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A NOTE ON OLFC CONTROL APPLIED IN A PARAMETRICALLY EXCITED PENDULUM

Angelo Marcelo Tusset
Bianca Marin
Rodrigo Tumolin Rocha
Frederic Conrad Janzen
Federal University of Technology – Paraná, 84016-210, Ponta Grossa – PR, Brazil
tusset@utfpr.edu.br; bi_marin@hotmail.com; digao.rocha@gmail.com; fcjanzen@hotmail.com

José Manoel Balthazar
ITA - Aeronautics Technological Institute, Marechal Eduardo Gomes Square, 12228-900, São José dos Campos – SP, Brazil
jmbaltha@gmail.com

Abstract. This paper presents the application of the Optimal Linear Feedback Control (OLFC) to suppress the chaotic behavior of a parametrically excited pendulum. The adopted active control strategy consists of two controls: a nonlinear feedforward control in order to keep the system controlled in a desirable orbit, and the feedback control, which may be obtained by considering the LQR control with the goal of bringing the system into a desired orbit. Numerical simulations showed the efficiency of the proposed control in the suppression of the chaotic behavior.

Keywords: Optimal Linear Feedback Control, chaos, parametrically excited pendulum, active control strategy.

1. INTRODUCTION

The dynamics of the parametrically excited pendulum has been currently investigated for some researchers (Clifford and Bishop, 1995, 1996; Lu, 2006; Lenci *et al*., 2008, Litak *et al*. 2008a). Without control action, the pendulum may exhibit different types of behaviour, ranging from periodic to chaotic oscillations (Kecik and Warminski, 2012), as verified by Xu *et al*. (2005), and Litak *et al*. (2010).

The authors (Stilling and Szyszkowski, 2002, Sieber *et al*., 2008, Yokoi and Hikihara, 2011, Wang and Jing, 2004, Tusset et al., 2015), used a controller for pendulum system with the objective of stabilizing chaotic motion in a desired periodic motion.

This paper proposes the movement control of a parametrically excited pendulum considering the optimal feedback control. The representation of the parametric pendulum model is illustrated in Fig. 1.
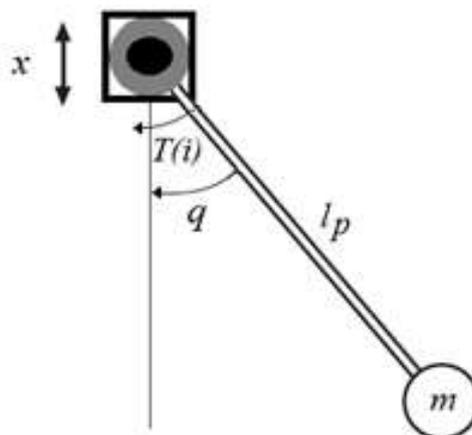

Figure 1: Schematic diagram of the parametric pendulum.

The mathematical model of the parametric pendulum, shown in Fig. 1, has its equations of motion as Eq. (1) (Xu *et al.*, 2005).

$$\ddot{q} + \gamma\dot{q} + (1 + \rho\cos(wt))\sin(q) = aT \tag{1}$$

where: q is the angle of pendulum.

Equation (1) can be rewritten in state-space form, given by Eq. (2).

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\gamma x_2 - (1 + \rho\cos(w\tau))\sin(x_1) + aT \tag{2}$$

where: $x_1 = q$ and $x_2 = \dot{q}$.

In order to eliminate the chaotic behaviour of Eq. (2), a control was designed by introducing a torque control signal ($aT = U$) in the pendulum system denoted in Eq. (3).

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\gamma x_2 - (1 + \rho\cos(w\tau))\sin(x_1) + U \tag{3}$$

where: $U = u^* + u$. The u^* is the feedforward control and u is the feedback control. For the system of Eq. (3), it is considered that:

$$u^* = \dot{x}_2^* + \gamma x_2^* + (1 + \rho\cos(w\tau))\sin(x_1^*) \tag{4}$$

where x_1^* and x_2^* are the desired trajectories.

