brasília, Faculdade UnB Gama, Brasília, Brasil.

[maanunes@unb.br](mailto:maanunes@unb.br)

**Abstract.** HVAC systems (heating, ventilating and air conditioning) are responsible for making an environment remain within a desirable range of temperature, humidity and cleanliness. To get the desire environment, the HVAC systems use a set of rigid ducts with different cross section. In addition to increasing the comfort in industrial spaces, is emphasized the importance of the structural analysis, referring to the vibratory behavior, in this way avoiding the occurrence of phenomena such as resonance. In the field of structural dynamics, the finite element method (FEM) is becoming increasingly important to the industry. Attempting to reduce the size and time response of different complex models on numerical software, component mode synthesis (CMS) is one of the main techniques for modelling complex or large sized systems. The present work uses an application of the CMS in modal analysis of an HVAC duct system. In order to analyze the structural vibrational behavior the CMS uses a modal superset that consists in to modelling individual components of a structure separately and then to couple them to form an assembled system. The principal objective of this article is to use the CMS technique for numerical estimation of natural frequencies and vibration modes of a duct system.

**Keywords:** Component mode synthesis, natural frequency, modal analysis, HVAC system.

### 1. INTRODUCTION

One of the main challenges for the industry is to eliminate or reduce to an acceptable level the vibration in mechanical systems, which can also produce noise affecting the comfort environment. Noise has a wide range of consequences for humans, from general disturbance, reducing personal comfort (such as lack of privacy or difficulty in talking) and even leading to serious health problems such as heart failure, hearing difficulties, stress, etc. (Howard and Cazzolato, 2015). HVAC system which principals objectives are heating, ventilating and air conditioning, has as function to control and maintain conditions of desirable temperature, humidity and cleanliness for people or organisms within a closed environment Figure 1.



Figure 1. HVAC duct system applied in the industry (ISOVER, 2015).

HVAC system design is a subdiscipline of mechanical engineering, based on the principles of thermodynamics, fluid mechanics, and heat transfer. This type of system is applicable to a wide variety of cases and situations aiming the transport of airflow for the temperature control. HVAC duct systems can have different sizes, cross sections, materials, etc.

With the advance of the technology and the necessity of getting everyday a faster and effectiveness model in numerical solution of structures, appears the finite element method (FEM). Even with the FEM, exist some complex simulations that increase the cost of time and processing for any numerical software, for this reason was created the component mode synthesis (CMS).

The modal synthesis techniques can be divided in analytical/numerical and experimental. The modal synthesis by residual flexibility (MSRF) it is one of these experimental techniques (Araújo, 1998), using the modal superset of residual flexibility and it has the advantage of being a mixed technique that can be used in numeric methods as in experimental methods. To use this technique it is essential that the modal base be mass normalized. Although, when the problem is purely experimental, the application of this technique becomes difficult because the physical matrices of the structure problem are unknown.

The objective of this work is to use the CMS for the estimative of the natural frequencies and modal shapes of an entire beam model with quarter-wavelength tube (QWT) geometry. The complete system is divided in two beams with different geometry as was modeled for Howard and Cazzolato, 2015. The entire system and the substructures will be calculated through FEM using the Hermitian beam element applied in the *MATLAB*<sup>®</sup> software. To validate the results, the entire system and the substructures will be modeled in *ANSYS*<sup>®</sup> software.

## 2. FEM ANALYSIS AND MODAL SYNTHESIS METHOD

### 2.1 FEM Analysis

The modeling of the resonator silencer of the type quarter-wavelength tube was done and implemented in *MATLAB*<sup>®</sup> using Hermitian beam element as a planar (2-D) frame element including the axial and bending deformation, as shown in Equation (1) e (2). Hence, the substructures were also modeled by this technique including axial and bending stiffnesses (Kwon and Hyochoong, 2000).

$$[K] = \frac{E}{l^3} \begin{bmatrix} Al^2 & 0 & 0 & -Al^2 & 0 & 0 \\ 0 & 12I & 6Il & 0 & -12I & 6Il \\ 0 & 6Il & 4Il^2 & 0 & -6Il & 2Il^2 \\ -Al^2 & 0 & 0 & Al^2 & 0 & 0 \\ 0 & -12I & -6Il & 0 & 12I & -6Il \\ 0 & 6Il & 2Il^2 & 0 & -6Il & 4Il^2 \end{bmatrix} \quad (1)$$

$$[M] = \frac{\rho Al}{420} \begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22l & 0 & 54 & -13l \\ 0 & 22l & 4l^2 & 0 & 13l & -3l^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13l & 0 & 156 & -22l \\ 0 & -13l & -3l^2 & 0 & -22l & 4l^2 \end{bmatrix} \quad (2)$$

Where:

- $I$ : Moment of inertia of cross-section.
- $E$ : Elastic Modulus.
- $A$ : Cross-sectional area.
- $\rho$ : Mass density per volume.
- $l$ : Length of the element.
- $K$ : Stiffness matrix.
- $M$ : Mass matrix.

