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STEADY HEAT TRANSFER SOLUTION IN A FINNED-TUBE RADIATOR CONFIGURATION

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Abstract. *The solution for the energy transfer in a finned-tube radiator configuration is derived, considering the surface radiative heat transfer and internal heat conduction. The radiative interaction between adjacent radiator elements has been included, and thus a resulting non-linear integro-differential system of equations arises to describe the combined conduction-radiation heat transfer problem. This system is normalized and shown to be represented by two dimensionless radiative parameters. Convergence analyses for the integro-differential system for various combinations of the non-dimensional parameters are presented and compared based on the Generalized Integral Transform Technique (GITT). The numerical results indicate the effectiveness of the hybrid method for the proposed solution scheme and it is shown how the convergence behavior is directly affected by the dimensionless radiative parameters.*

Keywords: *Finned-tube radiator, Combined conduction-radiation, GITT, integro-differential system.*

NOMENCLATURE

A, B, C, D, E	integral coefficients
f	filter
F	configuration factor
H	height
k	conductivity
L	length
N	norms
N_{C_L}	radiative parameter
t	thickness
T	temperature
x, y	spatial coordinates
X	eigenfunctions
Greek Symbols	
δ	Kronecker delta function
ξ, η	dimensionless spatial coordinates
γ, λ	functions
μ	eigenvalues
κ	geometric parameter
σ	Stefan-Boltzmann constant
φ	filtered solution
θ	dimensionless temperature
ϕ	angle
Subscripts	
i, j	summation indices
b	base
max	maximum
Overscripts	
—	transformed function
-	average function

1. INTRODUCTION

Combined conduction and radiation heat transfer are a subject commonly found in the literature, with a wider range of applications, such as thermal insulation systems, outer space radiators, thermal analysis in satellites and spacecraft thermal control systems. However, when dealing with combined conductive-radiative heat transfer, there is no need to explore afar outer space applications to find problems of this nature. Radiator systems for effective energy dissipation, the manufacture of glass and heat transfer in industrial furnaces represent some of the day-to-day applications, which prove the relevance of this theme in many research fields.

Aimed at the realm of combined conduction-radiation heat transfer, many studies can be mentioned, such as investigations into the influence of radiation on thermal performance of electronic cooling systems represented by aligned-fin heat sinks (Khor *et al.*, 2010), multi-dimensional conduction-radiation heat transfer problems through the use of purely numerical methods (Mishra *et al.*, 2012), comparison between Homotopy perturbation method and finite difference method for the solution of solar heat exchangers problems (Dehghan *et al.*, 2015), numerical analysis of room heated by two-panel radiators under different circumstances and conditions (Sevilgen and Kilic, 2011) or conductive-radiative heat transfer in two-dimensional complex geometries obtained by a modified discrete ordinates method (Sakami *et al.*, 1996).

Specifically regarding finned-tube configurations, Sparrow and Minkowycz (1962) pointed out that radiators of practical interest may consist of finned-tube configurations in which there is significant radiant interaction between radiator elements. The sophistication of the radiative portion of the analysis can vary considerably, depending on the accuracy required and the importance of radiation relative to heat conduction.

Regarding the Generalized Integral Transform Technique (Cotta, 1993), the applications of this approach are extensive, from solution of convection-diffusion problems (Sphaier *et al.*, 2011; Almeida and Cotta, 1995; Serfaty and Cotta, 1992), to conjugated heat transfer in microchannels (Knupp *et al.*, 2015), among others. However, when considering coupled conductive-radiative heat transfer, most works described in the literature exploit purely numerical methods to acquire the solution of the problem. This seems arbitrary as the hybrid analytical-numerical nature of the GITT method makes it a good fit for problems requiring a bit of numerical implementation, such as coupled conductive-radiative heat transfer. Although numerical approaches are common in combined conductive-radiative problems, there are some works which illustrates the use of the GITT approach, like natural convection and radiation combined in a non-heated surface of a thin plate (Naveira-Cotta *et al.*, 2011), where the direct problem is solved by means of the GITT.

Based on the importance of the field, the focus of this study is supported on the analysis of the surface radiation and inner conduction heat transfers, as it would be present in a vacuum, for example, for space applications. The consideration of a single radiator-fin configuration is developed in the mathematical formulation by the use of the Generalized Integral Transform Technique and the solution methodology of the resulting non-linear integro-differential system is implemented through the symbolic computation based software Mathematica (Wolfram, 2003).

2. PROBLEM FORMULATION

Previously to the problem formulation, some considerations need to be taken into account beforehand: steady state, black surfaces, base surfaces at prescribed temperature T_b, constant thermal properties and radiation emission losses are considered at both sides of the fin structure (closed-sandwich configuration). The parallel flat plate fins is arranged by two surfaces connected by perpendicularly-positioned fins, where both sides have radiation emission losses, as can be seen by the scheme presented in figure 1.