Substituting u^* (Eq. (4)) into Eq. (3) and defining the deviation of the desired trajectory as:

$$y = \begin{bmatrix} x_1 - x_1^* & x_2 - x_2^* \end{bmatrix}^T \tag{5}$$

The system of Eq. (3) may be rewritten in matrix form given by Eq. (6).

$$\dot{y} = Ay + G(y, x^*) + Bu \tag{6}$$

where:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -\gamma \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } G(y, x^*) = \begin{bmatrix} 0 \\ (\rho\cos(w\tau))\sin(x_1^*) - (\rho\cos(w\tau))\sin(y_1 + x_1^*) \end{bmatrix} \tag{7}$$

The feedback control (u) can be found by solving the following Eq. (8).

$$u = -R^{-1}B^T P\, y = -Ky \tag{8}$$

The State Dependent Riccati equation to obtain P, is given by:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{9}$$

2. COMPUTATIONAL PROCEDURE

The analyses of the system are carried out through numerical simulation that considers the following parameters: $\gamma = 0.1$, $\rho = 2.256281$, $w = 1.634171$, $x_1(0) = 0.314$ and $x_2(0) = 0$.

Defining the desired trajectory, as being a periodic orbit: $x_1^* = \rho\cos(w\tau) + \tau$, $x_1^* = -\rho w\sin(w\tau) + 1$, and matrices:

$$Q = \begin{bmatrix} 100 & 0 \\ 0 & 10 \end{bmatrix} \text{ and } R = \begin{bmatrix} 1 \end{bmatrix} \tag{11}$$

The feedback control u is obtained using $K = lqr(A, B, Q, R)$, which is a command of Matlab®, yielding:

$$u = -10(y_1) - 5.378138(y_2) = -10(x_1 - x_1^*) - 5.378138(x_2 - x_2^*) \tag{12}$$

3. RESULTS AND DISCUSSION

Figures 2 show the behaviour of Eq. (3) considering $U = 0$. The uncontrolled system has a chaotic behaviour, being that the respective Lyapunov's exponents are $\lambda_1 = 0.29$ and $\lambda_1 = -0.39$, thus proving the chaotic behaviour.
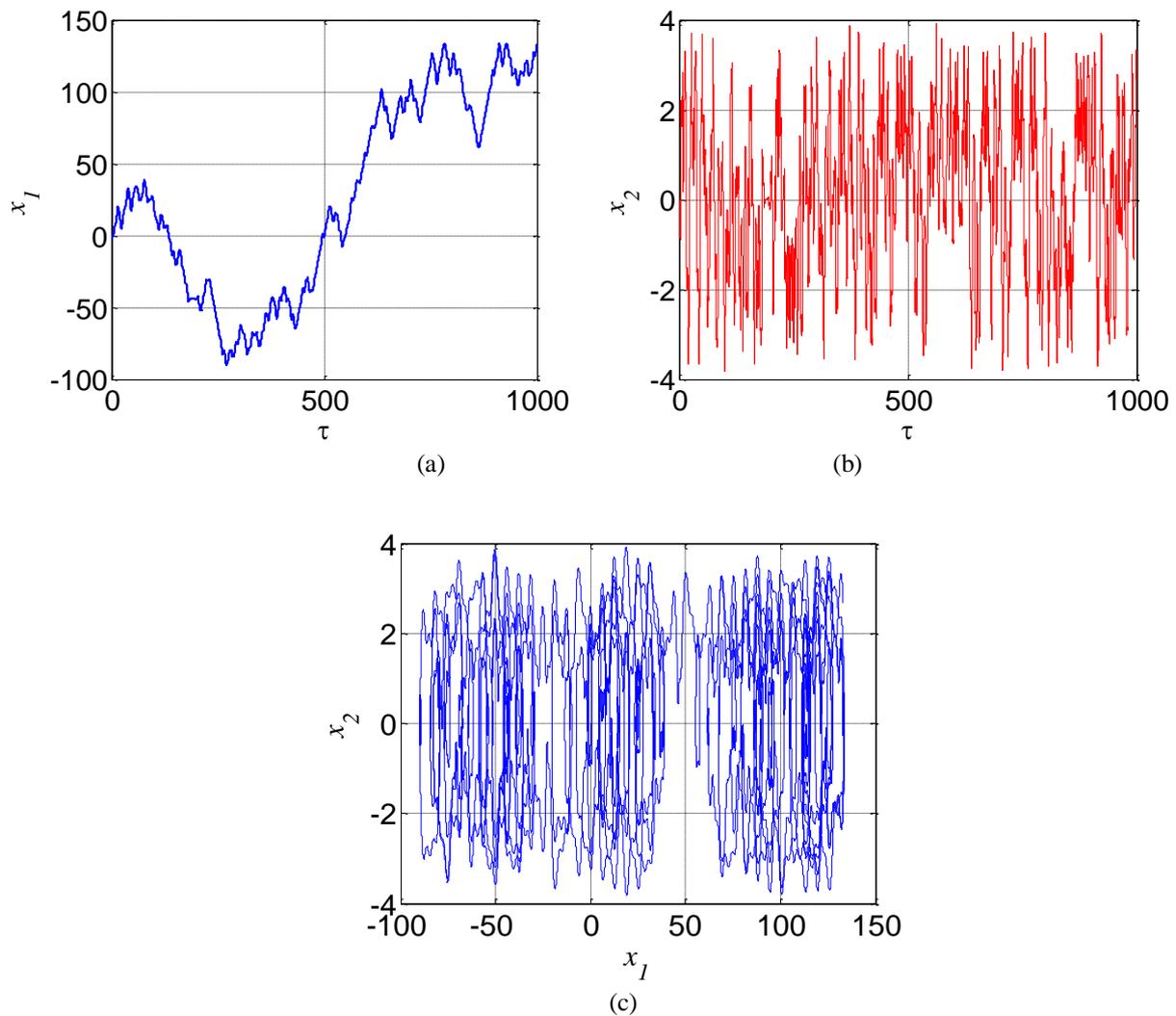