For the element degrees of freedom  $\{u_1, v_1, \theta_1, u_2, v_2, \theta_2\}$  as seen in Figure 2.

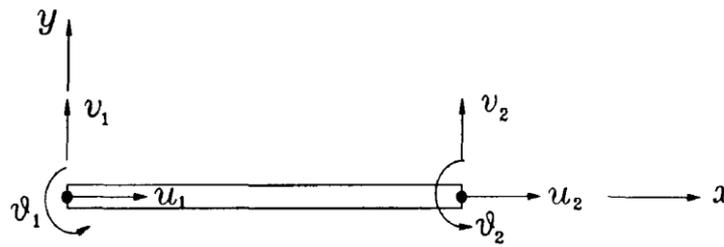


Figure 2. A Linear Frame Element.

## 2.2 Modal Synthesis Method

The process of modal synthesis using MSRF method was proposed for Araújo, 1998. The conception of the method is planted on three basic steps:

- The definition of the substructures that were divided from the complete system.
- The calculation of the modal superset (flexible modes more static modes) through an analytical or experimental formulation.
- Assemble and solve the global equations of motions according to the connectivity imposed.

Araújo, 1998, developed the procedure of analysis and calculation of the MSRF technique, which uses the modes of residual flexibility as a function of the inertia plots and rigidity of the dynamic matrix. Once the maintained modes and the eliminated modes are determined from the modal basis, define the calculation of the modes that will be used in the equation of motion for the two substructures to be connected. Solving the synthesized eigen problem, we have the eigenvalues and eigenvectors of the total system, which is conformed by the two substructures in the modal coordinates maintained.

When a modal synthesis is developed that uses the modal superset of residual flexibility, the researcher can define which modes he wants to remove from the modal basis. Because these modes are eliminated from the modal basis, they are used to define residual flexibility modes that improve the modal basis of the substructures, likewise depending on the number of modes maintained in the modal basis, they may be significant in identifying modes which can be removed. Araújo, 1998, proposed a methodology for the identification of modes to be maintained, called “Automatic Mode Elimination Criterion (AEC)”, based on the energy level of the contour of the substructures.

## 2.3 Resonator Silencer (Quarter-wavelength Tube)

Resonator silencers function by providing a high reactive impedance causing an incident acoustic wave to reflect upstream. They usually have no or little acoustic absorption material within the device. This is in contrast to absorptive silencers that rely on the use of acoustic absorbing material to attenuate incident acoustic waves.

Common geometries of resonator-type silencers are shown in Figure 3 and include (a) quarter-wavelength tube, (b) Helmholtz resonator, (c) expansion chamber, (d) contraction.

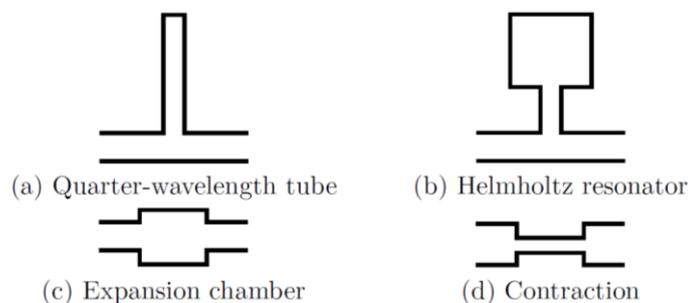
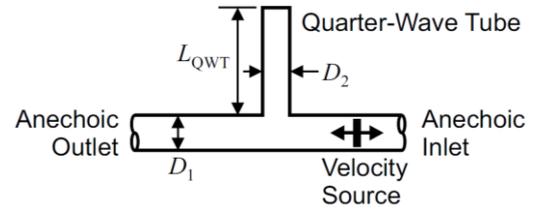


Figure 3. Geometries of resonator-type silencers.