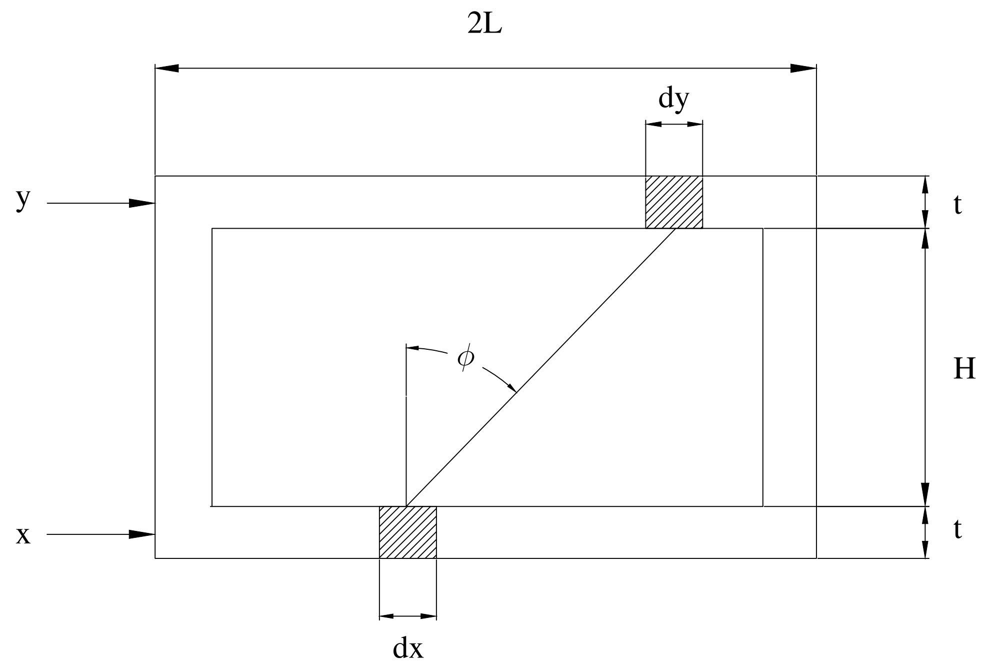

Figure 1: Configuration array for parallel flat plates.

Based on these assumptions, the energy balance yields an integro-differential equation with the corresponding boundary conditions, as follows:

$$k\,t\,\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} \;=\; 2\,\sigma\,T^4(x) \;-\; \int_{y=0}^{2\,L} \sigma\,T^4(y)\,\mathrm{d}F_{\mathrm{d}x-\mathrm{d}y} \;-\; (F_{\mathrm{d}x-1} +\, F_{\mathrm{d}x-2})\,\sigma\,T_b^4, \tag{1a}$$

$$T = T_b, \quad \text{at} \quad x = 0 \tag{1b}$$

$$\frac{\partial T}{\partial x} = 0, \quad \text{at} \quad x = L \tag{1c}$$

In equation (1a), k is the thermal conductivity, t is the thickness, σ is the Stefan-Boltzmann parameter and F is the configuration parameter. As the temperature distribution is symmetric about the $x = L$ plane, the equations can be solved only within $0 \leq x \leq 2L$.

The normalization process is established with the following dimensionless quantities:

$$\xi \,=\, \frac{x}{L}, \qquad \eta \,=\, \frac{y}{L}, \qquad \theta \,=\, \frac{T}{T_b}, \tag{2}$$

After applying the dimensionless variables, equation (1a) and the boundary conditions in equations (1b) and (1c) become:

$$\frac{1}{N_{C_L}}\frac{\mathrm{d}^2\theta}{\mathrm{d}\xi^2} \;=\; 2\,\theta^4(\xi) \;-\; \int_{\eta=0}^{1} \theta^4(\eta)\,\mathrm{d}F_{\mathrm{d}x-\mathrm{d}y} \;-\; (F_{\mathrm{d}x-1} +\, F_{\mathrm{d}x-2}), \tag{3a}$$

$$\left.\frac{\partial\theta}{\partial\xi}\right|_{\xi=1} \;=\; 0, \qquad \theta(0) \;=\; 1, \tag{3b}$$

The radiative dimensionless parameter N_{C_L} is defined by:

$$N_{C_L} \;=\; \frac{L^2\,\sigma\,T_b^3}{k\,t}. \tag{4}$$

As the set of equations and boundary conditions are established, a general relation for the configuration factors are obtained from Siegel and Howell (2002), considering the geometric scheme:

$$\mathrm{d}F_{dx-dy} = \frac{1}{2}\,\cos(\phi)\,\mathrm{d}\phi = \frac{1}{2}\,\mathrm{d}(\sin(\phi)) = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}y}(\sin(\phi))\mathrm{d}y \tag{5}$$

$$\mathrm{d}F_{dx-b} = \frac{1}{2}\,(\sin(\phi'') - \sin(\phi')) \tag{6}$$

One important remark related to the determination of configuration factors is based on the application of a wide fins setting, where the fin structure is considerably long in the direction perpendicular to the figure plane.

