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COBEM-2017-0331 NONLINEAR DYNAMICS ANALYSIS OF TRUSSES APPLIED IN AEROSPACE STRUCTURES

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Abstract. *In this work, we perform a simplified nonlinear dynamic analysis of an aerospace structure - a truss boom - to study the impact of our geometrically and material nonlinear model in its dynamic behavior. For this, we have implemented a numerical integration step-by-step algorithm for the resolution of non-linear dynamic systems based on the Newmark-beta algorithm, where the resolution of dynamic equilibrium iterations in each time step are performed by Newton-Raphson method. The boom structure, similar with the ADAMmast[®], is modelled as a 2D truss structure under the influence of a dissipative harmonic load to simulate the thruster effect in a hypothetical attitude control situation. Finally, we present a brief discussion of the program in the scope of bi-dimensional problems.*

Keywords: *Nonlinear dynamics, Planar truss, Aerospace Structures, Deployable Structures.*

1. INTRODUCTION

Since the dawn of space race, scientists seek innovative and reliable projects that can lead to a new era of space travels and exploration systems previously unknown. That sole purpose has motivated many new researches and improvements, especially in the field of material engineering and aerospace/mechanic engineer. Those new concepts, materials and design are mainly focused on high efficiency and safety – due to the high ratio cost/payoff of this industry. The current main limitation of the state of the art in aerospace appendages are the low length/weight ratio and the old-fashion deployment and package techniques. Nonetheless, the development of high performance composites – such as the Elastic Memory Composite (Fetchko, et al., 2004) and Gossamer Structures (Straubel, et al., 2011) - and new mathematical models/procedures that can describe the correct behavior of these lightweight structures can have an important role of new exploration missions.

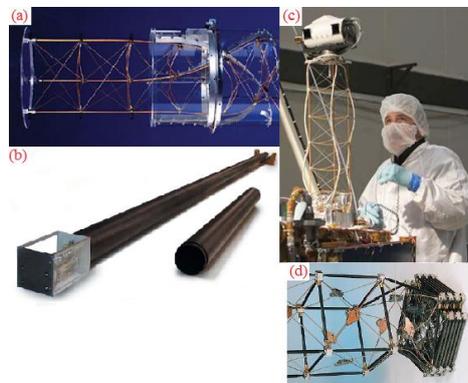


Figure 1. State of the art spacecraft boom concepts. [Image Credits: (a) ATK/ABLE Engineering, (b) Grumman 2010, (c) Lockheed Martin and (d) NASA]

These advancements together with the development of extremely fast and multi-core computers allowed missions such as the AstroMesh™ Reflector Program, Canadian RADARSAT-2 (terrestrial observation satellite containing structures Flexible and solar panels) and the most recent, IKAROS project (Tsuda, et al., 2011). In this context, the present work seeks the development of a new nonlinear model, based on the geometrically nonlinear space truss theory (Souza Lima and Brasil, 1998) that can take in account a material model that can works outside its linear region. Therefore, we have implemented a constitutive given by a third order polynomial, as shown below

$$\sigma^* = E_0 \varepsilon^* = \frac{E_0}{l} (1 + \gamma dl^2) dl \quad (1)$$

where E_0 is the elastic modulus of the material; γ a tuned parameter seeking the best fit to the real strain-stress curve and dl the difference between the initial and current length of a truss elemnt. Furthermore, this paper presents a simplified review of the Newmark's method in addition to a plain simulation of a planar truss structure, showing its effect on the time history obtained via geometrically nonlinear space truss theory.