The Table 1 shows a schematic of a quarter-wavelength tube (QWT) resonator silencer attached to a circular main exhaust duct and the relevant parameters used in the example of Howard and Cazzolato, 2015.

Table 1. Parameters used in the analysis of a circular duct with a quarter-wavelength tube.

Description	Parameter	Value	Units
Diameter main duct	$D_1$	0.1	m
Diameter QWT	$D_2$	0.05	m
Length QWT	$L_{QWT}$	1.5	m
Speed of sound	$c_0$	343.24	m/s
Density	$\rho_0$	1.2041	kg/m <sup>3</sup>
Velocity at inlet	$u_1$	0.001	m/s



Using those parameters was obtained a comparison of the transmission loss results calculated using *MATLAB*<sup>®</sup> and *ANSYS*<sup>®</sup> *Workbench* analysis. The analysis was repeated to calculate the transmission loss setting different values for the parameter  $D_2$  in 0.1m, so that the ratio of area of the quarter-wavelength tube to the main exhaust duct was  $N = 1$ . The Figure 2 shows that *ANSYS*<sup>®</sup> results overlay the theoretical prediction.

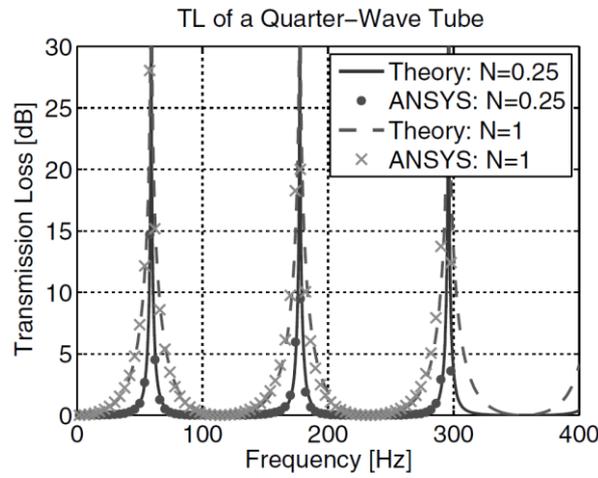


Figure 4. Transmission loss of a quarter-wavelength tube calculated theoretically and using *ANSYS*<sup>®</sup> *Workbench*, for ratios of areas between the quarter-wavelength tube and the main exhaust duct of  $N = 0.25$  and  $N = 1.0$ .

## 2.4 Modal Assurance Criterion (MAC)

The modal assurance criterion (MAC) index is used to know the correlation between modes of the synthesized structure and the real or complete model (Ewins, 1984). This value is obtained through the least squares deviations between the correlated modal bases. As the most satisfactory value is the index close to unity, meaning that the analyzed modal bases have a good correlation. If the index value is close to zero, the modal bases have a weak correlation. The MAC index is calculated according to Equation (3).

$$MAC(X_1, X_2) = \frac{(X_1^T X_2)^2}{(X_1^T X_1)(X_2^T X_2)} \quad (3)$$

$$\overline{MAC} = \frac{\sum_{j=1}^m MAC_j(s, x)}{m} \quad (4)$$

Where  $X_1$  e  $X_2$  are the real and synthesized eigenvectors.

Usually,  $MAC > 0.80$  result in a good correlation, while  $MAC < 0.40$  produces a poor correlation (Piana, 2016).

### 3. METHODOLOGY

The synthesis is going to be made through the MSRF method and was evaluated using discrete masses (Ordoñez, 2017). The Figure 3 shows the dimensions and the geometry of the quarter-wavelength tube used for Howard and Cazzolato, 2015, that is composed for two beams, which is constituted with a bigger beam with 40 elements and 41 nodes and a smaller beam with 15 elements and 16 nodes, this means that the complete system were composed for 55 elements and 56 nodes, the Figure 4 shows how was distributed the elements along the entire system. The modelling method was implemented in *MATLAB*<sup>®</sup> software. Remembering that the Hermitian beam element has three degree of freedom per node (displacement in  $x$ ,  $y$  and the rotation in the plane  $xy$ )