After some algebra, normalization and applying trigonometric relations on equations (5) and (6), the final form os the configurations factors between $\mathrm{d}x - \mathrm{d}y$ and the bases $\mathrm{d}x - 1$ and $\mathrm{d}x - 2$ are given by:

$$\mathrm{d}F_{\mathrm{d}x-\mathrm{d}y} \;=\; \frac{\kappa^2/2}{\left((\eta-\xi)^2+\kappa^2\right)^{3/2}}\,\mathrm{d}\eta, \tag{7a}$$

$$\mathrm{d}F_{\mathrm{d}x-1} \;=\; \frac{1}{2}\left(1 - \frac{\xi}{(\xi^2+\kappa^2)^{1/2}}\right), \tag{7b}$$

$$\mathrm{d}F_{\mathrm{d}x-2} \;=\; \frac{1}{2}\left(1 - \frac{2-\xi}{((2-\xi)^2+\kappa^2)^{1/2}}\right), \tag{7c}$$

Considering the angle relations for configuration factors, the final integro-differential equation can be seen below:

$$\frac{1}{N_{C_L}}\frac{\mathrm{d}^2\theta}{\mathrm{d}\xi^2} \;=\; 2\,\theta^4(\xi) \;-\; \frac{\kappa^2}{2}\int_0^1 \theta^4(\eta)\left[\frac{1}{\left((\eta-\xi)^2+\kappa^2\right)^{3/2}} \;+\; \frac{1}{\left((2-\eta-\xi)^2+\kappa^2\right)^{3/2}}\right]\mathrm{d}\eta\,+ \\ -\,1 \;+\; \frac{1}{2}\left[\frac{\xi}{(\xi^2+\kappa^2)^{1/2}} \;+\; \frac{(2\,-\,\xi)}{((2-\xi)^2+\kappa^2)^{1/2}}\right]. \tag{8}$$

In which κ is described by:

$$\kappa \;=\; \frac{h}{L}. \tag{9}$$

Some simplifications can be imposed though functions γ and λ, respectively expressed as:

$$\gamma(\xi,\eta) \;=\; \frac{\kappa^2}{2}\left[\frac{1}{\left((\eta-\xi)^2+\kappa^2\right)^{3/2}} \;+\; \frac{1}{\left((2-\eta-\xi)^2+\kappa^2\right)^{3/2}}\right], \tag{10a}$$

$$\lambda(\xi) \;=\; \frac{1}{2}\left[\frac{\xi}{(\xi^2+\kappa^2)^{1/2}} \;+\; \frac{(2-\xi)}{((2-\xi)^2+\kappa^2)^{1/2}}\right] \;-\; 1 \tag{10b}$$

3. PROPOSED SOLUTION SCHEME

The method of solution is based on the procedure of the Generalized Integral Transform Technique (GITT), which requires the expansion of function $\theta(x)$ in a series of eigenfunctions, based on the Sturm-Liouville eigenvalue problem. Before applying such method, a filtering process of non-homogeneous terms is proposed to improve the convergence rate, as seen below:

$$\theta(\xi) \;=\; \varphi(\xi) \;+\; f(\xi), \tag{11}$$

The filter problem is then established:

$$\frac{1}{N_{C_L}}\,\frac{\mathrm{d}^2 f}{\mathrm{d}\xi^2} = 2f(\xi) + \bar{\lambda} \tag{12}$$

$$\left.\frac{\partial f}{\partial \xi}\right|_{\xi=1} \;=\; 0, \qquad f(0) \;=\; 1, \tag{13}$$

where $\bar{\lambda}$ can be calculated by:

$$\bar{\lambda} = \int_0^1 \lambda(\xi)\,\mathrm{d}\xi \tag{14}$$

The solution of the filter can be obtained analytically:

$$f(\xi) = \frac{1}{2}\,(\bar{\lambda}+2)\,\left[\cosh(\sqrt{2\,N_{C_L}}\,\xi) - \sinh(\sqrt{2\,N_{C_L}}\,\xi)\,\tanh(\sqrt{2\,N_{C_L}})\right] - \frac{\bar{\lambda}}{2} \tag{15}$$

The separation process results in the following filtered system:

$$\frac{1}{N_{C_L}}\,\frac{\mathrm{d}^2\varphi}{\mathrm{d}\xi^2} \;=\; 2\,\left[(\varphi(\xi)+f(\xi))^4 - f(\xi)\right] \;-\; \int_{\eta=0}^{1}(\varphi(\eta)+f(\eta))^4\,\gamma(\xi,\eta)\,\mathrm{d}\eta + \lambda(\xi) - \bar{\lambda} \tag{16}$$

$$\left.\frac{\partial \varphi}{\partial \xi}\right|_{\xi=1} \;=\; 0, \qquad \varphi(0) \;=\; 0, \tag{17}$$