A complete overview of the state of the art in spacecraft deployable appendages can be found in Belvin, et al. (2016), where the author discuss about the challenge and limitations of the new deployable structures concepts and designs, thus justifying the recent studies and advances in the area. Similarly, Fetchko, et al. (2004) present a brief review of tdeployable booms, arguing that the technological advances in high performance materials are limited by the old-fashion structural boom designs. Fetchko, et al. (2004) also highlights that the development of new materials, such as, the Elastic Memory Composite (EMC), can be the solution for the limitations of old-fashion structural boom designs. Moreover, Wriggers (2008) presents the mathematical formulation of the Newmark's method, as well as, the Newton-Raphson iterative process and several linear/nonlinear examples. A direct approach of the method is presented in Brasil (1990), were we can find some non-linear application, along with a FORTRAN® implementation of the method.

2. NONLINEAR MODEL

This section has the purpose of introducing the geometrically nonlinear space truss theory presented in Souza Lima and Brasil (1998), as well as the modified nonlinear model developed in this work. As described before, this modification of the original exact theory is based on a material nonlinearity approach seeking to obtain the "real" stress-strain curve for a given material.

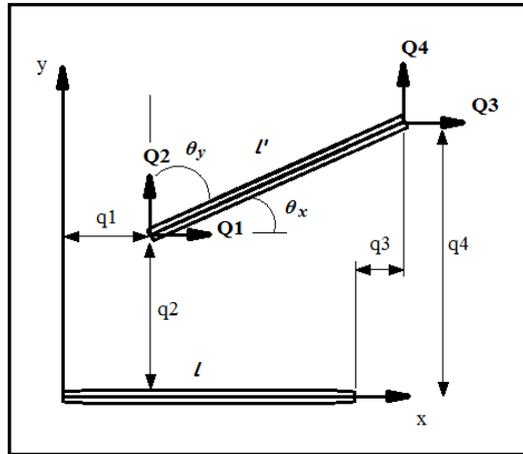


Figure 2. Planar truss described in the local reference frame. [Adapted from Souza Lima and Brasil (1998)]

Using as starting point the planar truss described in the Figure 2, it is possible to obtain the relationship between the nodal displacement q , the truss deformed length l' and the direction cosines, given by:

$$l + (q_3 - q_1) = l' \cos \theta_x \quad (2)$$

$$(q_4 - q_2) = l' \cos \theta_y \quad (3)$$

where l is the truss initial length; q_i is a component of the nodal displacement array for a specific direction and the angles θ_x and θ_y are the angles between the displaced position and the two reference axes. Assuming that the truss element follows the Hooke's law and an engineering strain relation, we can write an engineering strain relation to obtain the normal force action in the bar, given by:

$$N = E_0 A_0 \varepsilon = \frac{EA}{l} (l' - l) = \frac{EA}{l} dl \quad (4)$$

where E_0 is the material elastic modulus; A_0 is the initial cross-section area of the bar; ε is the engineering strain relation and dl is the difference between the deformed and the initial length. Thus, the values of the component of the nodal forces are:

$$Q = Nc = N\{-\cos\theta_x; -\cos\theta_y; \cos\theta_x; \cos\theta_y\} \quad (5)$$

Differentiating the nodal force array, Eq. (5), with respect to the nodal displacement, we can obtain the expression of the tangent stiffness matrix k_T , given by:

$$k_T = \frac{E_0 A_0}{l'} c^T c + \frac{N}{l'} a^T a \quad (6)$$

with a given by $[-[I]_{2 \times 2} \ [I]_{2 \times 2}]$. Nevertheless, it should be noted that Eq. (6) describes the tangent stiffness matrix in the local reference system, being necessary to carry out a transformation process to obtain its counterpart in the global reference system. Using the rotation formula, we have:

$$\overline{k_T} = T^T k_T T \quad (7)$$

where T is the usual rotation matrix, containing the direction cosines of the truss in the global reference axes. From Eq. (6), it is possible to obtain the global tangent stiffness matrix using the following summation process:

$$K_T = \sum A^T \overline{k_T} A \quad (8)$$

being A the ordinary incidence matrix of each bar. Adapting the original nonlinear model, via the addition of a nonlinear cubic parameter (Eq. (1)), we have that the modified normal force acting on the planar truss is given by:

$$N^* = E_0 A_0 \varepsilon = \frac{E_0 A_0}{l} dl (1 + \gamma dl^2) \quad (9)$$

with the new tangent stiffness matrix given by

$$k_T^* = \frac{E_0 A_0}{l'} (1 + 3\gamma dl^2) c^T c + \frac{N^*}{l'} a^T a \quad (10)$$

Finally, using the same procedure developed before (Eqs. (7) and (8)) it is possible to obtain the global stiffness matrix for the modified tangent stiffness matrix given by Eq. (10).

