

24th COBEM - 2017



24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-0463

3D NUMERICAL SIMULATIONS OF NATURAL CONVECTION IN CAVITIES USING ADAPTIVE MESH REFINEMENT

Bernardo Alan de Freitas Duarte

Aristeu da Silveira Neto

Universidade Federal de Uberlândia, av. João Naves de Ávila, 2121, Uberlândia.

be@jl.adm.br

Abstract. *This work presents 3D numerical simulations of natural convection using adaptive mesh refinement for one-phase flow. Temperature gradient was used as refinement criteria. Flow was represented by Navier-Stokes and energy equation which was solved for temperature. The physical model consisted of a differentially heated cavity with height aspect ratio of one or two. The model was applied to predict thermal exchange rate between the fluid and a wall kept at a fixed temperature. Nusselt number calculations and flow evolution were in good agreement with literature and demonstrated the validity of the present model. Temperature gradient criteria guided adequately mesh refinement evolution and promoted accurate results. Assessment of computational efficiency revealed simulations using adaptive mesh refinement (AMR) with lower costs than simulations using a uniform grid. AMR simulations were up to 34% faster compared to uniform grid simulations and required up to ten times less computational cells. Therefore, adaptive mesh refinement using temperature gradient as refinement criteria is suggested as a promising method for modeling natural convection.*

Keywords: *Nusselt number, thermal transfer rate, Oberbeck-Boussinesq approximation*

1. INTRODUCTION

The main objective of this paper was to quantify the efficiency of simulations using adaptive mesh refinement (AMR) compared to uniform grids in non-isothermal problems. The present work investigated numerical simulations of natural convection using uniform grids or AMR. Benchmarks between different mesh configurations were performed for cavities with height aspect ratio of one or two.

Dynamics of flow in non-isothermal conditions is a problem of great industrial and scientific interest. Thermal transport in complex flow phenomena is frequently encountered in a variety of industrial processes (Kizildag et al., 2013; Wan et al., 2001). Among such flows, some deal with thermal convection, which physical mechanisms are omnipresent in nature and in technological applications. Natural convection flows in cavities have attracted the attention of many researchers due to their presence in many industrial applications. Although the geometry of cavities is frequently simple, the flow may become very complex with the existence of turbulent structures in flow at high Rayleigh number. It is important to understand the underlying mechanisms that are related to the interaction between heat transfer and fluid dynamics. In addition, knowledge about these physical process of flow allows the improvement of industrial operations and may produce guideline to more efficient projects.

Computational simulations of natural convection provides several important information about flow behavior and thermal transfer rate in non-isothermal condition. On the other hand, results from computational simulations depends on the quality of the employed spatial discretization. Accurate numerical solution of partial differential equations rely on the discretization on a computational grid with sufficiently high resolution; however, simulations using uniform grids overly increase computational costs due to a large domain region unnecessarily refined. A uniform and fine grid is associated with a high computational cost which may limit the applicability of solving several complex problems of interest. Conversely, AMR is a computational tool allowing a criteria definition to guide a spatially non-uniform mesh refinement according to an indicator function, such as vorticity, temperature gradient or interface presence.

AMR may provide a strategy to solve complex problems using lower computational resources compared to uniform grids (Akhtar et al., 2013) and it reduces computational power requirement without affecting precision (Ningegowda et al., 2014). The relevance in using AMR in multiphase flows is particularly high since the interface region requires a fine grid due to high gradients calculations and the rest of the domain usually do not require a fine grid (Nikolopoulos et al., 2007)

Therefore, numerical simulations of natural convection in cavities were performed using adaptive mesh refinement in order to study the influence of grid configuration in thermal transfer problems. Time and computational power saved were quantified and conclusions were performed at the end of the present work.