Figure 2: (a) Time history of the states (x_1), (b) time history of the states (x_2), (c) phase plane.

3.1 Application of the OLFC

In Fig. 3 is observed the behaviour of Eq. (3) with the proposed control. The application of the OLFC control was efficient in driving the system to a periodic behaviour.
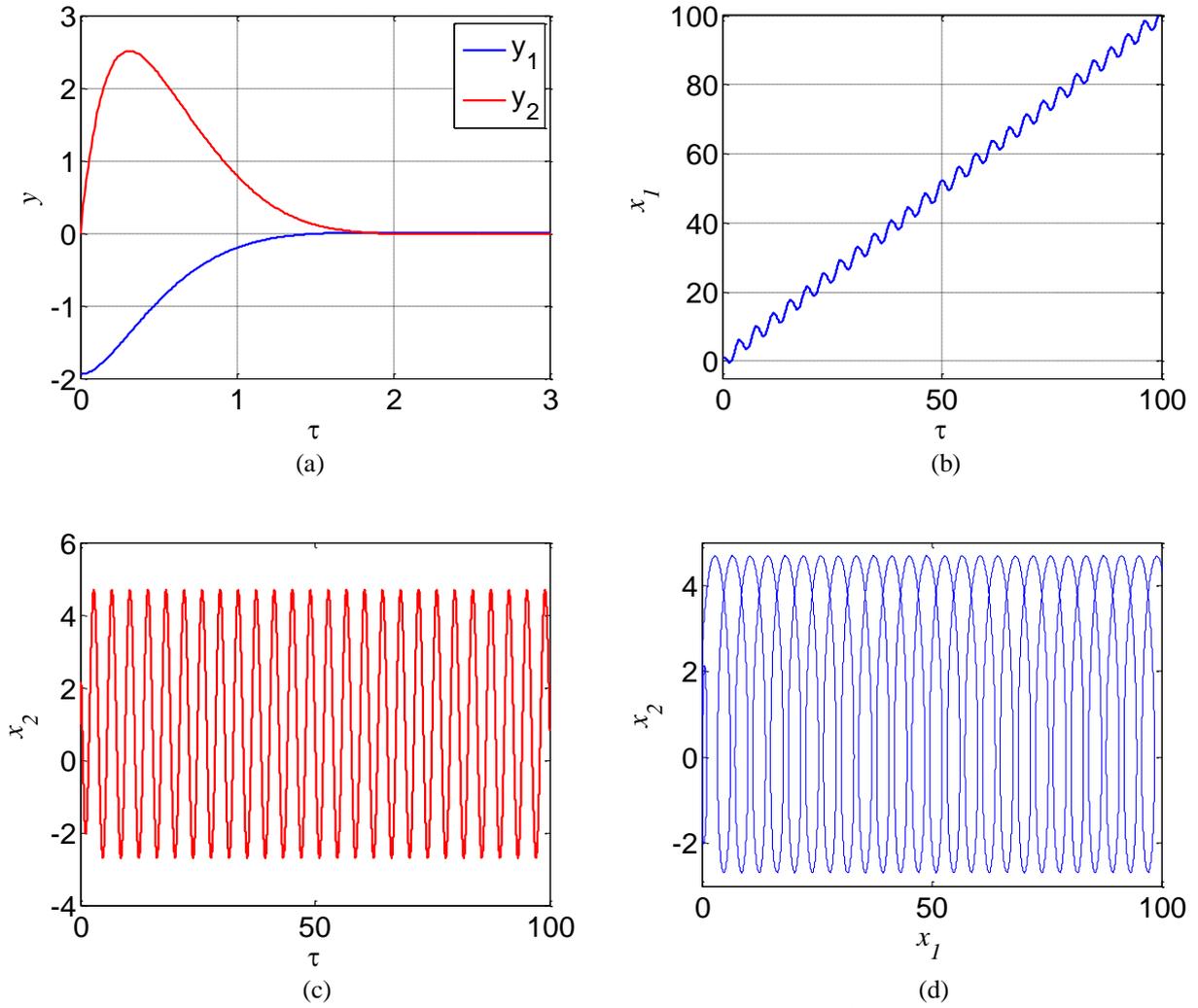
Figure 3: Movements of the pendulum with the proposed control: (a) Deviation of the desired trajectory. (b) Angle of the pendulum ($x_1 = y_1 + x_1^*$). (c) Angular velocity ($x_2 = y_2 + x_2^*$). (d) Phase diagram.