3. COMPUTATIONAL PROCEDURE

As previously described, the resolution of dynamical system used in this paper are solved through the Newmark's algorithm, with the dynamic equilibrium step obtained by Newton-Raphson. Thus, we can see the importance of a brief introduction for a correct understanding and implementation of these methodologies.

3.1 Newmark's Method

The main idea behind the Newmark's method is the development of a prediction step, where the generalized variables at a final time t_f – endpoint of the interval dt - are based on the values at the initial time t_i ; thus solving the motion equations in an incremental way. Based on this assumption, we have that for a given a generic dynamic equilibrium equation, as presented in Eq. (10),

$$g(\ddot{q}, \dot{q}, q) = f(t) \quad (11)$$

where $g(\ddot{q}, \dot{q}, q)$ corresponds to the reaction equations and $f(t)$ is the applied loads array. We initially must perform the prediction process, where the acceleration, velocity and generalized displacement fields at the final instant t_f are based on the values of initial instant t_i . This prediction is given by:

$$\delta q = q^{t_f} - q^{t_1} \quad (12)$$

$$q^{i_f} = b_0 \delta q - b_2 q^{i_1} - b_3 q^{i_1} \quad (13)$$

$$q^{i_f} = b_1 \delta q - b_4 q^{i_1} - b_5 q^{i_1} \quad (14)$$

The parameters b_i ($i = 0, \dots, 5$) are chosen by the user, to obtain the best precision and stability for each problem. For the next time step, we write the force residue \hat{f} at time t_f as an approximation of q^{t_f} , obtained from a previous iteration.

$${}^j \hat{f}^{t_f} = f^{t_1} - h({}^j q^{t_f}) \quad (15)$$

where h - function of the displacement $q(t_f)$ - is obtained from Eq. (11) by the introduction of those relations presented in Eqs. (12)-(14). For the next iteration, we look for a new approximation ${}^{j+1} q^{t_f}$ such that

$${}^{j+1} \hat{f}^{t_f} \approx 0 \quad (16)$$

The exact solution, of course, would make equation Eq. (16) an equality. Expanding the residue function \hat{f} in the neighborhoods of the assumed prediction in Taylor series, we have

$${}^{j+1} \hat{f}^{t_f} = {}^j \hat{f}^{t_f} + {}^j [\partial \hat{f} / \partial q]^{t_f} {}^{j+1} \delta q + \dots = 0 \quad (17)$$

Disregarding the superior order terms, we can rewrite Eq. (17) as

$$- {}^j [\partial \hat{f} / \partial q]^{t_f} {}^{j+1} \delta q = {}^j \hat{f}^{t_f} \quad (18)$$

or

$${}^j \hat{K} {}^{j+1} \delta q = {}^j \hat{f}^{t_f} \quad (19)$$

where $\hat{K}(q) = -[\partial \hat{f} / \partial q]$ is the gradient matrix of the process. These equilibrium iterations (Eq. (18)) are solved by Newton-Raphson method. A priori, the values of beta used in this paper are the same as those used by Brasil (1990), which seeks to guarantee the stability and the best convergence ratio of the process.