2. MATHEMATICAL MODEL

Oberbeck-Boussinesq approximation (OB) was included in the present work in order to model thermal effects on specific mass variations. OB has been used for most of what is known about natural convection (Gray, 1976); nonetheless, OB is restricted to single-phase fluid flows and it may be inappropriate to employ this mathematical approximation in applications where large effects of specific mass variations occurs (Harish et al., 2016). Since the effects of temperature variations were small, OB was employed in order to model the variations of specific mass due to temperature. The continuity equation is given by the following expression:

$$\vec{\nabla} \cdot \vec{v} = 0 \quad (1)$$

Equation (2) shows the momentum equation:

$$\rho_0 \frac{D\vec{v}}{Dt} = -\vec{\nabla} p + \vec{\nabla} \cdot \left[\mu \left(\vec{\nabla} \vec{v} + (\vec{\nabla} \vec{v})^T \right) \right] + (\rho - \rho_0) \vec{g} \quad (2)$$

Finally, the energy equation is given by the following equation:

$$\frac{DT}{Dt} = \alpha \nabla^2 T \quad (3)$$

Equation (4) presents the thermodynamic relation employed in the momentum equation to compute variable specific mass effects as a temperature function:

$$(\rho - \rho_0) \vec{g} = -\rho_0 \beta (T - T_0) \vec{g} \quad (4)$$

3. COMPUTATIONAL PROCEDURE

Simulations were performed using a three-dimensional cubic cavity with flow subjected to gravity acceleration action. A cavity was used as physical model due to simple geometry (Wan et al., 2001) and presence in several industrial applications (Kizildag et al., 2013). Figure 1 shows the physical model employed in the numerical simulations of the present work. Null velocities and null pressure gradient were imposed over all domain faces.

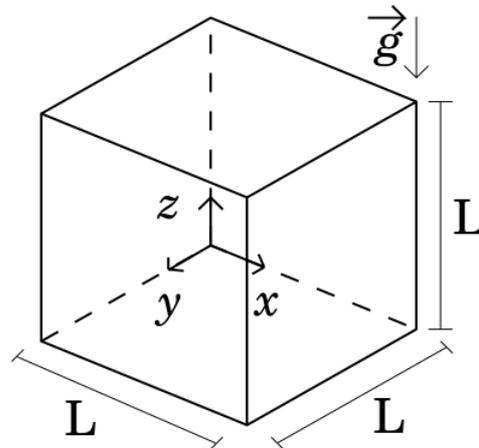


Figure 1. Physical model employed in the present paper

Domain boundaries were named according to geographic orientation which was illustrated in Fig. 2. The east and west walls have, respectively, a uniform high and low temperature. The south, north, bottom and top walls were considered adiabatic.

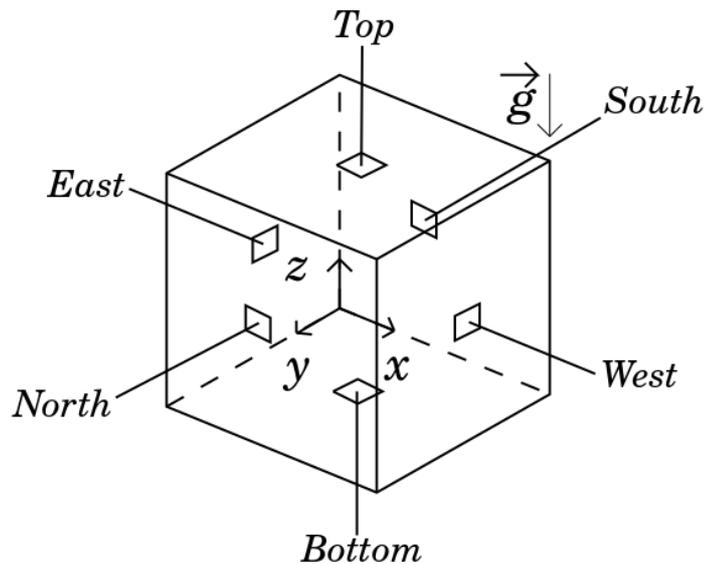


Figure 2. Domain boundaries names according to geographic orientation

Simulations were carried out using a structured and non-uniform three-dimensional cartesian grid. Two mesh refinement levels were employed in the numerical simulations and the base level presented $32 \times 32 \times 32$ cells. Velocity-pressure coupling was accomplished using a two-step projection method with an explicit treatment for advection terms and an implicit treatment for pressure and diffusion terms. Barton scheme (Centrella, 1984) is used for advective terms spatial discretization. Pressure and energy equations were solved implicitly using multigrid-multilevel solver. The transient equations were solved according to finite volume methodology using MFSim code, which has been developed in the last 10 years in cooperation with a large research group and Petrobras scientific support. All simulations were performed in parallel ambient in the cluster from the Fluid Mechanics Laboratory at the Federal University of Uberlândia (UFU), Brazil.

