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STOCHASTIC ANALYSIS OF 1-DOF VEHICLE SUSPENSION SYSTEM EMPLOYING ASYMMETRICAL DAMPING

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Abstract. *Ground vehicles are subject to the vibrations induced by the road in which they travel. The road needs to have acceptable roughness index in order to provide low impact without compromising tire contact. However, vehicles still need to be equipped with suspension systems to minimize the effects of road imperfections. For analysis of suspension systems, the irregularities contained in the road can be described by the road profile itself or by its statistical characteristics. Thus, ISO 8608:1995 specifies a uniform method for characterizing the vertical profile of roads, streets, highways and unpaved roads. The profile of the road can be generated through approximate stochastic functions through the power spectral density of the signal. In this work, the modeling of the road profile is shown according to ISO 8608:1995 for different roughness classes and a quarter-car model with asymmetrical damping is used to represent the vehicle. Sinusoidal approximation is used for the road profile. The main objective of this work is to analyze the suspension system employing asymmetrical damping, in which the stochastic excitation comes from a sinusoidal approximation and the road profile parameters are described by ISO 8608:1995.*

Keywords: *Stochastic road profile, asymmetrical damping, suspension system, vehicle dynamics*

1. INTRODUCTION

The search for improvements in passenger comfort and safety is constant in the automotive industry. In this context, several technological innovations related to the improvement of performance and comfort were introduced to the suspension systems in the last decades. Several studies involving numerical and experimental simulation in suspension systems are applied in order to improve comfort. An issue that must be taken into account when the study is done numerically is the characterization of the road profile that will determine the source of excitation of the system. However, there are certain characteristics which must be taken into account in order that the method used can accurately represent the characteristics of an actual road.

In order to inspect and classify the quality of the roads, laser profilers are used. The laser profilometer is an evolution of the rod profilometer and allows to measure deviations in elevation of the surface of the road in relation to an ideal profile. Based on this longitudinal profile, the various indicators of surface regularity of the road are calculated (Barella *et al.*, 2005). The use of this equipment assists in the capture of the data referring to the road to which one wishes to study or obtain its characteristics.

Generally in numerical simulations for dynamic analysis of passenger vehicles, the profile of the pavement can be represented by functions such as step functions, harmonic, ramp, triangular waves and others. Although these cited functions provide a basis for studies, they are crude models to represent the real features along a road. However, it is common and accepted that the profile of a common road is more complex. In order to have a more realistic profile of the road, a stochastic model can be used (Rill, 2011; Wong, 2001).

Thereby, to characterize a road profile that contains the characteristics of a real road and consequently provides a good approximation to be used, ISO 8608:1995 determines guidelines for the parameters to be used. Several authors have used analytical expressions based on this standard as a way of characterizing the excitation in vehicular suspension system analysis (Tamboli and Joshi, 1999; Verros *et al.*, 2005; Paraforos *et al.*, 2016).

Generally in models of suspension systems, linear symmetric damping is used. However, vehicular dampers present a nonlinear behavior. Furthermore, the damping force during the extension phase is different of the compression phase. This type of damper can shift the oscillation mean position of the body coupled to the damper when excited harmonically (Rajalingham and Rakheja, 2003; Silveira *et al.*, 2017). This effect has been studied by several authors and is called jacking down effect.

Therefore, it should be keep in mind that improving passenger comfort of the vehicle depends on the techniques involved in research into suspension systems. The use of analytic expressions that characterize the road helps in understanding the effects that the irregularities of the road cause in relation to the comfort of the passengers. Observing different studies involving the global dynamics of a vehicle, it can be concluded that the improvement of comfort should be considered as a global aspect.

The main objective of this work is to analyze the suspension system employing asymmetrical damping, in which the stochastic excitation comes from a sinusoidal approximation and the road profile parameters are described by ISO 8608:1995.

2. VEHICLE MODELING

The base of the system represents the road on which the vehicle moves, whose irregularities cause the forces that excite the system. The system shown in Figure 1 is comprised of mass m, the spring with elastic constant k, and the viscous damper with damping coefficient $c^{\pm}$, whose damping coefficient during the extension phase is given by c^{+} and damping coefficient during compression phase is given by c^{-}.

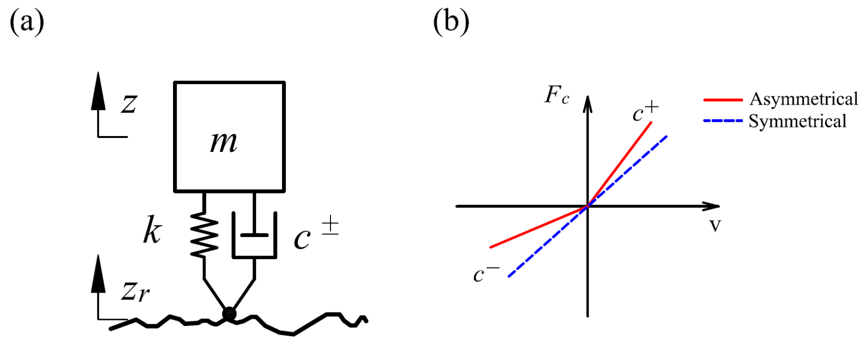

Figure 1. Schematic representation of the (a) 1-DOF quarter-car model with asymmetrical damping and (b) damping force characteristics of asymmetrical damper in relation to the relative velocity.

