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NONLINEAR DYNAMICS AND CHAOS OF A SMA-HYBRID COMPOSITE OSCILLATOR

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Abstract. Composite materials have been widely used because its great capability to join high strength and stiffness in lightweight structures. Shape memory alloy hybrid composite (SMAHC) is an interesting alternative to build adaptive composites that increase even more the applicability of this kind of material. The aim of this paper is to study nonlinear dynamics of an oscillator with SMAHC element. A polynomial constitutive model is employed to describe the SMA fiber thermomechanical behavior while the matrix material is modelled as linear elastic element. A homogenized model is developed for the SMAHC that is employed in a mechanical oscillator in order to analyze the dynamical response. Numerical simulations are carried out evaluating system responses and the influence of SMA fiber volume fractions. Results show that even 0.1% of SMA volume fraction may be responsible to define a chaotic or periodic response.

Keywords: Shape memory alloys, composite materials, chaos, vibration

1. INTRODUCTION

Modern engineering is always looking for new advanced materials that can increase system capabilities. Ashby and Bréchet (2003) identified two possible approaches to address this problem: the development of new materials or the creation of hybrid materials that combine characteristics of existing materials. The connection between smart and composite materials can increase even more the applicability of composite materials, creating adaptive hybrid composites. In this regard, the joining of shape memory alloys (SMAs) with composites creates the Shape Memory Alloy Hybrid Composites (SMAHC) that can be employed for different purposes as shape or vibration control (Lester *et al.*, 2015).

SMA belongs to a class of smart materials that have unique characteristics due to solid phase transformation. The two main phenomena involved on SMA thermomechanical behavior are pseudoelasticity and shape memory effect (SME). Pseudoelasticity takes place in high temperatures where the microstructure is austenitic in a stress-free state. Under this condition, when a mechanical load is applied, microstructure is transformed to detwinned martensite and after unloading the reverse transformation occurs (Lagoudas, 2008). This response is associated with a hysteresis loop that dissipates energy and can be exploit for dynamical applications (Savi, 2015).

This paper deals with a dynamical analysis of a system with a composite bar with SMA fibers embedded in a matrix with linear-elastic behavior. A single degree of freedom oscillator is analyzed considering that the restitution force is provided by an SMAHC. Equations of motion are solved using Runge-Kutta method. Nonlinear dynamics is

analyzed exploring the complex response of the oscillator. Lyapunov's exponents, bifurcation diagram and phase portraits are used to present the system response.

2. DYNAMICAL MODEL

SMA thermomechanical behavior can be described by different constitutive models. Based on the discussion presented by Paiva and Savi (2006), the polynomial model proposed by Falk (1983) is chosen for the SMA fibers. The SMA free energy potential is defined as

$$\psi_{sma}(\varepsilon, T) = \frac{a}{2}(T - T_M)\varepsilon_{sma}^2 - \frac{b}{4}\varepsilon_{sma}^4 + \frac{b^2}{24a(T_A - T_M)}\varepsilon_{sma}^6 \quad (1)$$

where a and b are material properties and T_A and T_M are the temperatures above austenite and martensite phases are stables, respectively.

By definition, $\sigma_{sma} = \partial\psi_{sma} / \partial\varepsilon_{sma}$. Hence, the stress-strain-temperature (σ - ε - T) relation on SMA fibers are described by

$$\sigma_{sma} = a(T_{sma} - T_M)\varepsilon_{sma} - b\varepsilon_{sma}^3 + \frac{b^2}{4a(T_A - T_M)}\varepsilon_{sma}^5 \quad (2)$$

Considering the linear elastic constitutive model of the matrix,

$$\sigma_m = E_m \varepsilon_m \quad (3)$$

where E_m is the matrix elastic modulus.

The composite bar equivalent constitutive model considers three assumptions: matrix and fibers are in temperature equilibrium; both phases have the same strain by kinematical hypothesis; sharing applied stress provides equilibrium. Under these assumptions,

$$T_{sma} = T_m = T \quad (4)$$

$$\varepsilon_{sma} = \varepsilon_m = \varepsilon \quad (5)$$

$$V_{sma}\sigma_{sma} + (1 - V_{sma})\sigma_m = \sigma \quad (6)$$

where V_{sma} is the SMA fiber volume fraction.