The GITT method then proceeds with the solution of the Helmholtz-type eigenvalue problem:

$$X''(\xi) + \mu_i^2\,X_i = 0 \tag{18}$$

$$X_i'(1) = 0 \quad \text{and} \quad X_i(0) = 0, \tag{19}$$

which gives,

$$X_i(\xi) = \sin(\mu_i \xi) \qquad \text{with} \qquad \mu_i = \left(i - \frac{1}{2}\right)\pi, \quad i = 1, 2, 3, ... \tag{20}$$

where X_i are eigenfunctions and μ_i are eigenvalues. Considering the concept of orthogonality between the eigenfunctions, the following relation is satisfied:

$$\int_0^1 X_i(\xi)\,X_j(\xi)\,\mathrm{d}\xi = \delta_{i,j}\,N_j = \delta_{i,j}\,N_i \tag{21}$$

where $\delta_{i,j}$ is the Kronecker delta and the norms N_i are given by:

$$N_i = \int_0^1 X_i^2(\xi)\,\mathrm{d}\xi = \frac{1}{2} \tag{22}$$

The definition of the inverse-transform pair is also required:

$$\varphi(\xi) = \sum_{i=1}^{\infty} \frac{\overline{\varphi}_i \, X_i(\xi)}{N_i} \tag{23}$$

$$\overline{\varphi}_i = \int_0^1 \varphi(\xi) \, X_i(\xi) \, \mathrm{d}\xi \tag{24}$$

The procedure involves the multiplication of equation (16) by the eigenfunctions X_i and integrating in the domain $0 \le \xi \le 1$, resulting in:

$$-\frac{\mu_i^2}{N_{C_L}} \overline{\varphi}_i = A_i + \sum_{j=1}^{\infty} B_{i,j} \, \overline{\varphi}_j + \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} C_{i,j,k} \, \overline{\varphi}_j \, \overline{\varphi}_k + \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty} D_{i,j,k,l} \, \overline{\varphi}_j \, \overline{\varphi}_k \, \overline{\varphi}_l + \\ + \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty} E_{i,j,k,l,m} \, \overline{\varphi}_j \, \overline{\varphi}_k \, \overline{\varphi}_l \, \overline{\varphi}_m \tag{25}$$

where the integral coefficients A, B, C, D and E are defined as:

$$A_i = \int_0^1 X_i(\xi) \int_0^1 \gamma(\xi,\eta) \, f(\eta)^4 \, \mathrm{d}\eta \, \mathrm{d}\xi + 2 \int_0^1 f(\xi)^4 \, X_i(\xi) \, \mathrm{d}\xi + \int_0^1 (\lambda(\xi) - \bar{\lambda}) \, X_i(\xi) \, \mathrm{d}\xi + \\ - 2 \int_0^1 f(\xi) \, X_i(\xi) \, \mathrm{d}\xi \tag{26}$$

$$B_{i,j} = 4 \int_0^1 X_i(\xi) \int_0^1 \gamma(\xi,\eta) \, f(\eta)^3 X_j(\eta) \, \mathrm{d}\eta \, \mathrm{d}\xi + 8 \int_0^1 f(\xi)^3 \, X_j(\xi) \, X_i(\xi) \, \mathrm{d}\xi \tag{27}$$

$$C_{i,j,k} = 6 \int_0^1 X_i(\xi) \int_0^1 \gamma(\xi,\eta) \, f(\eta)^2 \, X_j(\eta) \, X_k(\eta) \, \mathrm{d}\eta \, \mathrm{d}\xi + 12 \int_0^1 f(\xi)^2 \, X_j(\xi) \, X_k(\xi) \, X_i(\xi) \, \mathrm{d}\xi \tag{28}$$

$$D_{i,j,k,l} = 4 \int_0^1 X_i(\xi) \int_0^1 \gamma(\xi,\eta) \, f(\eta) \, X_j(\eta) \, X_k(\eta) \, X_l(\eta) \, \mathrm{d}\eta \, \mathrm{d}\xi + 8 \int_0^1 f(\xi) \, X_j(\xi) \, X_k(\xi) \, X_l(\xi) \, X_i(\xi) \, \mathrm{d}\xi \tag{29}$$

$$E_{i,j,k,l,m} = \int_0^1 X_i(\xi) \int_0^1 \gamma(\xi,\eta) \, X_j(\eta) \, X_k(\eta) \, X_l(\eta) \, X_m(\eta) \, \mathrm{d}\eta \, \mathrm{d}\xi + \\ + 2 \int_0^1 X_j(\xi) \, X_k(\xi) \, X_l(\xi) \, X_m(\xi) \, X_i(\xi) \, \mathrm{d}\xi \tag{30}$$

The solution is then obtained by the truncation of the system with a finite number of terms. The computation time is dependent of the numerical implementation of the integral coefficients, so some numerical work is required with the **NIntegrate** function from the *Mathematica* platform. From the solution of $\overline{\varphi}_i$, φ is then calculated with the inversion formula from the GITT procedure. The filter f plus the filtered solution φ gives the dimensionless temperature distribution $\theta(\xi)$.