According to the classical vibration theory (Nayfeh and Balachandran, 2008; Rao, 2004), the dynamical equation for the system represented in Figure 1, can be written as:

$$m\ddot{z} + c^{\pm}(\dot{z} - \dot{z}_r) + k(z - z_r) = 0 \tag{1}$$

in which the damping coefficient of extension phase (c^{+}) and the damping coefficient of compression phase (c^{-}) is define as: c^{+} if $\dot{z} - \dot{z}_r \geq 0$ and c^{-} if $\dot{z} - \dot{z}_r < 0$.

The parameters used here are based on values for a medium size passenger vehicle (Silveira *et al.*, 2014), $m = 1500$ kg, $k = 44000$ N/m. For the simulations with symmetrical damping, the coefficients for extension and compression are $c^{+} = 4000$ and $c^{-} = 4000$ Ns/m, and the asymmetrical system receives different values for compression and extension according to an asymmetry ratio β, defined as $\beta = c^{+}/c^{-}$.

3. ROAD PROFILE

The ISO 8608:1995 proposes a qualification of the road roughness (Class A - H) based on the PSD according to the longitudinal profile. The values of PSD that ISO has proposed for road roughness is shown in table 1.

Table 1. Degree of roughness expressed in terms of Ω

Road class	Degree of roughness $\Phi(\Omega_0)^1$ 10^{-6} m^3		
	Lower limit	Geometric mean	Upper limit
A (very good)	-	1	2
B (good)	2	4	8
C (average)	8	16	32
D (poor)	32	64	128
E (very poor)	128	256	512
F	512	1024	2048
G	2048	4096	8192
H	8192	16384	-
$\Omega_0 = 1$ rad/m			

The power spectral density (PSD) of the road presents a constant drop in its magnitude according to the spatial frequency (Rill, 2011). According to ISO 8608:1995, random road profiles can be approximated by a PSD as follows:

$$\Phi(\Omega) = \Phi(\Omega_0)\left(\frac{\Omega}{\Omega_0}\right)^{-w}, \tag{2}$$

in which $\Omega = 2\pi/L$ rad/m denotes the spatial angular velocity and L is the wavelength. The value of PSD at the reference spatial angular velocity $\Omega_0 = 1$ rad/m is described by $\Phi_0 = \Phi(\Omega_0)$. The drop in the magnitude is modeled by the roughness coefficient of waviness w (according to ISO 8608:1995, $w = 2$). The integral of the PSD over a given frequency band calculates the average power of the signal over this frequency band.

According to this standard, roads, whether paved or not, can be classified from A to H, the classes are defined by reference to the degree of roughness (Φ_0) as shown in Table 1 . Class A having $\Phi_0 = 1 \times 10^{-6}$ m^3, for example, classifies roads as very good, while $\Phi_0 = 256 \times 10^{-6}$ m^3 represents rather rough roads (Tyan *et al.*, 2009; Rill, 2011). Figure 2 shows the relation between the PSD and the angular spatial frequencies.

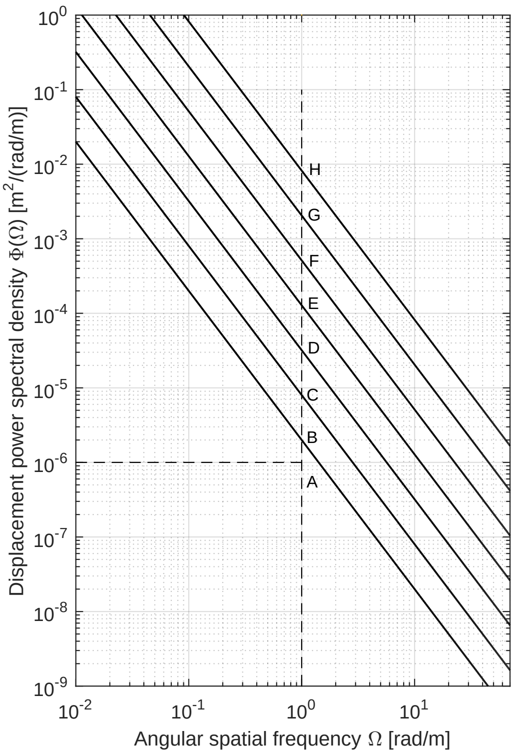

Figure 2. Relation between the PSD value ($\Phi(\Omega)$) and the angular spatial frequencies (Ω) for road surface classification (A - H) according to ISO 8608:1995.