4. RESULTS AND DISCUSSION

First, the numerical model was validated with literature according to isotherms behavior and thermal transfer rate at the heated wall. Then, benchmarks between simulations with uniform grid and AMR were conducted for cavities with height aspect ratio of one or two.

5. Numerical model validation

The numerical model was initially validated for natural convection in one-phase flow using Oberbeck-Boussinesq approximation with Prandtl number of 0.71 and for a range of Rayleigh numbers from $Ra=10^3$ to $Ra=10^6$. Isotherms were analyzed in the central xz -plane in Fig. 3 and compared qualitatively to Wan et al. (2001) in Fig. 4.

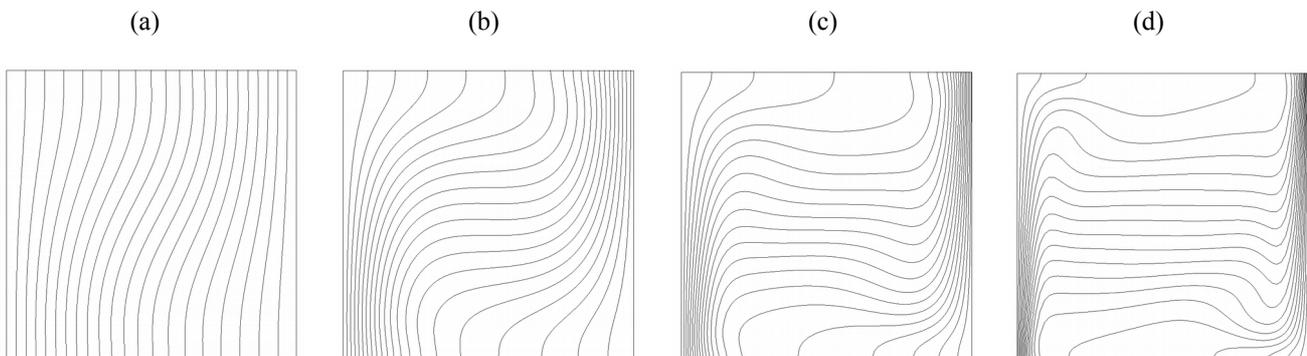


Figure 3. Isotherms from the present work with (a) $Ra=10^3$; (b) $Ra=10^4$; (c) $Ra=10^5$; (d) $Ra=10^6$

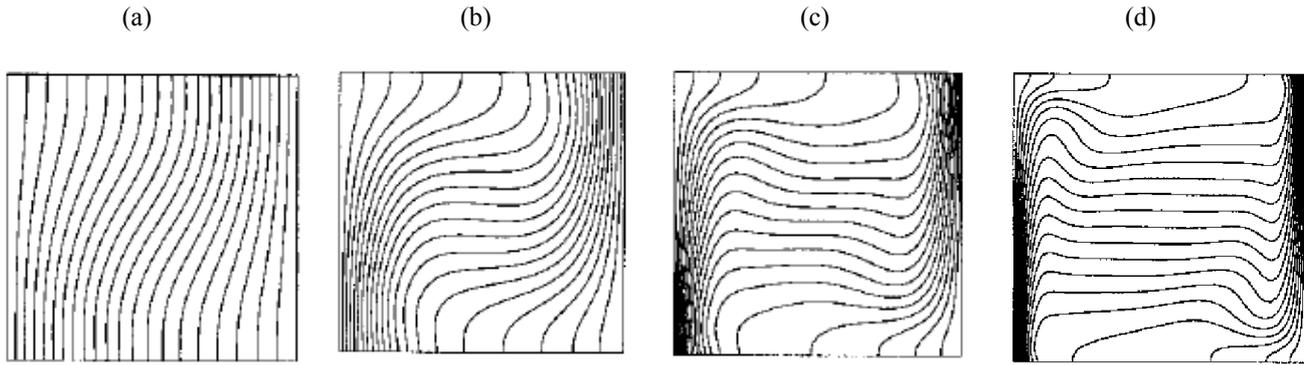


Figure 4. Isotherms from Wan et al. (2001) with (a) $Ra=10^3$; (b) $Ra=10^4$; (c) $Ra=10^5$; (d) $Ra=10^6$

Isotherms from the present work were qualitatively similar to the isotherms from Wan et al. (2001). As Rayleigh number increases, the flow becomes more turbulent and isotherms assumed progressively more complex aspect due to flow advection, as described previously by Padilla et al. (2013).