3.2 Proposed control with parametric errors

In order to determine the effects of uncertainties on the performance of the controller, it is estimated that an error of $\pm 20\%$ is encountered in the parameters (γ , ρ and w), with a strategy similar used by Tusset *et al*. (2015).

In Figs. 4, 5 and 6, the robustness of the control to maintain the system in a periodic orbit are observed, considering the proposed control with variation in parameters: $\gamma_u = 0.1(0.8 + 0.4 r(t))$, $\rho_u = 2.256281(0.8 + 0.4 r(t))$, $w_u = 1.634171(0.8 + 0.4 r(t))$, where $r(t)$ is a random number $r(t) = [0,1]$.

Considering that $e_c = \begin{bmatrix} x_{1c} - x_{1c}^u & x_{2c} - x_{2c}^u \end{bmatrix}^T$ and x_c are the states obtained with the control without parametric uncertainties and x_c^u the states obtained with the control with parametric uncertainties.

In Fig. 4 can be observed the robustness of the control in keeping the system in the same orbit obtained with the control without uncertainties, considering the uncertainties only in the feedback control (u), considering the matrix: $A = \begin{bmatrix} 0 & 1 \\ 0 & -0.1(0.8 + 0.4 r(t)) \end{bmatrix}$.
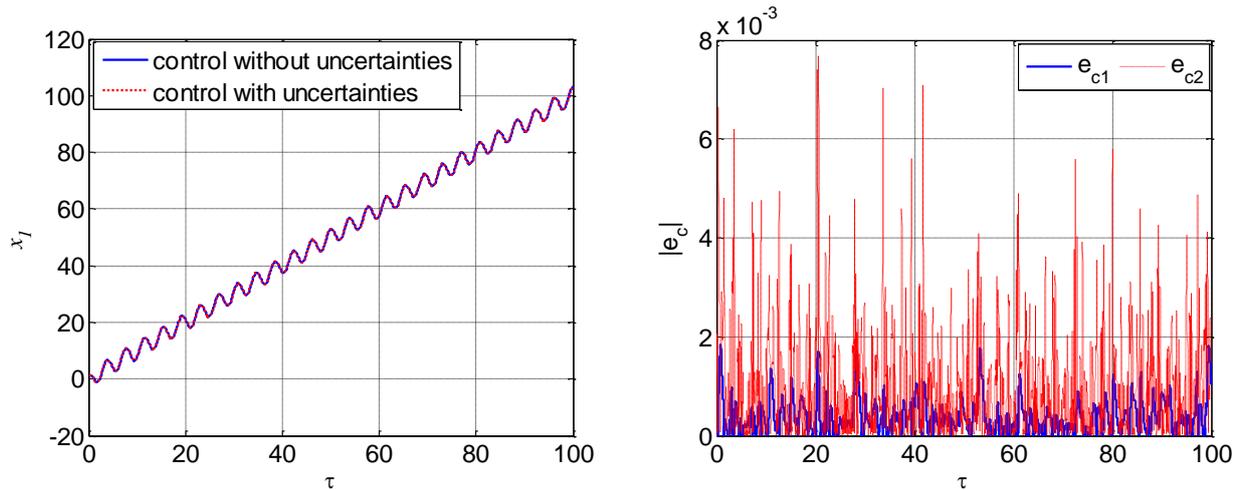
Figure 4: Movements of the pendulum with the proposed control with parametric errors in feedback control (u): (a) Angle of the pendulum. (b) Deviation of the desired trajectory $e_c = \begin{bmatrix} x_{1c} - x_{1c}^u & x_{2c} - x_{2c}^u \end{bmatrix}^T$

As can be seen in Fig. 4 even with uncertainties in the feedback control the control was robust with an absolute maximum error of approximately $|e_{c1}| \approx 0.001834$ and $|e_{c2}| \approx 0.007678$.

In Fig. 5, it is possible to observe the robustness of the control in keeping the system in the same orbit obtained with the control without uncertainties, considering the uncertainties only in the feedforward control (u^*), considering the feedforward control in form: $u^* = \dot{x}_2^* + \gamma_u x_2^* + (1 + \rho_u \cos(w_u \tau)) \sin(x_1^*)$.
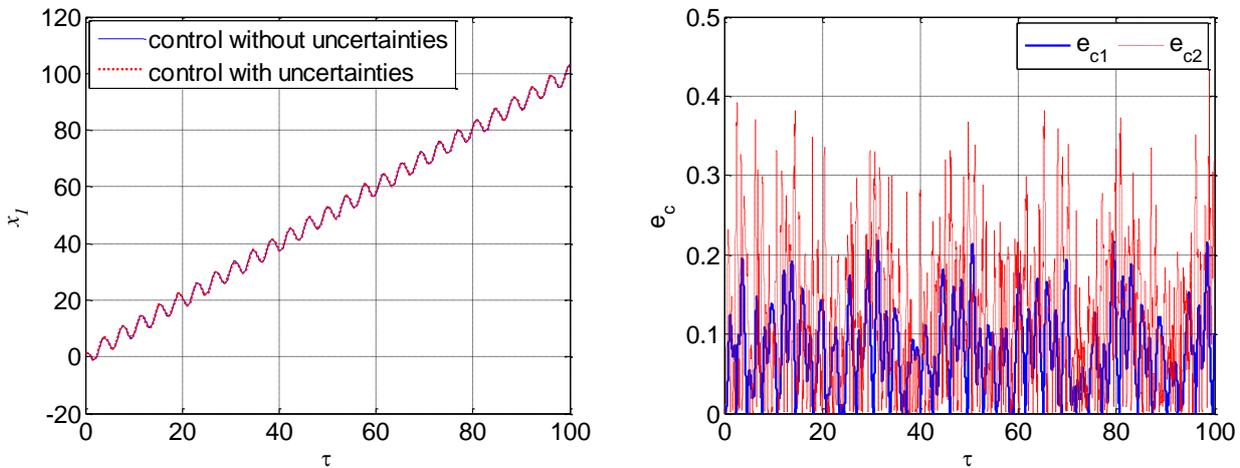

Figure 5: Movements of the pendulum with the proposed control with parametric errors in feedback control (u*): (a) Angle of the pendulum. (b) Deviation of the desired trajectory $e_c = \begin{bmatrix} x_{1c} - x_{1c}^u & x_{2c} - x_{2c}^u \end{bmatrix}^T$.