4. SINUSOIDAL APPROXIMATION

Assuming that the vehicle travels at a constant speed v in a determined road segment, the road profile can be generated randomly and approximated by a superposition of N harmonic terms, in which, $N \to \infty$ (Rill, 2011; Verros *et al.*, 2005). This sinusoidal approximation in relation to the road length s can be written in the form:

$$z_r(s) = \sum_{i=1}^{N} A_i \sin(\Omega_i s + \Psi_i) \tag{3}$$

in which each sine wave is determined by its amplitude A_i and its wavenumber Ω_i. By means of different sets of uniformly distributed phase angles (Ψ_i) in a range between 0 and 2π, different profiles can be generated (Rill, 2011).

Equation 3 describes the profile as a function of the length (s) of the road, however, this profile can also be described as a function of time (t). Knowing that the vehicle is moving at a constant speed $\frac{ds}{dt} = v$, the vehicle's instantaneous position is given by $s = vt$, assuming the initial position $s = 0$ at time $t = 0$.

Thus, replacing the expression $s = vt$ in Equation 3, the road profile in relation to the time can be written in the form:

$$z_r(t) = \sum_{i=1}^{N} A_i \sin(\Omega_i\, v\, t + \Psi_i) \tag{4}$$

where the amplitude A_i is described by:

$$A_i = \sqrt{2\Phi(\Omega_i)\Delta\Omega}, \qquad i = 1(1)N \tag{5}$$

and the frequency range $(\Delta\Omega)$ is defined as follows:

$$\Delta\Omega = \frac{\Omega_N - \Omega_1}{N - 1} \tag{6}$$

In order to keep the frequency range shown in ISO 8608, in this work the frequency range $(\Delta\Omega)$ is considered from $\Omega_1 = 0.0087$ rad/m to $\Omega_N = 17.7716$ rad/m. The amplitudes $A_i, i = 1(1)N$ were calculated by Equation 5 and the function `rand` of MATLAB ® was used to produce a random distribution of phase angles (Ψ) in a range from 0 to 2π.

5. DATA ANALYSIS OF ROAD PROFILE

According to Dodds and Robson (1973) the road profile can be classified as a Gaussian random process. However, the upper limit at the sum in Equation 3 has to be considerably large to avoid that the signal presents a predominantly harmonic characteristic. Figure 3 shows the influence of N in the time history and at the histogram of the road profile obtained using Equation 3.

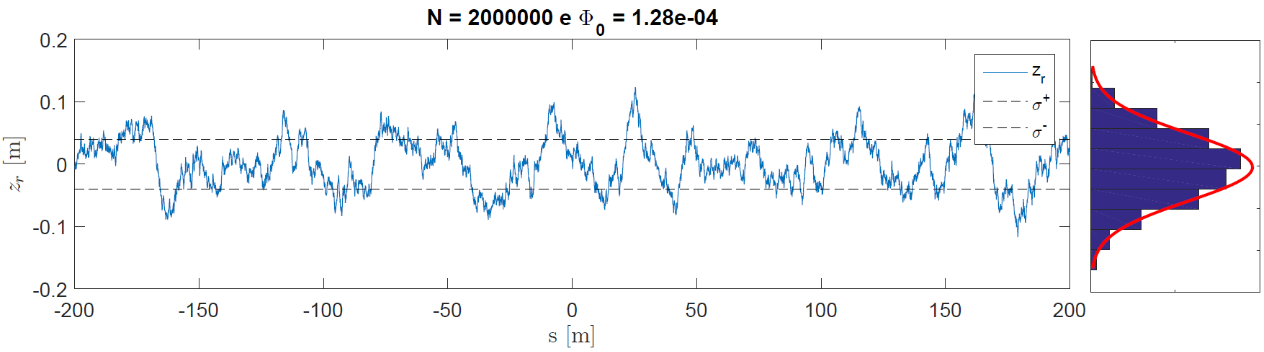

Figure 3. Comparison between the time history and the histogram considering a road profile class D with $\Phi(\Omega_0) = 128 \times 10^{-6}$ m^3.

Figure 4-a shows the relation between the number of harmonic terms N and the value of kurtosis of the signal. The values converge to 3 when $N > 20000$, indicating that this is a good value for N, as kurtosis close to 3 indicates normal distribution (Bulmer, 1979). It is known that the computational time is directly affected by the number of harmonic terms N. The time t considered to generate the road profile also influences the response because the wavenumber Ω_i contains low values of frequency and the sinusoidal road profile takes into account these low frequencies. Thus, depending on the time considered the distribution can be affected by this low values.