In order to present a quantitative analysis and report an accurate validation, thermal transfer rate was evaluated by the mean Nusselt number computed at the heated wall. Nusselt number was calculated at the east wall and then compared to literature. Mean Nusselt number at the east wall was computed after the achievement of flow permanent regime and good agreement was found between the present work and literature, as shown in Tab. 1.

Table 1. Nusselt number at east wall

Reference	$Ra = 10^3$	$Ra = 10^4$	$Ra = 10^5$	$Ra = 10^6$
Present work	1.071	2.061	4.390	8.901
Padilla et al. (2013)	1.072	2.068	4.427	8.805

The numerical model was adequately validated since the numerical results presented good agreement with literature. Next subsections comprises benchmarks between simulations with uniform grids and AMR.

6. Benchmark between simulations with uniform grid and AMR for a cavity with height aspect ratio of one

Assessment of AMR simulations were conducted and compared to uniform grid simulations. Simulations were performed considering $Ra=10^3$ and $Pr=0.71$. Table 2 presents results from the simulations with different grid configurations.

Table 2. AMR efficiency for the simulation of $Ra = 10^3$ with the cavity of height aspect ratio of one

Grid configuration	Number of cells	Time (h)
Uniform grid	2,097,152	96
AMR	250,000	72

AMR simulations promoted time saving by limiting the need of a fine uniform grid in the whole domain. In addition, large computational costs were spared using AMR because of the considerable decreased number of computational cells needed. Finally, accurate thermal transfer results were obtained when using AMR. The number of cells employed in AMR simulations were almost ten times lower than the uniform grid simulation and about of 30% of time was saved compared to the uniform grid simulation.

Figure 5 illustrates the adaptive mesh refinement used and the temperature field in the final time of the simulation.

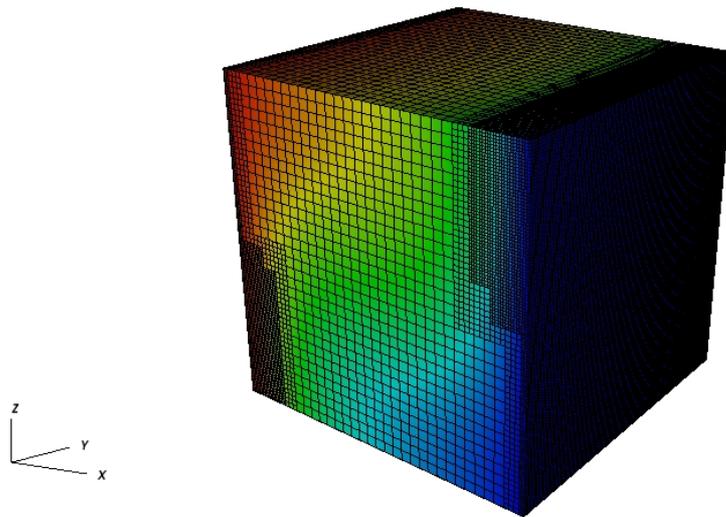


Figure 5. Temperature field and adaptive mesh in simulation's final time.

According to the refinement criteria used, temperature gradient was considered an adequate parameter to guide mesh refinement since it has provided accurate results near the walls where thermal transfer was measured and saved significant computational resources by reducing the number of cell in the domain center. Compared to other refinement criteria, such as vorticity magnitude, temperature gradient may be considered more suitable to natural convection problems since vorticity is present in several regions of the domain. Figure 6 shows velocity field at simulation's final time, where the gradients of velocity are evident nearly in all the domain. Therefore, vorticity magnitude would be not an adequate refinement criteria since a fine mesh would be placed unnecessarily almost in every place of the computational domain.