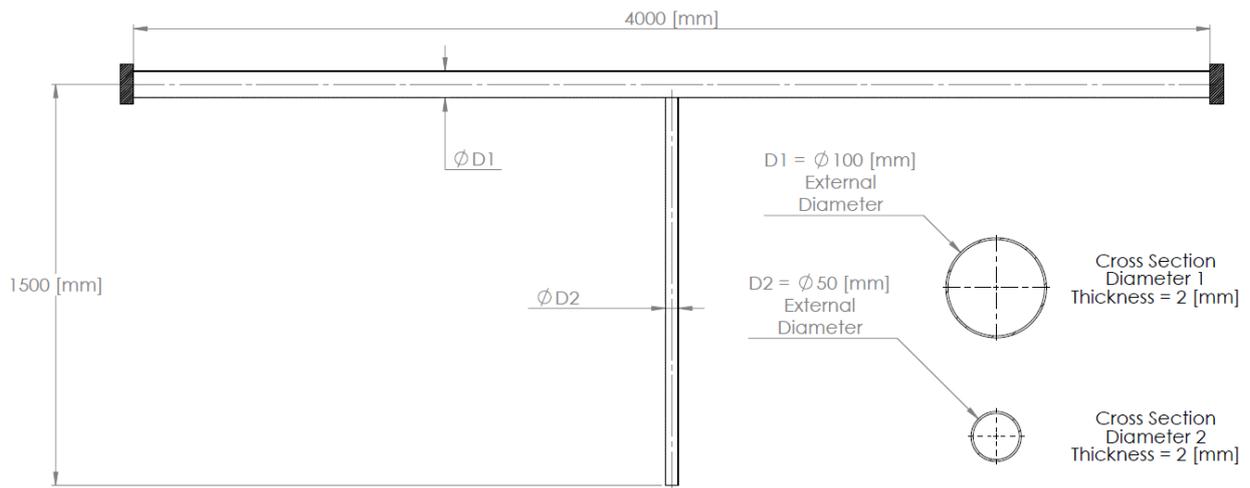


Figure 5. Model of the quarter-wavelength tube silencer.

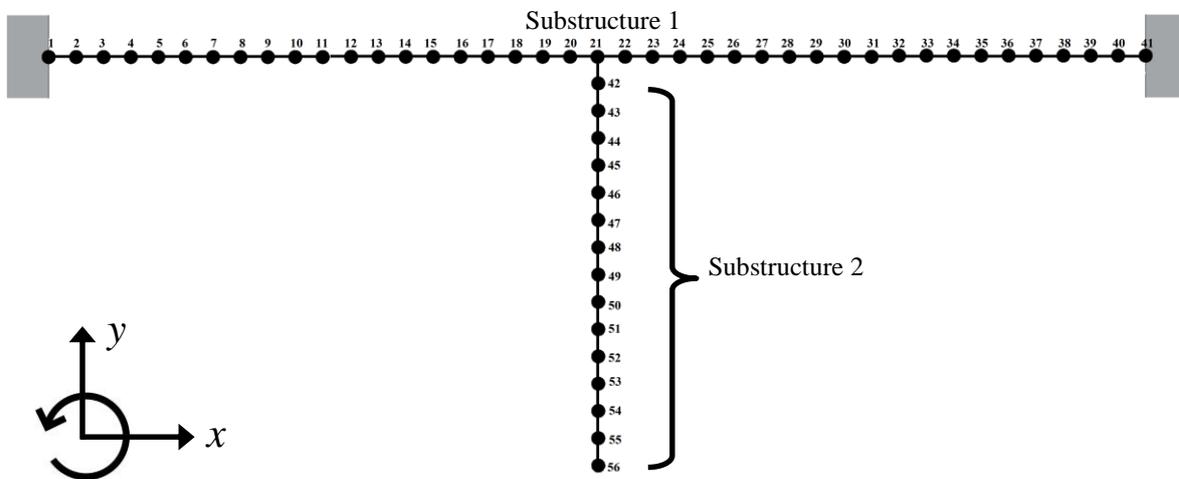


Figure 6. Finite element model of the quarter-wavelength tube silencer.

Table 2 shows material properties of the beam used in the analysis in SAE 1010.

Table 2. Material's properties of the beam.

Density ( $\rho$ )	7870 [Kg/m <sup>3</sup> ]
Modulus of Elasticity ( $E$ )	200 [GPa]
Poisson's Coefficient ( $\nu$ )	0,3
Thickness ( $h$ )	0,002 [m]

As it can see in the Figure 6, the boundary conditions of the first substructure are clamped-clamped, which means that the node 1 and 41 are constrained in its three degree of freedom, while the boundary conditions of the second substructure are free-free.