As can be seen in Fig. 5 even with uncertainties in the feedforward control the control was robust with an absolute maximum error of approximately $|e_{c1}| \approx 0.2179$ and $|e_{c2}| \approx 0.4321$.

In Fig. 6 is observed the robustness of the control in keeping the system in the same orbit obtained with the control without uncertainties, considering the uncertainties in feedback control (u) and in the feedforward control (u^*), considering the feedforward control in form: $u^* = \dot{x}_2^* + \gamma_u x_2^* + (1 + \rho_u \cos(w_u \tau)) \sin(x_1^*)$, and the matrix: $A = \begin{bmatrix} 0 & 1 \\ 0 & -0.1(0.8 + 0.4r(t)) \end{bmatrix}$.
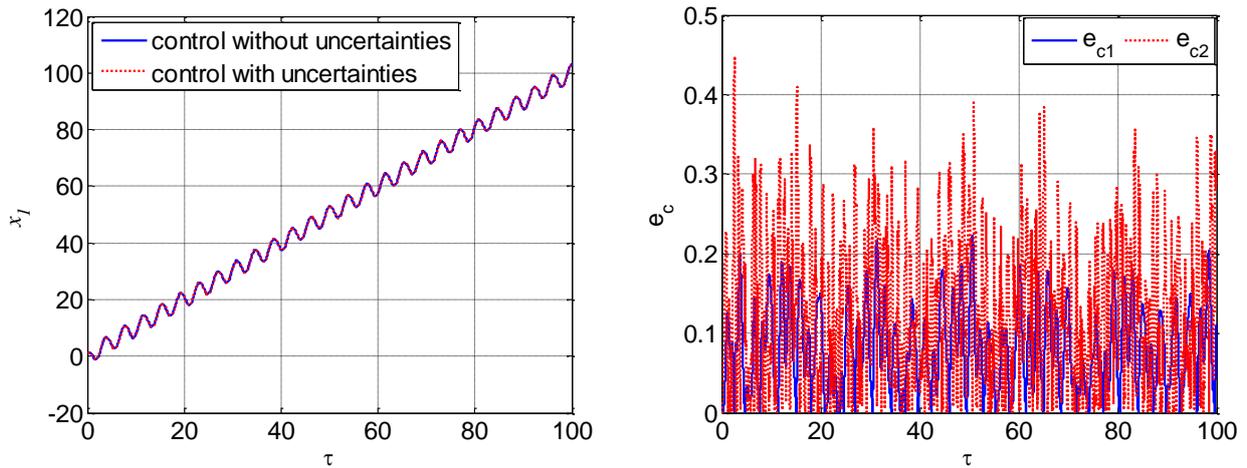
Figure 6: Movements of the pendulum with the proposed control with parametric errors in feedback control and feedforward: (a) Angle of the pendulum. (b) Deviation of the desired trajectory $e_c = \begin{bmatrix} x_{1c} - x_{1c}^u & x_{2c} - x_{2c}^u \end{bmatrix}^T$.

As can be seen in Fig. 6 even with uncertainties in the feedforward control the control was robust with an absolute maximum error of approximately $|e_{c1}| \approx 0.223$ and $|e_{c2}| \approx 0.4476$.

Conclusions

As can be seen in both Figs. 3, with the proposed association of feedforward control (u^*) and feedback (u), it was possible to control the oscillations of a parametrically excited pendulum in a pre-defined periodic orbit.

As can be observed in Figs. 4, the proposed control with only parametric errors in the feedback control (u) maintains a very low error as shown in Fig. 4b, and the system remains robust, as shown in Fig. 4a.

Related to the sensitivity of the feedforward control (u^*), it can be observed in Fig. 5a that the proposed control is robust, however the error increases significantly with respect to the error only in the feedback control (u) if there is an increase of 11768.19% for e_{c1}, and 5527.768% for e_{c2}.

As can be observed in Fig. 6, the proposed control is more sensitive to parametric variations in the feedforward control (u^*).

4. ACKNOWLEDGEMENTS

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