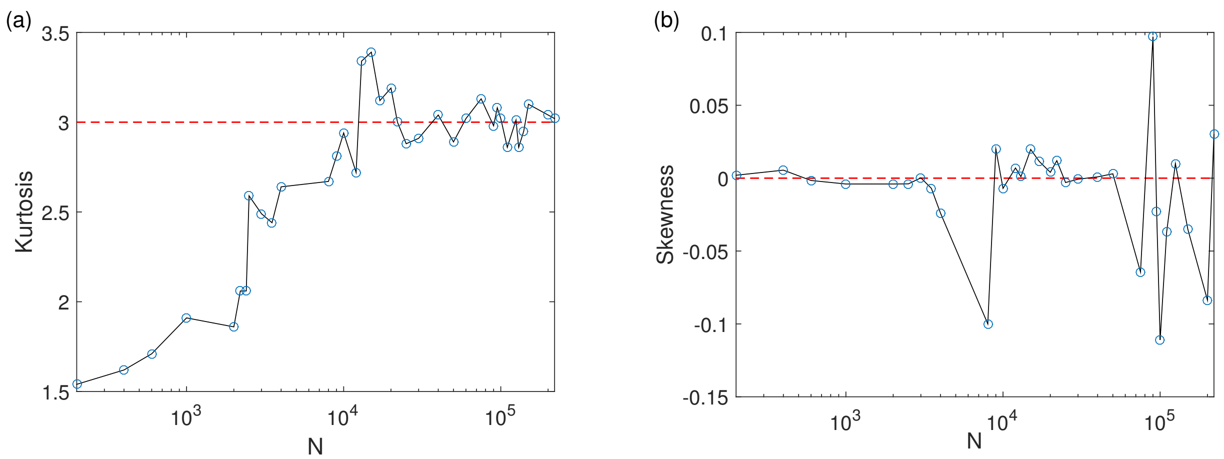

Figure 4. Relation between the value of harmonic terms N and the value of kurtosis (a) and the value of skewness (b) for a road profile considering a road profile class C with $\Phi(\Omega_0) = 16 \times 10^{-6}$ m^3 and $t = 3000$ s.

In view of this, the road profiles used in this work where generated by equation 4 using $N = 10000$ and the $t = 3000$ s with $step = 0.001$ s.

Figure 5 shows three histograms and probability mass function (PMF) for road classes A, B and C considering randomly phase angles (Ψ). The values of kurtosis and skewness are also indicated for each case. The values of kurtosis are not exactly 3 for all cases, it can be explained by the fact of the function used to generate the phase angles were uniformly distributed. Thus, depending on the seed used the value of kurtosis can be closer or further of the ideal value 3. The skewness close to zero indicate a symmetry of the distribution for all cases.

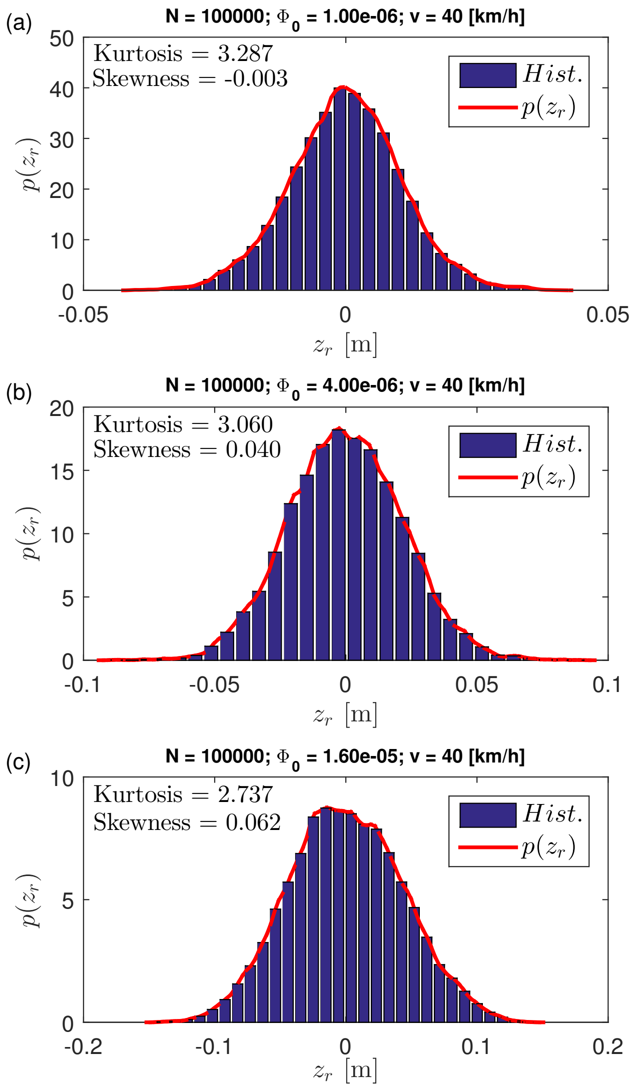

Figure 5. Histogram and probability mass function (PMF) for (a) road class A, (b) road class B and (c) road class C considering randomly phase angles (Ψ).

