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ENERGY HARVESTING BY APPLYING A HYSTERETIC DAMPED NES DEVICE INTO A PORTAL FRAME PLATFORM OF TWO-DEGREES-OF-FREEDOM

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Abstract. *This work presents energy harvesting application using a two-degrees-of-freedom portal frame platform with a nonlinear energy-sink device coupled to its supported beam, considering its horizontal motion. The nonlinear energy-sink (NES) device consists of a mass-spring-damper system, whose damping is viscoelastic, i.e., considered as a hysteretic damping. The portal frame has two-to-one internal resonance, which presents saturation phenomenon, partially transferring the vibration energy from one to other mode of vibration. The presence of the phenomenon allows the energy harvesting from the column vibration, which will receive the surplus vibration because of saturation. The energy harvesting is investigated in this case using a nonlinear piezoelectric material. The governing equations of motion of the system was derived by using the energy method of Lagrange considering the whole system coupled each other and nonlinear. The results showed with the hysteretic damping of the NES, the system became chaotic and the energy harvesting almost doubled.*

Keywords: *hysteretic damping, nonlinear energy-sink, saturation phenomenon, portal frame structure, energy harvesting*

1. INTRODUCTION

Recently, the efforts to improve the energy harvesting of environment sources have been increased substantially. Many of them are constantly explored and there is a powerful level of energy to be extracted. The kinect energy is one of the most powerful means of energy due to its easily way to be found and explored. Generally, the kinect energy is extracted from the vibration of some structures which are, many times, developed to this reason (Rocha, 2016). However, there are many kinds of devices which are capable to convert this kind of energy into electrical energy.

Among such possible energy harvesting devices, the piezoelectric materials have been used as a common energy transducer due to their significantly response for stimulus of different physical natures (Preumont, 2006; Priya and Inman, 2009). In energy harvesting area, these materials have been used as a low-power energy device converting vibration energy into electrical energy, whose technique of energy harvesting have been studied by (Stephen, 2006; Syta, *et al.*, 2015; Stanton, *et al.*, 2010; Jalili, 2009), among others. However, PZTs have been demonstrated to be nonlinear in the aspect of relation between strain constant and the electric field (DuToit and Wardle, 2007; Twiefel, *et al.*, 2008), which was experimentally verified (Crawley and Anderson, 1990). Besides, an approximation between the results of a nonlinear model to the experimental results was developed considering two coefficients that are linear and nonlinear piezoelectric coefficients, whose approximation is an adjust to the experimental curve (Tripplet and Quinn, 2009). The nonlinear coefficient influences the final result of the energy harvesting, i.e., depending on its value, there will be gain or loss of energy (Rocha, 2016; Iliuk, *et al.*, 2013a, 2013b, 2014; Balthazar, *et al.*, 2014). An overview of the nonlinearities presented by the piezoelectric material was carried out (Daqaq, *et al.*, 2014).

Introducing the PZT transducer on a vibrating structure, it is possible to convert such vibration energy into electric energy. However, depending on the configurations of the structure, it may possess certain nonlinearity presenting a different kind of behaviour. Particularly, a structure of two-degrees-of-freedom with quadratic stiffness, when excited in resonance by an external excitation and possesses a two-to-one internal resonance between its two modes of vibration, i.e., a phenomenon, called saturation, may occurs. Besides those resonant conditions, when the amplitude of excitation is small, only the second mode is excited. As the amplitude reaches a critical value, which depends on the damping, the excited mode becomes saturated and the vibration energy "spills over" into the other mode, and transfer part of that vibration energy into the other mode that begins to vibrate. This is the saturation phenomenon described by many authors, for example, in Refs. (Rocha, 2016; Nayfeh, 2000; Nayfeh and Mook, 2008; Mook, *et al.*, 1985; Mankala and Quinn, 2004; Quinn, 2007; Rocha, *et al.*, 2016a, 2017a; Golnaraghi, 1991; Oueini, 1999; Oueini, *et al.*, 1997; Pai *et al.*, 1998, 2000; Balthazar, *et al.*, 2003; Felix, *et al.*, 2005, 2014; Tusset, *et al.*, 2015).