Once it was established the parameters for the FEM for the entire system and the substructures, it was calculated the natural frequencies and mode shapes. Getting as a result for each degree of freedom in all the nodes one natural frequency and mode shape.

The next step in the numerical procedure was made the component mode synthesis with the program *MATLAB*<sup>®</sup> to get the frequencies of the system, using the results obtained through the *MATLAB*<sup>®</sup> program for the natural frequencies and mode shapes of each substructure. For this problem it is going to be used the first 40 natural frequencies and mode shapes from the substructure 1 and the first 15 natural frequencies and mode shapes from the substructure 2, aiming to maintain a proportion in the size and quantity of modes of each substructure. In this case the last frequency of each substructure was chosen to define residual flexibility modes, this frequency and mode shape are the higher value in this problem and it is the frequency with less probability to appear in this kind of structural system.

To validate the result, this same model was simulated with the software *ANSYS*<sup>®</sup> - *APDL 16.0*, which is the most common technique, used for extract the natural frequencies and the vibration modes. In the same way as the FEM in *MATLAB*<sup>®</sup> using the element type BEAM188 and the quantity of element how was described, the structure was divided in substructures to compare each structure results with the analytical result. Also the entire system was modelled through numerical analysis to compare the result of the synthesis and the numerical result of the entire system with *ANSYS*<sup>®</sup>.

#### 4. RESULTS

The results obtained by modal analysis of the substructures for the first five natural frequencies on *MATLAB*<sup>®</sup> and *ANSYS*<sup>®</sup> are tabulated in Table 3:

Table 3. First five natural vibration's frequencies of the substructures.

N° of Mode	Eigenvalue of the System <i>ANSYS</i> <sup>®</sup> [Hz]	Eigenvalue of the System FEM <i>MATLAB</i> <sup>®</sup> [Hz]	Relative Error [%]	MAC Index
<b>Clamped-Clamped Beam (Horizontal: 4m)</b>				
1	38,47	38,88	1,07	1,0000
2	104,73	107,18	2,33	0,9999
3	202,05	210,11	3,99	0,9997
4	327,55	347,32	6,03	0,9993
5	478,46	518,84	8,44	0,9984
Mean Values			4,37	0,9884
<b>Free-Free Beam (Vertical: 1.5m)</b>				
1	0,00	0,00	0,00	1,0000
2	0,00	0,00	0,00	1,0000
3	0,00	0,00	0,00	1,0000
4	134,46	135,51	0,78	1,0000
5	365,35	373,55	2,24	1,0000
Mean Values			0,60	1,0000

Table 4 shows the relationship between the first ten natural frequencies of the synthesized structure and the fully calculated structure both using the FEM as Hermitian beam element as a planar (2-D) frame element in *MATLAB*<sup>®</sup>.

Table 4. First ten natural vibration's frequencies of the entire system and the synthesized system.

Mode N°	Theoretical Frequency <i>MATLAB</i> <sup>®</sup> [Hz]	Frequency Synthesized <i>MATLAB</i> <sup>®</sup> [Hz]	Relative Error [%]	MAC
1	20,49	20,49	0,000	0,9998
2	32,08	32,08	0,000	0,9718
3	105,39	105,39	0,000	0,9706
4	135,56	135,56	0,000	0,9991
5	186,70	186,70	0,000	0,9550
6	339,52	339,52	0,000	0,9673
7	379,51	379,51	0,000	0,9936
8	464,15	464,15	0,000	0,9019
9	610,77	610,77	0,000	0,9966
10	709,69	709,69	0,000	0,9567
Mean Values			0,000	0,9712

It was synthesized 40 frequencies in the first substructure and 15 frequencies in the second substructure, including the constrained degrees of freedom in both substructures; it means that the total frequencies obtained on the synthesized system were 47 frequencies. Because it was eliminated one mode shape to define the residual flexibility modes and the first six frequencies that represented the constrained degrees of freedom of the system which means that the value was zero. The mean value of the entire answer including all the 47 frequencies obtained was 1,12% in the relative error and the Figure 4 shows the behaviour of the MAC index along the response.