6. POWER SPECTRAL DENSITY

The method used to estimate the power spectral density (PSD) of a signal is the method called the Welch method (Welch, 1967). The Welch method is based on the division of the signal into segments with defined length and these segments, usually presents a 50% of overlap. Each segment is typically windowed, and then the DFT is performed in the data segment of the windows. Finally, the squared magnitudes of the DFT are described by their means, for each discrete frequency. Figure 6 shows that the number of harmonic terms affect both the PSD and road profile randomly generated. Low quantity of harmonic terms N presents low peaks of frequencies in the signal and its profile presents a predominantly harmonic .

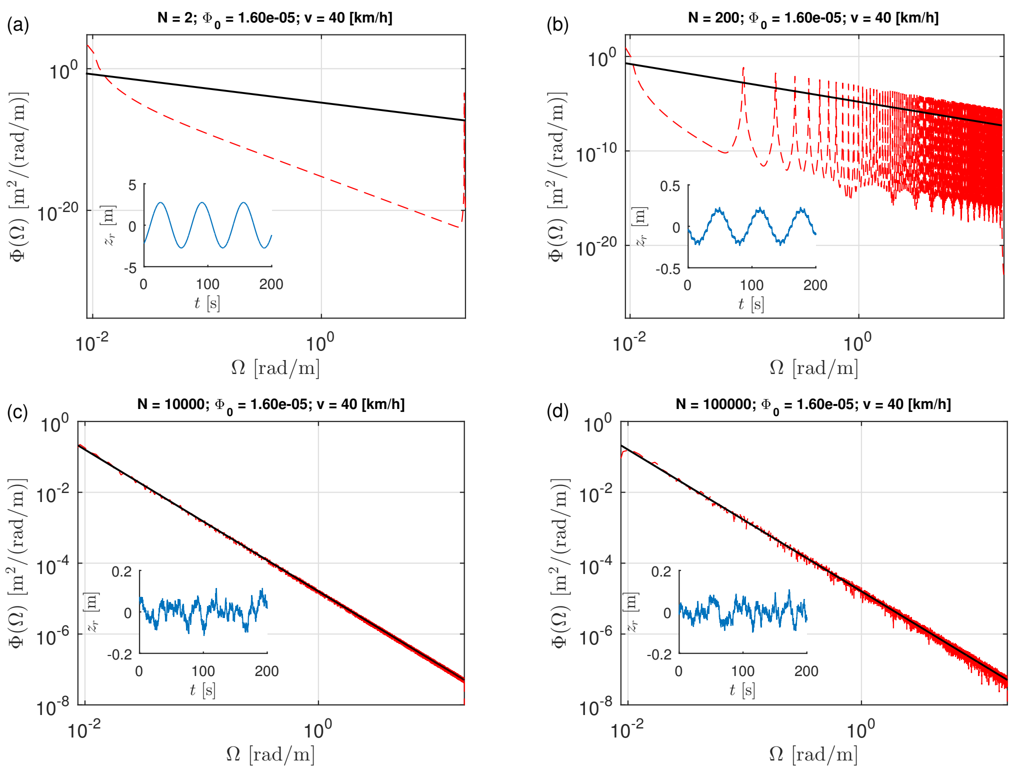

Figure 6. Analytic and estimated PSD using the Welch method using (a) $N = 2$, (b) $N = 200$, (c) $N = 10000$ and (d) $N = 100000$ for a road class C considering randomly phase angles (Ψ).

This characteristic can be observed at Figure 6-a and the response that presents a stochastic profile and a PSD closer to the PSD show in ISO 8608:1995 is shown in Figure 6-d.

With purpose in show the response of the Welch method for different road classes, Figure 7 shows a comparison between the analytic and the estimated PSD for road profile A, B and C.

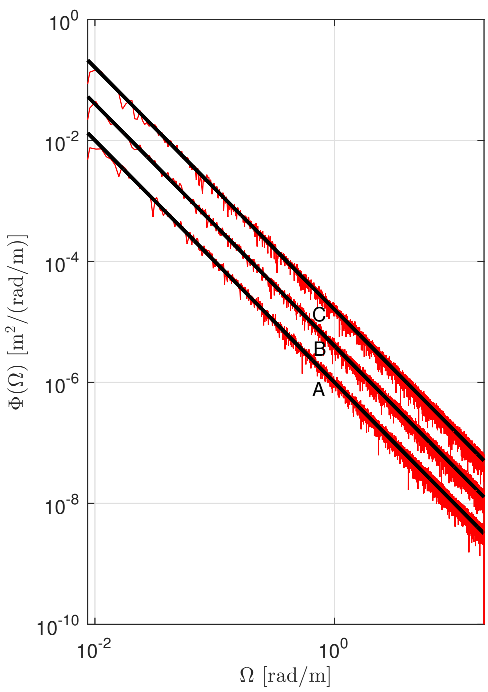

Figure 7. Relation between the analytic and PSD value using welch method ($\Phi(\Omega)$) and the angular spatial frequencies (Ω) for road surface classification (A - C) according to ISO 8608:1995.