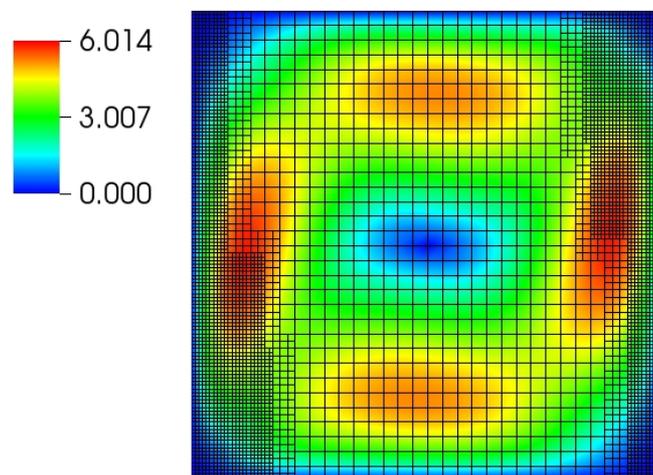


Figure 6. Central xz-slice with velocity field and the employed adaptive mesh in simulation's final time

Therefore, natural convection simulations in cavities with height aspect ratio of one presented great advantages using AMR compared to uniform grids. Temperature gradient was considered an ideal criteria refinement and accurate results confirmed the quality of mesh resolution.

7. Benchmark between simulations with uniform grid and AMR for a cavity with height aspect ratio of two

Natural convection simulations were performed in a cavity with height aspect ratio of two using AMR or uniform grid. The physical model employed was exactly the same from the previous subsection; however, the cavity presented now height aspect ratio of two. Table 3 shows data from AMR and uniform grid simulations performed.

Table 3. AMR efficiency for the simulation with $Ra = 2 \times 10^3$ considering cavity's height aspect ratio of 2

Grid configuration	Number of cells	Time (h)
Uniform grid	4,194,304	121
AMR	423,872	115

AMR simulations employed significantly less computational cells compared to the uniform grid simulation. The number of cells used in AMR simulation was approximately ten times lower than the number used in uniform grid simulations. On the other hand, the time required to run the simulations was very similar between the two different mesh configurations. Figure 7 shows the temperature field and the mesh at simulation's final time.

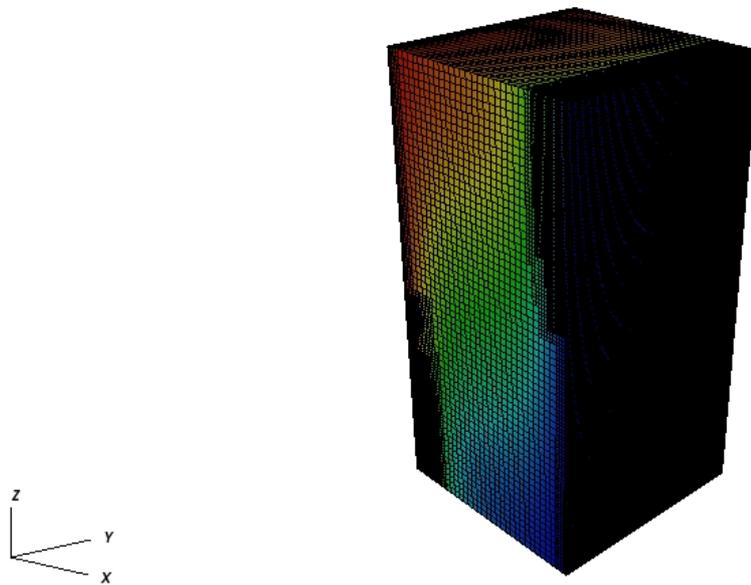


Figure 7. Temperature field and mesh configuration at simulation's final time.

Figure 7 exhibited a large central region of the domain with a course grid and only small regions close to the heated walls using a fine grid. Again, temperature gradient is presented as an adequate refinement criteria for natural convection problems since a large region in the domain center was solved with a course grid and a fine mesh was used near the high gradients, particularly near the walls, where thermal transfer rate is computed. Height aspect ratio did not influenced on the mean number of cells employed in AMR, since, the number of cells using AMR was 10 times lower compared to the uniform grid simulation for height aspect ratio of one and two. On the other hand, time required to run the simulations using AMR or uniform grid were not similar for both height aspect ratio probably due to numerical difficulties of convergence in multigrid-multilevel solver. Figure 8 shows velocity field in order to exhibit the gradients of velocity spreaded in several regions of the domain.