Hence, the one-dimensional constitutive model of the bar is defined as follows

$$\sigma = \left[V_{sma} a (T - T_M) + (1 - V_{sma}) E_m \right] \varepsilon - V_{sma} b \varepsilon^3 + V_{sma} \frac{b^2}{4a(T_A - T_M)} \varepsilon^5 \quad (7)$$

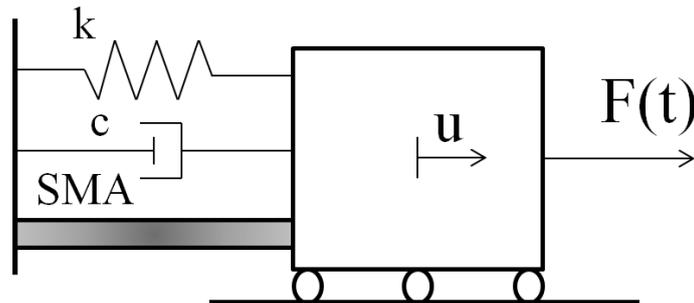


Figure 1: Scheme of 1D oscillator analogous to a composite bar.

Based on this, it is possible to analyze the dynamical system defining an oscillator where the restitution force is provided by the SMA hybrid composite. A schematic picture of the oscillator is presented in Fig. 1 where the elastic spring represents matrix and the linear viscous damping is associated with intrinsic dissipations of the system. A harmonic external load of the form $F(t) = mf_0 \cos(\Omega t)$ are also considered and small strain hypothesis is adopted, resulting in the following equation of motion,

$$m\ddot{u} + c\dot{u} + \frac{A}{L} [V_{sma}a(T - T_M) + (1 - V_{sma})E_m]u - \frac{A}{L^3}V_{sma}bu^3 + \frac{A}{L^5}V_{sma} \frac{b^2}{4a(T_A - T_M)}u^5 = mf_0 \cos(\Omega t) \quad (8)$$

By defining:

$$\alpha_1 = -\left(\frac{A}{mL^3}\right)V_{sma}b \quad (9)$$

$$\alpha_2 = \left(\frac{A}{mL^5}\right)V_{sma} \left[\frac{b^2}{4a(T_A - T_M)} \right] \quad (10)$$

$$\xi = \frac{c}{2m\omega^2} \quad (11)$$

$$\omega^2 = \left(\frac{A}{mL}\right)[V_{sma}a(T - T_M) + (1 - V_{sma})E_m] \quad (12)$$

The equations of motion is rewritten as follows

$$\ddot{u} + 2\xi\omega^2\dot{u} + \omega^2u + \alpha_1u^3 + \alpha_2u^5 = f_0 \cos(\Omega t) \quad (13)$$

3. RESULTS AND DISCUSSION

This section discusses numerical simulations of the SMA composite oscillator. Fiber volume fraction is a primary point on the composite materials design. Considering $T = 298.0K$ and $f_0 = 10m/s^2$, Fig. 2 presents the bifurcation diagram according to the V_{sma} variation. In general, it is observed a wide range of chaotic responses for $0.60 < V_{sma} < 0.95$ with some periodic windows.

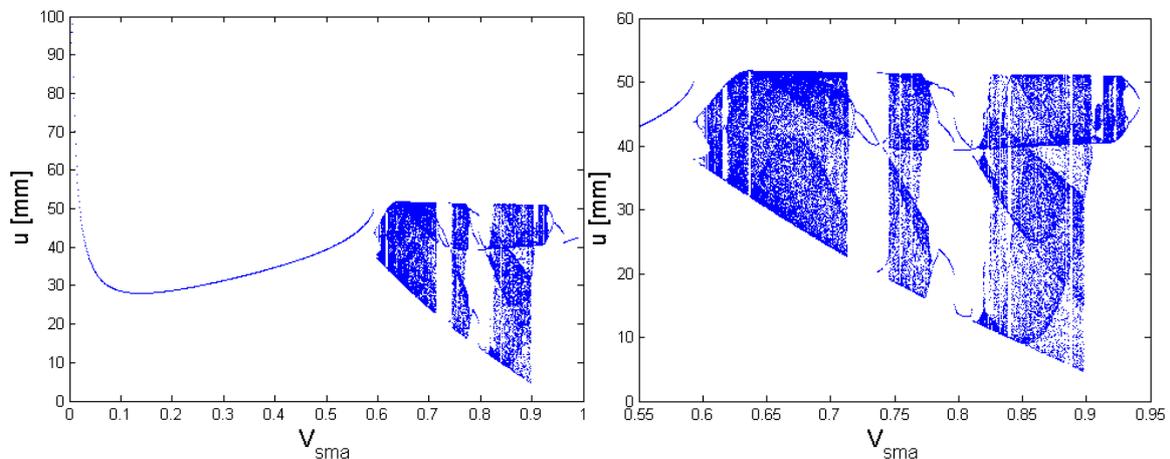


Figure 2: Bifurcation diagram according to the fiber volume fraction.