4. RESULTS AND DISCUSSION

The first set of results is based on the graphical analysis of the temperature distribution in the considered finned-tubed radiator configuration. Figure 2 shows the temperature behavior based on different values of dimensionless parameters N_{C_L} and κ. As can be examined, the decrease of κ results in a higher temperature drop comparing to the solution with a higher κ. This is to be expected as κ is located in the denominator of both radiative parameters γ and λ, which increases their values. As the value of N_{C_L} increases, so is the temperature drop and the effect of κ on the temperature distribution also increases.

Another representation of the results is based on the convergence analysis for different values of the radiative parameters N_{C_L} and κ. This analysis represents an error evaluation of the GITT approach in solving the considered non-linear integro-differential system of equations. Nevertheless, the convergence rate can be investigated with a total number of 15 terms in tables 1, 2 and 3 which show the convergence behavior for each position with parameters $N_{C_L} = 0.25, 0.75$ and 5.00, and $\kappa = 1$ and $1/3$.

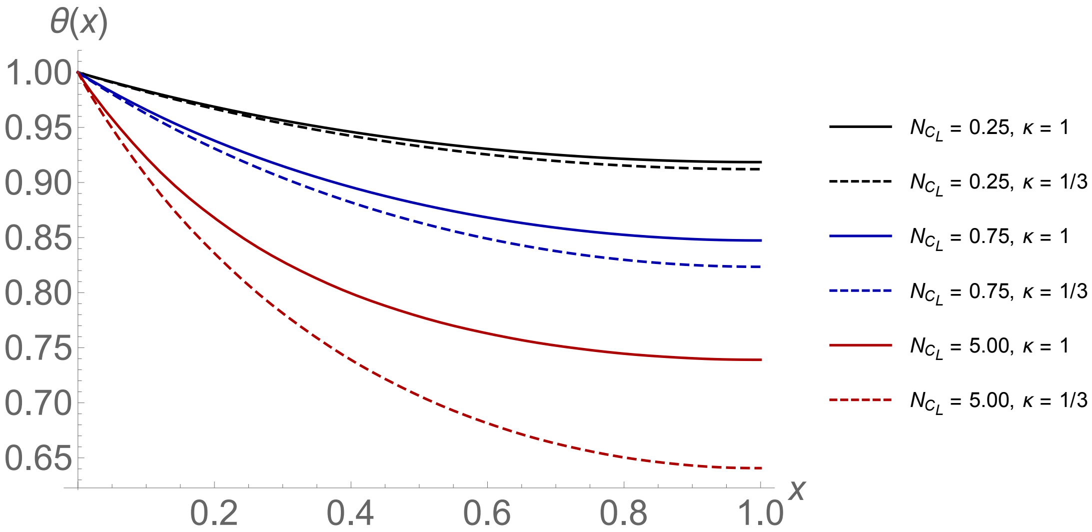

Figure 2: Temperature distribution for different dimensionless radiative parameters.

Table 1: Convergence analysis in different positions for radiative parameters $N_{C_L} = 0.25$ and different κ.