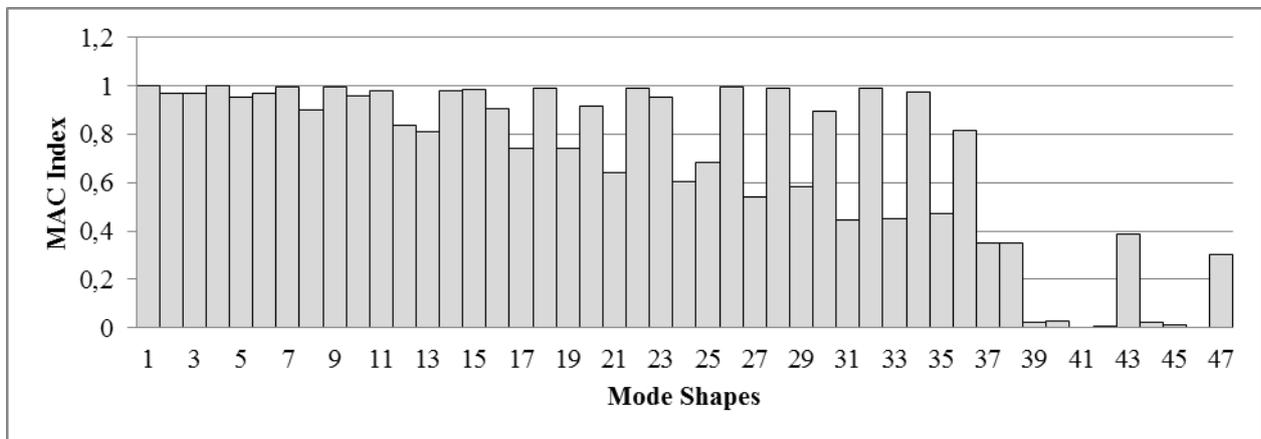


Figure 7. MAC Index behaviour between the answers in *MATLAB*<sup>®</sup>.

Table 5 shows the relationship between the first ten natural frequencies of the synthesized structure using the FEM in *MATLAB*<sup>®</sup> and the fully calculated structure using *ANSYS*<sup>®</sup> - *APDL 16.0* with BEAM188 as element type.

Table 5. First ten natural vibration's frequencies of the entire system in *ANSYS*<sup>®</sup> and the synthesized system in *MATLAB*<sup>®</sup>.

Mode N°	Theoretical Frequency <i>ANSYS</i> <sup>®</sup> [Hz]	Frequency Synthesized <i>MATLAB</i> <sup>®</sup> [Hz]	Relative Error [%]	MAC
1	20,43	20,49	0,290	0,9972
2	31,76	32,08	1,017	0,9655
3	103,28	105,39	2,045	0,1202
4	133,32	135,56	1,683	0,7105
5	179,76	186,70	3,859	0,9627
6	323,48	339,52	4,958	0,0045
7	364,33	379,51	4,167	0,5418
8	428,88	464,15	8,224	0,9320
9	605,79	610,77	0,823	0,9258
10	650,72	709,69	9,063	0,0321
Mean Values			3,613	0,6192

With the Table 5 it could be seen that if it is improve the “mesh” or in a simple way, if it is increase the number of elements that describes the system, the modal base will be better described. This is presented in the Table 6, making the comparison between the answers in *MATLAB*<sup>®</sup> using 400 elements in the first substructure and 150 elements in the second substructure. The Figure 8 shows the behaviour of the MAC index along the modes.

Table 6. First ten natural vibration's frequencies of the entire system and the synthesized system.

Mode N°	Theoretical Frequency <i>MATLAB</i> <sup>®</sup> [Hz]	Frequency Synthesized <i>MATLAB</i> <sup>®</sup> [Hz]	Relative Error [%]	MAC
1	20,49	20,49	0,001	1,0000
2	32,08	32,08	0,000	0,9976
3	105,39	105,39	0,000	0,9977
4	135,56	135,56	0,000	1,0000
5	186,70	186,70	0,000	0,9992
6	339,51	339,51	0,000	0,9982
7	379,49	379,49	0,000	0,9998
8	464,14	464,14	0,000	0,9989
9	610,64	610,64	0,000	0,9997
10	709,61	709,61	0,000	0,9982
Mean Values			0,000	0,9989