7. 1-DOF OSCILLATOR WITH ASYMMETRICAL DAMPING

As shown by Balike *et al.* (2013), the harmonically excited suspension system causes an upward or downward displacement in the mean position of the sprung mass. Figure 8 shows the mean acceleration ($\mu_{\ddot{z}s}$) and the mean displacement (μ_{zs}) of sprung mass in relation of the asymmetry ratio β for a road profile ISO class A with three different vehicle velocities.

This phenomenon is also observed by the mean displacement and acceleration of the sprung mass when the system is excited by a stochastic function. As the objective of this work is to evaluate the response to several values of damping coefficient for extension and compression, the ratio of asymmetry β is considered in the analysis. The mean acceleration

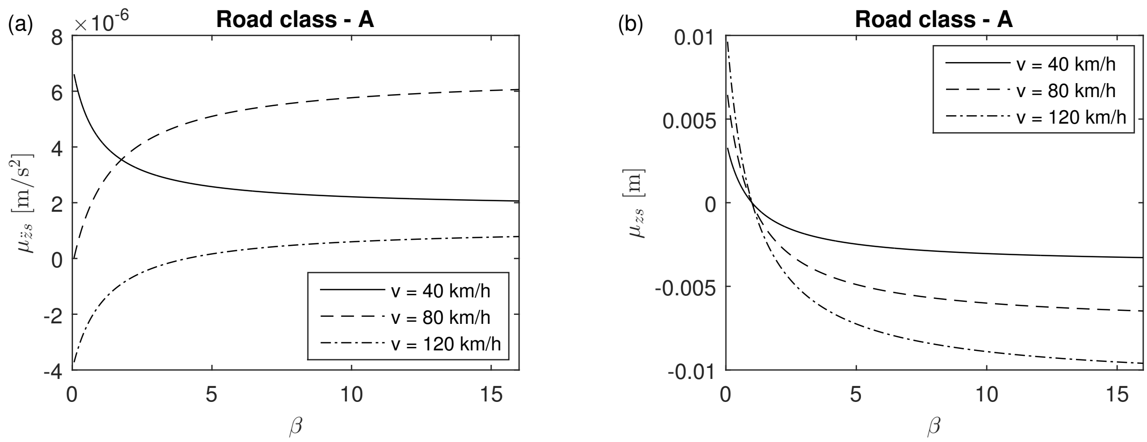

Figure 8. mean acceleration ($\mu_{\ddot{z}s}$) (a) and the mean displacement (μ_{zs}) (b) of sprung mass in relation of the asymmetry ratio β for a road profile ISO class A with three different vehicle velocities.

of sprung mass increases or decreases if the ratio considered also increases, however, it depends on the vehicle velocity (Figure 8-a). The crossing between the positive and negative mean displacement values occurs when $\beta = 1$ independent of the velocity considered. It is also observed that the greater the velocity furthest from the mean reference position ($\mu_z = 0$) the suspended mass oscillates, depending on the value of β (Figure 8-b).

Almost the same behavior characteristics in the mean acceleration and displacement can be observed for an ISO road class C (Figure 9). However, the mean displacement becomes greater when observed the extreme values of asymmetry ratio compared with a road class A.

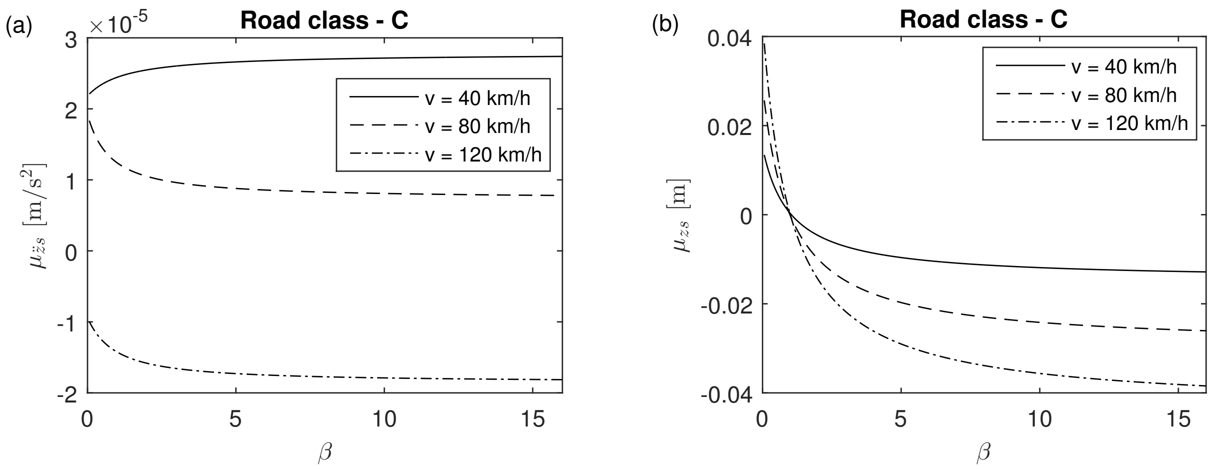

Figure 9. Mean acceleration (a) and the mean displacement (b) of sprung mass in relation of the asymmetry ratio β for a road profile ISO class C with three different vehicle velocities.