$N_{C_L} = 0.25, \kappa = 1$										
i_{max}	$\xi = 0.1$	$\xi = 0.2$	$\xi = 0.3$	$\xi = 0.4$	$\xi = 0.5$	$\xi = 0.6$	$\xi = 0.7$	$\xi = 0.8$	$\xi = 0.9$	$\xi = 1.0$
1	0.98097	0.96549	0.95307	0.94328	0.93572	0.93002	0.92591	0.92315	0.92156	0.92104
2	0.98237	0.96798	0.95612	0.94622	0.93791	0.93100	0.92547	0.92139	0.91888	0.91803
3	0.98281	0.96860	0.95656	0.94622	0.93748	0.93040	0.92505	0.92140	0.91932	0.91865
4	0.98300	0.96878	0.95652	0.94602	0.93733	0.93047	0.92527	0.92153	0.91923	0.91844
5	0.98310	0.96881	0.95643	0.94596	0.93740	0.93055	0.92522	0.92144	0.91924	0.91854
6	0.98316	0.96879	0.95638	0.94599	0.93744	0.93050	0.92520	0.92149	0.91925	0.91848
7	0.98319	0.96876	0.95638	0.94602	0.93742	0.93049	0.92523	0.92147	0.91924	0.91852
8	0.98320	0.96874	0.95639	0.94602	0.93740	0.93052	0.92521	0.92147	0.91925	0.91850
9	0.98321	0.96873	0.95641	0.94601	0.93741	0.93051	0.92521	0.92148	0.91924	0.91851
10	0.98321	0.96873	0.95641	0.94600	0.93742	0.93050	0.92522	0.92147	0.91925	0.91850
11	0.98321	0.96873	0.95641	0.94601	0.93741	0.93051	0.92521	0.92148	0.91924	0.91851
12	0.98321	0.96874	0.95640	0.94601	0.93741	0.93051	0.92521	0.92147	0.91925	0.91850
13	0.98321	0.96874	0.95640	0.94601	0.93741	0.93051	0.92522	0.92147	0.91924	0.91851
14	0.98320	0.96874	0.95640	0.94601	0.93741	0.93051	0.92521	0.92148	0.91925	0.91850
15	0.98320	0.96874	0.95640	0.94601	0.93741	0.93051	0.92522	0.92147	0.91925	0.91851
$N_{C_L} = 0.25, \kappa = 1/3$										
i_{max}	$\xi = 0.1$	$\xi = 0.2$	$\xi = 0.3$	$\xi = 0.4$	$\xi = 0.5$	$\xi = 0.6$	$\xi = 0.7$	$\xi = 0.8$	$\xi = 0.9$	$\xi = 1.0$
1	0.97919	0.96239	0.94906	0.93865	0.93071	0.92480	0.92058	0.91777	0.91617	0.91565
2	0.98111	0.96582	0.95324	0.94268	0.93370	0.92611	0.91992	0.91529	0.91241	0.91143
3	0.98173	0.96670	0.95386	0.94268	0.93309	0.92524	0.91931	0.91530	0.91303	0.91231
4	0.98201	0.96695	0.95381	0.94239	0.93287	0.92534	0.91962	0.91548	0.91289	0.91200
5	0.98215	0.96700	0.95368	0.94230	0.93297	0.92546	0.91956	0.91535	0.91292	0.91215
6	0.98223	0.96697	0.95361	0.94235	0.93303	0.92539	0.91952	0.91542	0.91293	0.91207
7	0.98227	0.96693	0.95361	0.94240	0.93299	0.92538	0.91957	0.91540	0.91291	0.91212
8	0.98229	0.96690	0.95363	0.94240	0.93297	0.92541	0.91955	0.91540	0.91293	0.91208
9	0.98230	0.96688	0.95365	0.94238	0.93299	0.92540	0.91954	0.91541	0.91291	0.91211
10	0.98231	0.96688	0.95366	0.94237	0.93300	0.92539	0.91956	0.91539	0.91293	0.91209
11	0.98230	0.96688	0.95365	0.94237	0.93299	0.92540	0.91955	0.91540	0.91292	0.91210
12	0.98230	0.96689	0.95364	0.94238	0.93298	0.92540	0.91955	0.91540	0.91292	0.91209
13	0.98230	0.96690	0.95364	0.94238	0.93299	0.92540	0.91955	0.91540	0.91292	0.91210
14	0.98229	0.96690	0.95364	0.94238	0.93299	0.92540	0.91955	0.91540	0.91292	0.91209
15	0.98229	0.96690	0.95364	0.94237	0.93299	0.92540	0.91955	0.91540	0.91292	0.91210

As can be seen in table 1 and 2, a maximum of 5 fully converged digits for as low as 12 terms can be observed for some positions, which means a error estimates of 10^{-5}. In this set of results, no apparent difference is noticed with different

Table 2: Convergence analysis in different positions for radiative parameters $N_{C_L} = 0.75$ and different κ.