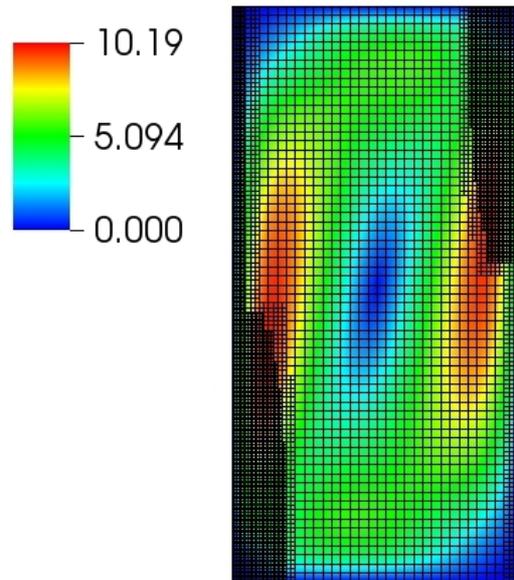


Figure 8. Velocity field and mesh configuration at simulation's final time.

According to the velocity gradients seen in Fig. 8, vorticity would be not suitable to guide refinement mesh in time in natural convection problems since it would unnecessarily refine large regions in the center of the domain. Therefore, the different height aspect ratio of the cavity did not changed the capability of temperature gradient to guide correctly mesh refinement in time. In addition, accurate results were obtained between AMR and uniform grid for all the simulations performed.

8. CONCLUSIONS

Adaptive mesh refinement used in the simulations of natural convection promoted time saving, by limiting the need of uniform grid in the whole domain. Large computational costs were saved when using adaptive mesh refinement because of the considerable reduction on the number of cells needed.

In addition, accurate results of thermal transfer rate were found when using adaptive mesh. Temperature gradient was a relevant refinement criteria to the numerical simulations of natural convection since it promoted the reduction of computational cells and saved time and computational power required.

9. ACKNOWLEDGMENTS

The authors gratefully acknowledge financial support from Petrobras, CNPQ, Fapemig and Capes. The authors are also grateful to the Mechanical Engineering Graduate Program from the Federal University of Uberlândia (UFU).

10. REFERENCES

- Akhtar, M. and Kleis, S., 2013. "Boiling flow simulations on adaptive octree grids". *International journal of multiphase flow*, Vol. 53, p. 88-99. DOI:10.1016/j.ijmultiphaseflow.2013.01.008.
- Centrella, J.M. and Wilson, J.R., 1984. "Planar numerical cosmology. II - The difference equations and numerical tests". *The Astrophysical Journal Supplement Series*, Vol. 54, p. 229-249. DOI:10.1086/190927.
- Gray, D.D. and Giordini, A., 1976. "The Validity of the Boussinesq Approximation for Liquids and Gases". *International journal of heat and mass transfer*, Vol. 19, p. 545-551. DOI:10.1016/0017-9310(76)90168-X.
- Harish, R. and Venkatasubbaiah, K., 2016. "Non-Boussinesq approach for turbulent buoyant flows in enclosure with horizontal vent and forced inlet port". *Applied Mathematical Modelling*, Vol. 40, p. 927-941. DOI:10.1016/j.apm.2015.05.013 .
- Kizildag, D. and Rodriguez, I. and Oliva, A. and Lehmkuhl, O., 2014. "Limits of the Oberbeck-Boussinesq approximation in a tall differentially heated cavity filled with water". *International journal of heat and mass transfer*, Vol. 68, p. 489 -499. DOI:10.1016/j.ijheatmasstransfer.2013.09.046.
- Ningegowda, B.M. and Premachandran, B., 2014. "A Coupled Level Set and Volume of Fluid method with multi-directional advection algorithms for two-phase flows with and without phase change". *International Journal of Heat and Mass Transfer*, Vol. 79, p. 532-550. DOI: 10.1016/j.ijheatmasstransfer.2014.08.039

- Nikolopoulos, N. and Theodorakakos, A. and Bergeles, G., 2007. "A numerical investigation of the evaporation process of a liquid droplet impinging onto a hot substrate". *Intenational Journal of heat and mass transfer*, Vol. 50, p. 303-319. DOI:10.1016/j.ijheatmasstransfer.2006.06.012.
- Padilla, E. and Lourenco, M. and Silveira-Neto, A., 2013. "Natural convection inside cubical cavities: numerical solutions with two boundary conditions". *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, Vol. 35, p. 275-283. DOI:10.1007/s40430-013-0033-y.
- Wan, P. and Patnaik, B. and Wei, G., 2001. "A new benchmark quality solution for the buoyancy-driven cavity by discrete singular convolution". *Numerical heat transfer*, Vol. 40, p. 199-228. DOI:10.1080/104077901752379620.

11. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.