4. NUMERICAL RESULTS

Based on the nonlinear structural model developed in Section 2, as well as, the numerical procedure presented in Section 3, it is possible to simulate a simplified 2D structure, showing the benefits of this approach. For this, we develop a planar truss model analogous to the ADAMmast[®] boom – a flight proven appendage used in the SRTM-Mission (Shuttle Radar Topography Mission). The model is based on a 2.5 m (height) by 0.25 m (width) boom, modelled as a 95 truss elements, shown in Fig. 3, under the influence of a dissipative harmonic load (Eq. (20)) applied at the uppermost nodes of the model, simulating a hypothetical attitude control via thrusters.

$$P(t) = A_0 \sin(\omega t) e^{-t} \quad (20)$$

where A_0 is the load amplitude and ω is the load frequency. In this case, a realistic amplitude value is desired, since we are trying to simulate a valid condition, hence, we use the work of Dankanich (2015) – Green Liquid Propulsion, to define a maximum value of 100N. Meanwhile, a frequency of 10π rad/s is set to demonstrate a fast pace control conditions. As described previously, the gamma value used in this work was based on a third order polynomial approximation of the original strain-stress curve for steel. For this, we used the command best fit in excel to determine the value of $\gamma = -20 \text{ m}^{-2}$.

Table 1. Model dimensions and Material properties.

Model Properties	Nomenclature	Value
Cross Section Area [m ²]	A_0	1.25×10^{-04}
Width [m]	w	$0.25 \times 10^{+00}$
Height [m]	h	$2.50 \times 10^{+00}$
Density [kg/m ³]	ρ	$7.50 \times 10^{+03}$
Elastic Modulus [N/m ²]	E_0	$210 \times 10^{+09}$
Poisson's coefficient	ν	$0.30 \times 10^{+00}$

In this first step, we define the model material as a steel alloy (properties presented in Table 1); mainly because its historical use in the aerospace application along with the fact that its stress-strain curve can be easily obtained in the literature. Additionally, other model properties – such as, Poisson coefficient and cross-section area – can be found in Table 1.

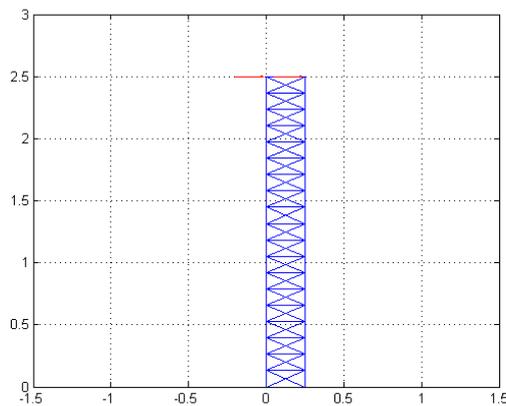


Figure 3. Aerospace boom modelled as a 2D truss structure, being the red arrows a representation of the applied force.

Figure 4 represents a superposition of all nodes displacement in a time interval $t = [0 - 0.21]$ s, showing us the harmonic behaviour of the structure – for the load profile showed in Eq. (20). Moreover, Fig. 4 also show us that the maximum displacement of this structure occurs on the rooftop nodes - as expected - with a magnitude of 1.1987 mm.