					$N_{C_L} = 0.75$, $\kappa = 1$					
i_{max}	$\xi = 0.1$	$\xi = 0.2$	$\xi = 0.3$	$\xi = 0.4$	$\xi = 0.5$	$\xi = 0.6$	$\xi = 0.7$	$\xi = 0.8$	$\xi = 0.9$	$\xi = 1.0$
1	0.95817	0.92644	0.90297	0.88610	0.87436	0.86649	0.86147	0.85846	0.85690	0.85642
2	0.96314	0.93532	0.91382	0.89656	0.88217	0.86997	0.85987	0.85217	0.84730	0.84562
3	0.96468	0.93749	0.91535	0.89657	0.88066	0.86783	0.85839	0.85223	0.84890	0.84787
4	0.96535	0.93810	0.91524	0.89586	0.88013	0.86809	0.85915	0.85269	0.84857	0.84713
5	0.96569	0.93821	0.91493	0.89566	0.88039	0.86837	0.85900	0.85237	0.84864	0.84748
6	0.96588	0.93815	0.91477	0.89577	0.88052	0.86822	0.85892	0.85255	0.84867	0.84730
7	0.96598	0.93806	0.91475	0.89588	0.88044	0.86819	0.85903	0.85248	0.84862	0.84741
8	0.96603	0.93798	0.91480	0.89588	0.88039	0.86826	0.85898	0.85248	0.84867	0.84734
9	0.96605	0.93794	0.91485	0.89583	0.88043	0.86825	0.85897	0.85251	0.84863	0.84739
10	0.96606	0.93793	0.91487	0.89581	0.88045	0.86822	0.85900	0.85248	0.84866	0.84736
11	0.96605	0.93794	0.91486	0.89583	0.88043	0.86824	0.85898	0.85251	0.84864	0.84738
12	0.96605	0.93796	0.91484	0.89585	0.88042	0.86825	0.85898	0.85249	0.84865	0.84736
13	0.96603	0.93797	0.91483	0.89585	0.88043	0.86823	0.85900	0.85249	0.84864	0.84738
14	0.96602	0.93798	0.91483	0.89584	0.88044	0.86824	0.85898	0.85250	0.84865	0.84737
15	0.96601	0.93799	0.91484	0.89583	0.88043	0.86824	0.85899	0.85249	0.84865	0.84738
					$N_{C_L} = 0.75$, $\kappa = 1/3$					
i_{max}	$\xi = 0.1$	$\xi = 0.2$	$\xi = 0.3$	$\xi = 0.4$	$\xi = 0.5$	$\xi = 0.6$	$\xi = 0.7$	$\xi = 0.8$	$\xi = 0.9$	$\xi = 1.0$
1	0.95216	0.91586	0.88898	0.86964	0.85617	0.84714	0.84135	0.83790	0.83609	0.83553
2	0.95856	0.92725	0.90289	0.88300	0.86605	0.85136	0.83897	0.82940	0.82329	0.82119
3	0.96063	0.93019	0.90495	0.88300	0.86398	0.84844	0.83693	0.82944	0.82543	0.82419
4	0.96154	0.93101	0.90480	0.88203	0.86327	0.84877	0.83796	0.83006	0.82497	0.82317
5	0.96200	0.93116	0.90438	0.88176	0.86361	0.84916	0.83775	0.82962	0.82505	0.82365
6	0.96225	0.93108	0.90416	0.88191	0.86379	0.84896	0.83764	0.82986	0.82509	0.82340
7	0.96239	0.93096	0.90414	0.88206	0.86368	0.84891	0.83779	0.82977	0.82502	0.82355
8	0.96246	0.93086	0.90421	0.88206	0.86361	0.84901	0.83772	0.82977	0.82509	0.82345
9	0.96249	0.93081	0.90428	0.88200	0.86366	0.84899	0.83771	0.82981	0.82504	0.82352
10	0.96250	0.93080	0.90430	0.88197	0.86369	0.84895	0.83775	0.82977	0.82508	0.82347
11	0.96249	0.93081	0.90428	0.88199	0.86367	0.84898	0.83772	0.82980	0.82505	0.82351
12	0.96248	0.93083	0.90425	0.88201	0.86365	0.84899	0.83772	0.82979	0.82507	0.82348
13	0.96246	0.93085	0.90424	0.88201	0.86367	0.84897	0.83774	0.82979	0.82506	0.82350
14	0.96245	0.93086	0.90424	0.88200	0.86368	0.84897	0.83772	0.82980	0.82507	0.82349
15	0.96244	0.93085	0.90424	0.88201	0.86369	0.84897	0.83773	0.82980	0.82507	0.82350

values of κ with smaller value of N_{C_L} in terms of convergence rate.

Table 2 shows the convergence with a higher value of N_{C_L} (5.00) and different values of κ, where a total of 4 fully converged digits can be observed, representing a error estimate of 10^{-4} in some positions. For higher values of N_{C_L}, a clear trend can be recognized where lower values of κ worsens the convergence estimates. Comparing all values of N_{C_L}, the error also decreases in general with lower values of N_{C_L}.

5. CONCLUSIONS

The current paper presented a mathematical formulation for a steady combined conductive-radiative heat transfer problem, considering a finned-tube radiator in a closed-sandwich configuration. The resulting integro-differential system was normalized and shown to be characterized by two dimensionless parameters, namely κ and N_{C_L}. The computational implementation of the GITT method presented for the considered problem showed the required robustness and precision, as expected. As the error estimates displayed adequate results, it was carried out a further investigation of the temperature distribution and the effects of both parameters N_{C_L} and κ. With the same κ-values, an increase of N_{C_L} showed a higher temperature drop, while for the same N_{C_L}-values, an decrease of κ showed a higher temperature drop. Regarding the integral transform solution, it is worth mentioning that a good choice for the filter problem is crucial in optimizing the convergence rate. For the presented cases, the filter problem was shown to be adequate, once converged graphical solutions were obtained with truncated series with less than 10 terms. However, a finer choice for the filter can emerge as a better fit for the convergence analyses, with more fully converged digits obtained.