Figure 10 shows how the asymmetry ratio influences the distribution on the amplitude values of sprung mass. Although the distribution of road profile stays at zero as mean the distribution of sprung mass amplitude tends to move to positive side if $\beta < 1$ and to negative side for $\beta > 1$.

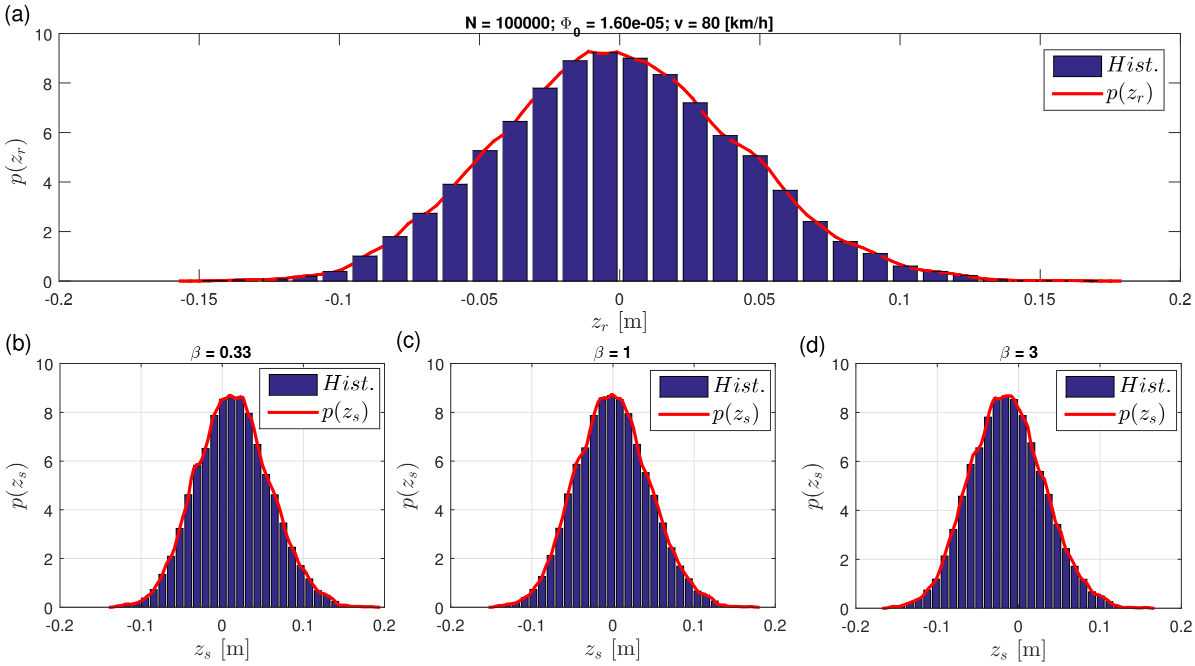

Figure 10. Histogram and probability mass function (PMF) for road class C (a), sprung mass with $\beta = 1$ (b) and sprung mass with $\beta = 3$ considering randomly phase angles (Ψ) and keep constant the vehicle velocity in 80 km/h.

8. STOCHASTIC MEAN DISPLACEMENT ANALYSIS

It is known that systems employing asymmetrical damping under harmonic excitation changes the mean position of the mass oscillation. The harmonic function is usually used as a form of excitation so that the system is forced into the same frequencies of the natural frequencies of the model (Silveira *et al.*, 2014; Fernandes *et al.*, 2015; Silveira *et al.*, 2017).

Considering a random road profile it is possible to see the same effect, depends on the vehicle velocity and the asymmetry ratio β. Figure 11 also shows that the mean displacement is not influenced by the mean road profile. Both amplitudes, road (z_r) and sprung mass (z_s), presented a normal distribution with different mean positions.