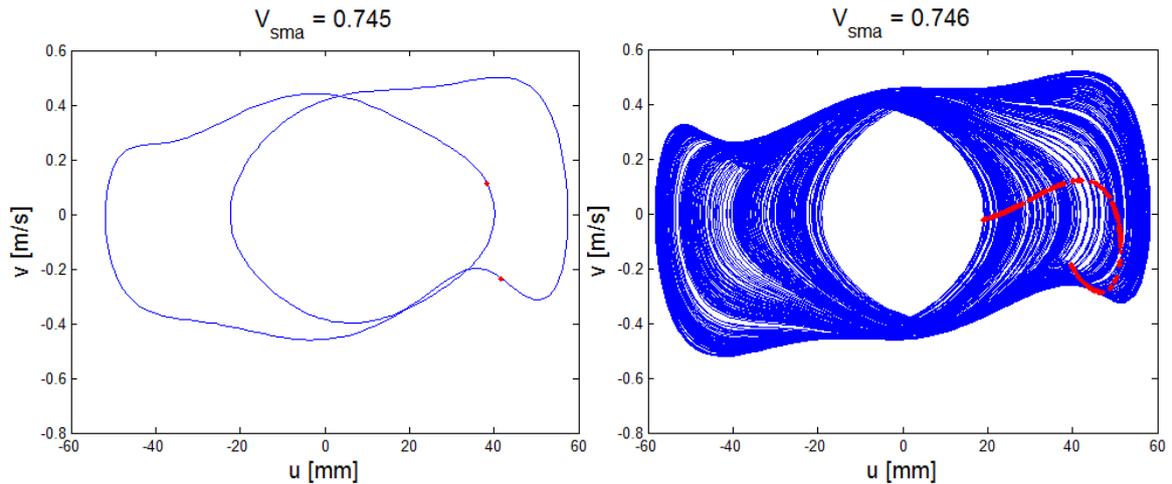


Figure 3: Phase portrait and Poincaré sections.

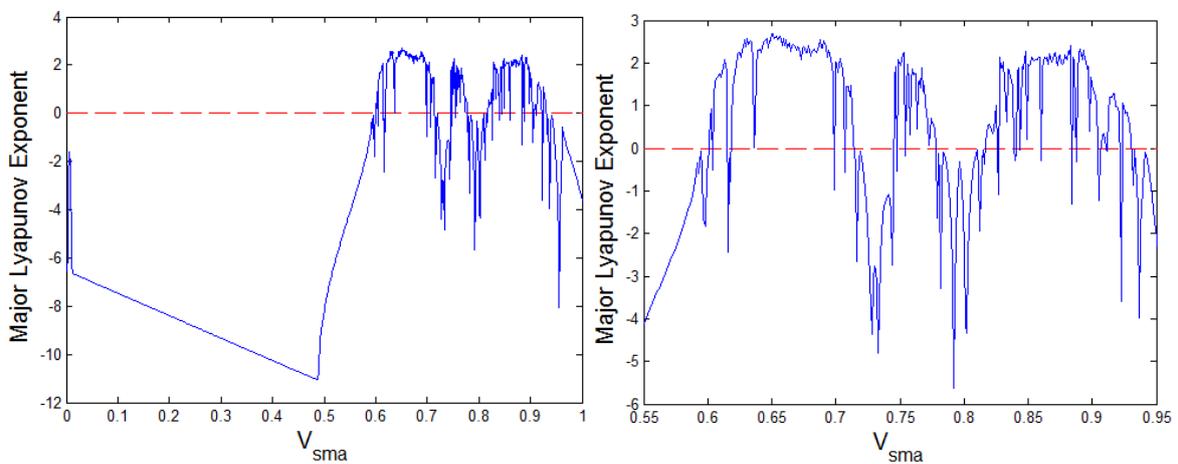


Figure 4: Lyapunov spectrum.

It is observed that the variation of 0.1% on the fiber volume fraction may modify the system response changing from chaotic to periodic response. The phase portraits and the Poincaré sections of V_{sma} equal to 0.745 and 0.746 are showed Fig. 3. Greater Lyapunov exponent is plotted in Fig. 4 where it is possible to identify the existence of positive values that assures the system chaotic behavior. It should be pointed out that volume fraction changes the composite stiffness, changing the stress level and therefore, phase transformation on SMA fiber. These changes reduces system nonlinearity, dramatically changing system response.

4. CONCLUSIONS

Dynamical behavior of shape memory alloy hybrid composite is investigated using a SMA oscillator. The SMAHC oscillator has a rich response with great influence of SMA fiber volume fraction. Numerical simulations show that small variations of SMA volume fraction can dramatically alter the system response. The addition of more laminas with linear stiffness occur a change of a chaotic to periodic behavior due to the reduction of nonlinear effects.

5. ACKNOWLEDGEMENTS

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