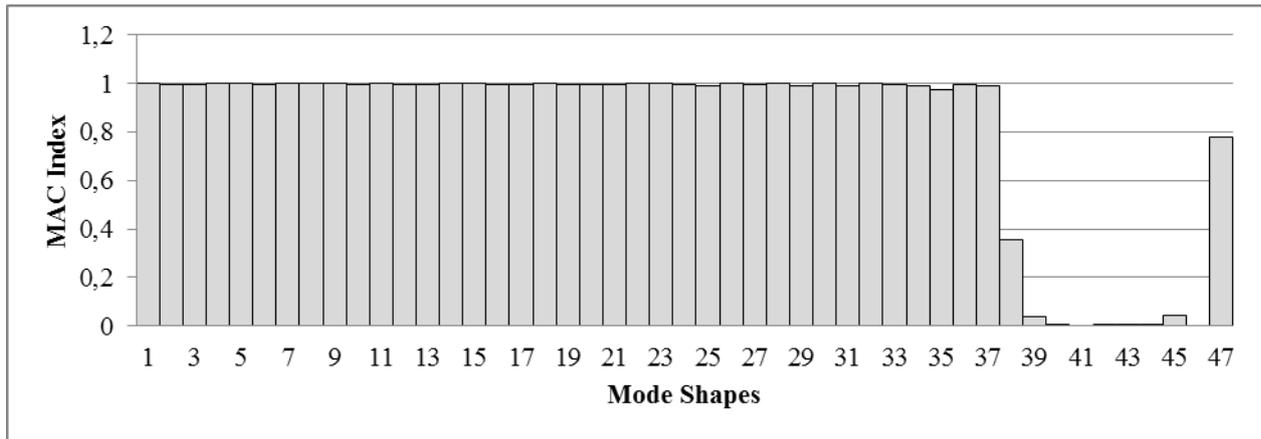


Figure 8. MAC index behaviour between the answers in *MATLAB*<sup>®</sup> with more elements describing the systems.

## 5. CONCLUSIONS

The Hermitian beam element as a planar (2-D) frame element including the axial and bending deformation presented effectiveness to get the entire system frequencies and also the use of this element to get a synthesized system through the MSFR method, with a relative error near to zero and a MAC near to the unity, this can be seen in the Table 4 and Table 6.

The results of identification of the modes of substructures using the MAC method showed that the MAC index is higher at 0,60 and some of this values near to the unity between *ANSYS*<sup>®</sup> and *MATLAB*<sup>®</sup>, with a mean relative error smaller at 4%.

According to Table 6 and Figure 8 it can be seen that if the system has a better described base modal, the result will be more accurate with almost all the MAC index values near to the unity.

Figure 4 show that the acoustic resonance frequencies are approximately 58 Hz, 177 Hz and 290 Hz, which means that if the structural natural frequencies are near to these values, it will appear noise due to vibration of the structural part. One way to avoid this phenomenon it will be just change the length of the bigger tube (substructure 1) or modify the position of the constraint.

## 6. REFERENCES

- Araújo, C.A., 1998. *Modelagem de Sistemas Dinâmicos Através da Síntese Modal de Componentes*. Ph.D. thesis, Campinas, São Paulo: Departamento de Projetos Mecânicos, Faculdade de Engenharia Mecânica, Unicamp.
- Ewins, D.J., 1984. *Modal Testing: Theory and Practice*. England: John Wiley & Sons Inc, London.
- Howard, C.Q. and Cazzolato, B.S., 2015. *Acoustic Analyses Using MATLAB<sup>®</sup> and ANSYS<sup>®</sup>*. Taylor & Francis Group, LLC. CRC Press.
- ISOVER, 2015. "HVAC Ducts Handbook" ISOVER Technical Insulation. 15 Aug. 2017 < <http://www.isover-technical-insulation.com/documentation/hvac-ducts-handbook> >.
- Kwon, Y.W. and Hyochoong, B., 2000. *Acoustic Analyses Using MATLAB<sup>®</sup> and ANSYS<sup>®</sup>*. Taylor & Francis Group, LLC. CRC Press.
- Ordoñez, J.M.B. and Nunes, M.A.A., 2017. "Método de Síntese Modal de Componentes em Sistemas HVAC Visando Análise de Vibração Estrutural". In *Proceedings of the 13th Ibero-American Congress of Mechanical Engineering – CIBEM2017*. Lisboa, Portugal.
- Piana, G., Lofrano, E., Carpinteri, A., and Paolone, A. 2016. "Experimental Modal Analysis of Straight and Curved Slender Beams by Piezoelectric Transducers". *CrossMark. Meccanica* 51:2797-2811, pp. 1-15.

## 7. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.