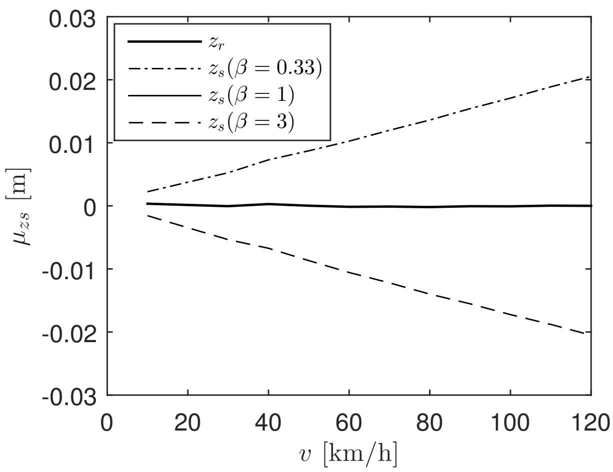

Figure 11. Mean displacement of sprung mass considering stochastic road profile class C for different vehicle velocity and asymmetry ratio $\beta = 0.33$, $\beta = 1$ and $\beta = 3$.

Figure 12 shows the mean amplitude of sprung mass considering a stochastic road profile class C with several values of β. The transparent plane cross $\beta = 1$, indicating that for this value of β the mean displacement of sprung mass is close to zero. For larger values of asymmetry ratio the mean displacement becomes positive and in opposition becomes negative for low values of asymmetry ratio.

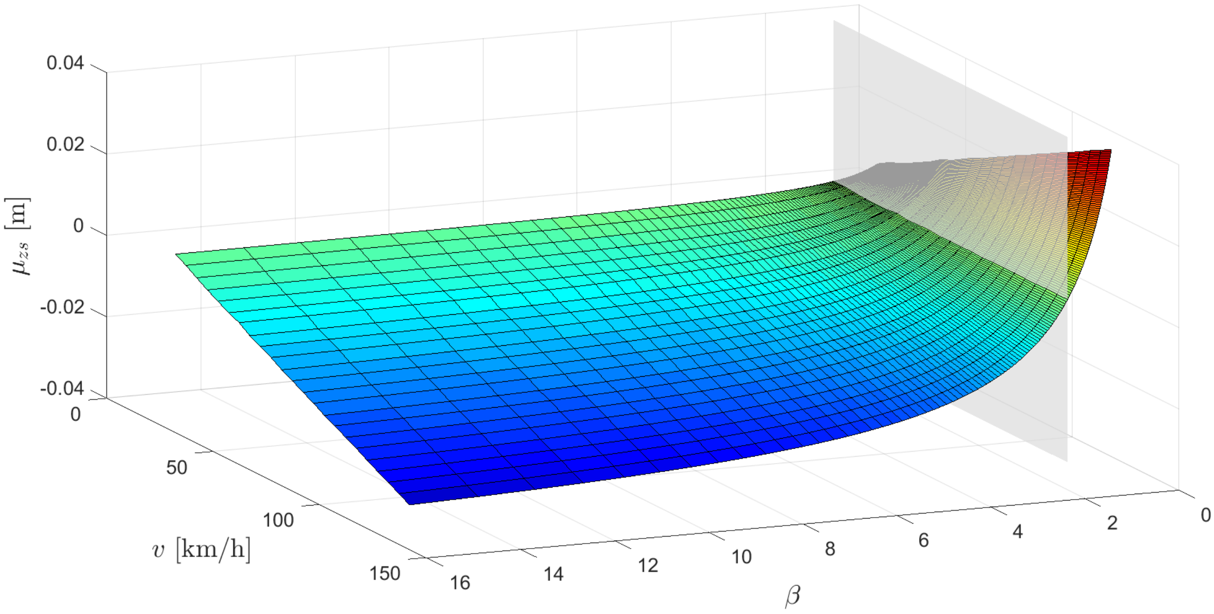

Figure 12. Mean displacement of sprung mass considering stochastic road profile class C for different vehicle velocity and several asymmetry ratio β.

Some comparisons using harmonic and stochastic road profile with different classes were done considering different asymmetry ratio. It was seen that there are a relation between the value of the mean displacement of sprung mass in both cases. The mean position is strongly influenced by the vehicle velocity and asymmetry ratio.

9. CONCLUSIONS

This work presented the response of a suspension system with asymmetric damping considering random road profile. The modeling of the road profile was based on the PSD of the signal as shown in ISO 8608:1995, and the road profile is composed by the sum of harmonics. The power spectral density obtained by the Welch method accurately estimates the PSD regardless of vehicle speed, number of harmonics and also the roughness index adopted.

The number of harmonic terms N and the simulation time have strong influence on the profile approximation, and may result in a PSD not compatible with the analytic PSD described by ISO. The shift in the mean position of sprung mass (jacking down effect) can be observed both in the mean displacement as in the amplitudes distribution of sprung mass.

The results from a sinusoidal road profile shows that the mean position of sprung mass presents a behavior similar to the system under stochastic road profile. As greater as the velocity of the vehicle, the mean position becomes higher depending on the asymmetry ratio.

By increasing the vehicle velocity, the behavior of mean displacement is apparently linear and presents values close to the system harmonically excited in higher frequencies. Therefore, the mean displacement of the stochastic system with asymmetrical damping can be approximated by the results of an equivalent system under harmonic excitation, similar to what is used by Silveira *et al.* (2017).

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