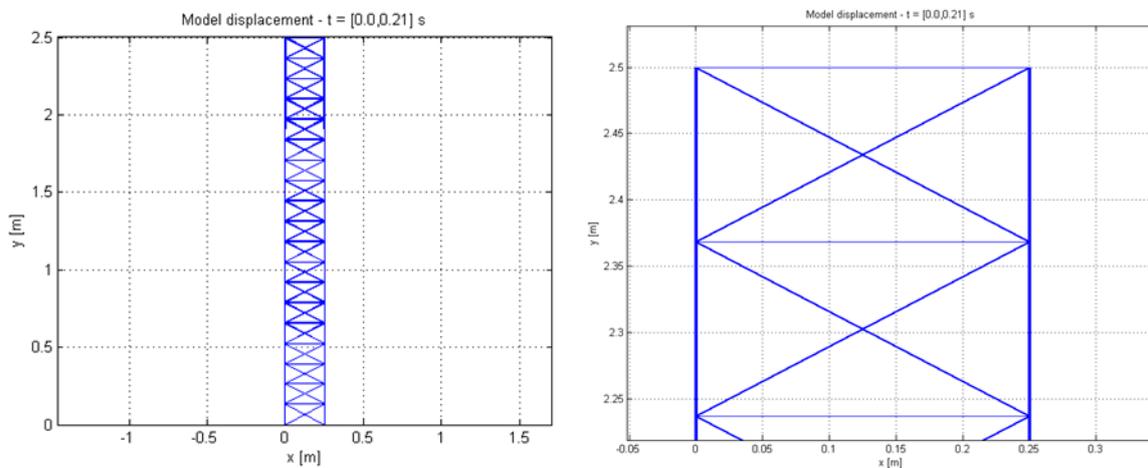


Figure 4. Superposition of the model displacement for a time interval $t = [0 - 0.21]$ s (left), and zoom of the model top elements (right).

In addition, Figure 5 show us the structure time history for the uppermost nodes - x and y displacements, where it became clear that the maximum displacement of the structure presents a value of 1.1937 mm and occurs in the x direction – same direction as the applied load. We can see that the results presented for the modified model are valid, since presents an analogous behaviour with the one obtained via exact theory .

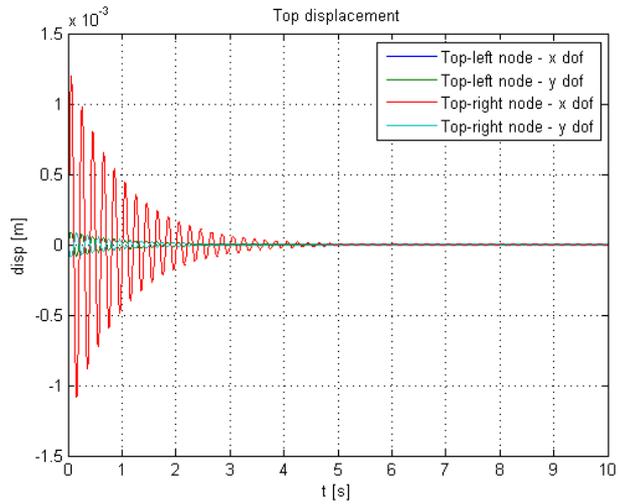


Figure 5. Time history of the rooftop for the given dissipative harmonic load.

From Figure 6, its possible to observe that modified model approach results in a valid description of the lightweight structure, considering the minute difference - 10^{-12} m – from the geometrically nonlinear theory know as exact truss theory. However, it may be noteworthy that this load condition may be unfitted to lead the structure to a strain regime outside the linear region. A possible solution for this problem in the adoption of a new material, e.g. a composite, that has a specific density lower than 0.9 kg/m and have a stiffness EA lower than the steel as an alternative to a new load condition.

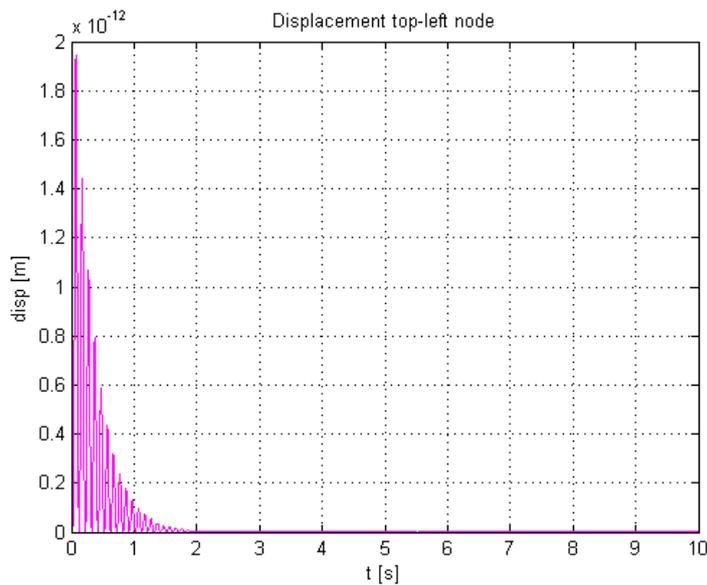


Figure 6. Time history difference between the geometrically nonlinear model and the modified material and geometric nonlinear model.