Table 3: Convergence analysis in different positions for radiative parameters $N_{C_L} = 5.00$ and different κ.

$N_{C_L} = 5.00, \kappa = 1$										
i_{max}	$\xi = 0.1$	$\xi = 0.2$	$\xi = 0.3$	$\xi = 0.4$	$\xi = 0.5$	$\xi = 0.6$	$\xi = 0.7$	$\xi = 0.8$	$\xi = 0.9$	$\xi = 1.0$
1	0.86487	0.78725	0.74911	0.73694	0.74056	0.75238	0.76678	0.77974	0.78855	0.79166
2	0.89772	0.84587	0.82091	0.80651	0.79307	0.77677	0.75820	0.74055	0.72783	0.72319
3	0.91038	0.86367	0.83326	0.80602	0.77994	0.75876	0.74610	0.74193	0.74254	0.74341
4	0.91641	0.86906	0.83201	0.79944	0.77531	0.76137	0.75345	0.74637	0.73956	0.73656
5	0.91960	0.87001	0.82907	0.79762	0.77782	0.76417	0.75200	0.74328	0.74022	0.74005
6	0.92137	0.86943	0.82748	0.79876	0.77915	0.76272	0.75123	0.74509	0.74056	0.73826
7	0.92234	0.86853	0.82734	0.79983	0.77839	0.76241	0.75236	0.74447	0.74009	0.73941
8	0.92284	0.86782	0.82787	0.79984	0.77789	0.76315	0.75186	0.74448	0.74061	0.73870
9	0.92306	0.86743	0.82836	0.79938	0.77825	0.76300	0.75179	0.74479	0.74018	0.73921
10	0.92311	0.86733	0.82853	0.79918	0.77851	0.76272	0.75212	0.74445	0.74054	0.73886
11	0.92307	0.86741	0.82841	0.79934	0.77833	0.76294	0.75189	0.74471	0.74029	0.73913
12	0.92298	0.86757	0.82821	0.79953	0.77819	0.76301	0.75192	0.74459	0.74047	0.73893
13	0.92287	0.86773	0.82811	0.79953	0.77830	0.76285	0.75203	0.74460	0.74036	0.73909
14	0.92276	0.86783	0.82813	0.79941	0.77839	0.76289	0.75191	0.74467	0.74042	0.73897
15	0.92266	0.86786	0.82822	0.79936	0.77832	0.76297	0.75196	0.74458	0.74040	0.73907
$N_{C_L} = 5.00, \kappa = 1/3$										
i_{max}	$\xi = 0.1$	$\xi = 0.2$	$\xi = 0.3$	$\xi = 0.4$	$\xi = 0.5$	$\xi = 0.6$	$\xi = 0.7$	$\xi = 0.8$	$\xi = 0.9$	$\xi = 1.0$
1	0.83754	0.74093	0.68980	0.66889	0.66677	0.67492	0.68706	0.69871	0.70684	0.70975
2	0.87582	0.80906	0.77276	0.74835	0.72509	0.69900	0.67122	0.64591	0.62808	0.62165
3	0.89118	0.83057	0.78744	0.74718	0.70837	0.67633	0.65591	0.64727	0.64590	0.64626
4	0.89853	0.83710	0.78584	0.73907	0.70268	0.67952	0.66488	0.65261	0.64210	0.63770
5	0.90238	0.83823	0.78229	0.73688	0.70572	0.68288	0.66309	0.64884	0.64287	0.64190
6	0.90447	0.83755	0.78043	0.73825	0.70730	0.68115	0.66216	0.65099	0.64326	0.63975
7	0.90560	0.83651	0.78027	0.73951	0.70639	0.68078	0.66348	0.65024	0.64269	0.64109
8	0.90618	0.83569	0.78088	0.73952	0.70582	0.68164	0.66290	0.65026	0.64330	0.64026
9	0.90644	0.83525	0.78146	0.73899	0.70624	0.68147	0.66282	0.65060	0.64280	0.64084
10	0.90650	0.83513	0.78164	0.73876	0.70653	0.68115	0.66319	0.65022	0.64321	0.64044
11	0.90645	0.83523	0.78151	0.73894	0.70633	0.68140	0.66293	0.65051	0.64292	0.64075
12	0.90635	0.83541	0.78129	0.73916	0.70617	0.68147	0.66297	0.65038	0.64313	0.64052
13	0.90623	0.83559	0.78117	0.73916	0.70630	0.68130	0.66310	0.65039	0.64300	0.64070
14	0.90610	0.83570	0.78119	0.73903	0.70640	0.68134	0.66296	0.65047	0.64307	0.64056
15	0.90600	0.83574	0.78130	0.73897	0.70632	0.68143	0.66301	0.65036	0.64305	0.64068

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