Finally, figure 7 represents a complementary simulation, where the original boom structure is maintained, altering only the material model - in this case, an aluminum 2024 alloy is used. As we can see, this new assumption increases the upper displacement to a value of 3.42 mm, as expected.

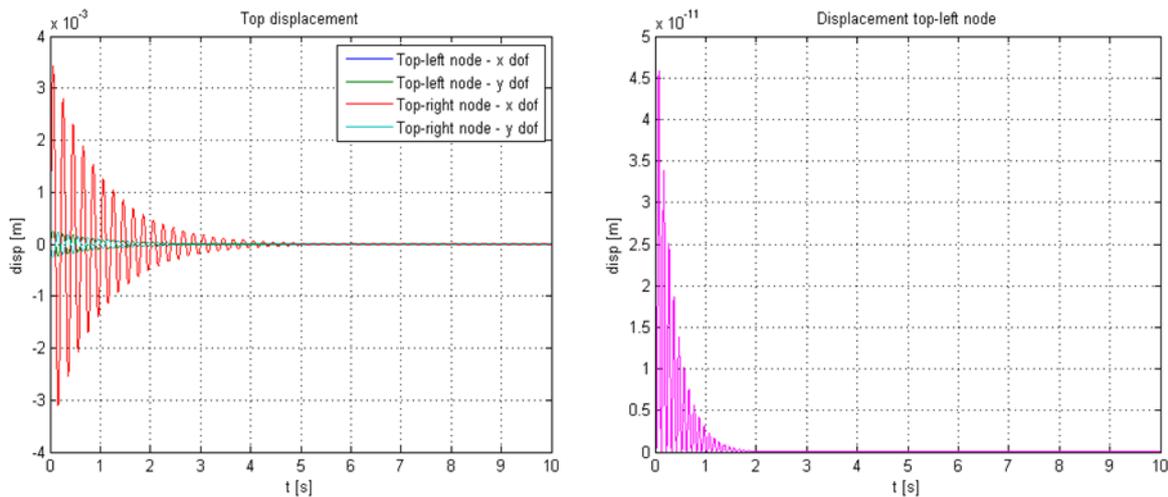


Figure 7. Time history of the upper nodes for an aerospace aluminum 2024 model. The leftmost image presents the nodal displacement; meanwhile, the rightmost curve shows the difference between the geometrically nonlinear model and the modified material and geometric nonlinear model.

Furthermore, we also see an increase (10x) in the difference between the two models, the geometrically nonlinear model and the modified material and geometric nonlinear model, implying that a change in the model material – using in this case high performance composites, for example – can lead to a greater differentiation between a geometric nonlinear assumption and a material nonlinear one.

5. CONCLUSION

As presented in the introduction, the main roadblock of our new era of space travels and exploration are the current limitations in the old-fashion deployment techniques and packaging designs. To overcome this limitation, new mathematical models that can quantify and predict the dynamic behaviors, especially with respect to lightweight and flexible structures, along with new high-performance materials are needed. In this context, we seek the development of a new nonlinear model, based on the geometrically nonlinear space truss theory (Souza Lima and Brasil, 1998) that can take in account the material nonlinearities.

Considering the results presented in section 4, we can see the viability of the developed model in the nonlinear analysis of lightweight structures, nevertheless, improvements must be made to make the mathematical model more reliable and effective. Furthermore, a series of case studies must be performed to demonstrate the difference between the dynamical response given by our model to the one obtained via a linear or geometrically nonlinear approach. In this context, future works trying to improve this methodology, as well as, studying its viability are the focus of our research.

6. ACKNOWLEDGEMENTS

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