External excitation in environment are very common due to wind, vehicle traffic, and etc. In this way, to simulate such excitations, electromechanical devices were developed to be used as an external exciter. One of them is the electro-dynamical shaker which has been used by several authors considering a harmonic kind input voltage (Xu, *et al.*, 2005,2007; Lee, *et al.*, 2008; Avanço, *et al.*, 2015; Felix, *et al.*, 2016; Tusset, *et al.*, 2017).

Many kinds of structures may present undesired dynamical behaviours, for example, chaotic behavior, or even a continuously periodic behavior although with lower amplitude. With that, there are some kind of control strategies which could eliminate the chaotic behaviour or even increase the amplitudes of periodic vibrations. A most used control strategy is the passive control strategy using a nonlinear energy-sink (Vakakis, *et al.*, 2008; Gendelman, *et al.*, 2001; Rocha, *et al.*, 2016b, 2016c, 2017b), which is a passive device, without the need of any electronic component, and is coupled directly on a structure, making possible to pump energy into the system. These kinds of device have demonstrated a very good performance from control chaotic behavior to improve amplitudes of vibration, due to its spring-mass-damper configuration.

Therefore, this work presents an energy harvesting application using a two-degrees-of-freedom portal frame platform with a nonlinear energy-sink device coupled to the supported beam, considering its horizontal motion.

Next section shows the mathematical and physical model to be studied and its governing equations of motion.

2. PHYSICAL MODEL AND MATHEMATICAL MODELLING

The two-degrees-of-freedom portal frame platform consists in the same of (Rocha, 2016) which is a supported beam of length L pinned to two columns with height h that are clamped in a base with mass m_0. The beam and columns are considered as a concentrated mass, which are M and m, respectively, whose masses are of two-degrees-of-freedom that is the vertical and horizontal motion. The coordinate q_1 is related to the horizontal displacement in the sway mode, with natural frequency ω_1, and q_2 to the mid-span vertical displacement of the beam in the symmetric mode, with natural frequency ω_2. The linear stiffness of the columns and the beam can be evaluated by a Rayleigh-Ritz procedure using cubic trial functions. Geometric nonlinearity is introduced by considering the shortening due to bending of the columns and of the beam.

The nonlinear energy-sink (NES) device consists in a mass m_4 coupled to the mid-span of the beam through a nonlinear cubic spring $k_n = k_{nl}x^3$ and a damp Hi, whose damping is viscoelastic, i.e., considered as a hysteretic damping ("i" imaginary). In addition, its motion is directed to the horizontal motion in order to tune the horizontal displacement of the portal frame, looking for a way to improve the energy harvesting. The application of the NES in a base-excited portal frame was studied in (Rocha, *et al.*, 2016b), showing the influence of the NES in the energy harvesting.

Therefore, a piezoelectric material is coupled to a column in order to convert the high amplitudes of column vibration energy into electric energy. The piezoelectric material is considered as a nonlinear device proposed by (Tripplet and Quinn, 2009). The authors defined an approximation of the theoretical model to experimental results as $d(q_1) = \theta(1+\Theta|q_1|)$, where θ is the linear piezoelectric strain coefficient and Θ is the nonlinear piezoelectric strain coefficient. Its representation is a RC circuit with capacitance C_p, resistance R_p and electrical charge Q_p.

In this case, the base is excited by a non-ideal source which consists of an electro-dynamical shaker, considering its electrical and mechanical parts. The electrical part of the shaker consists of an electrical circuit RL with resistance R_0, inductance L_0 and an electric charge Q_{s0}. The input voltage of the shaker represents the induced external force in the base, that is $F_{ext} = e_0 cos\omega_n t$, where e_0 is de amplitude of the external excitation and ω_n is the frequency of the external excitation. The external frequency is set near resonance with the second mode, which is the twice of the frequency of the first mode, i.e., $\omega_n \approx \omega_2 + \sigma_2$ and $\omega_2 \approx 2\omega_1 + \sigma_1$. Such resonant conditions are set to saturation phenomenon appear. The mechanical part of the shaker consists of its base with mass m_0, stiffness k_0 and damping c_0 with displacement represented by S_0.

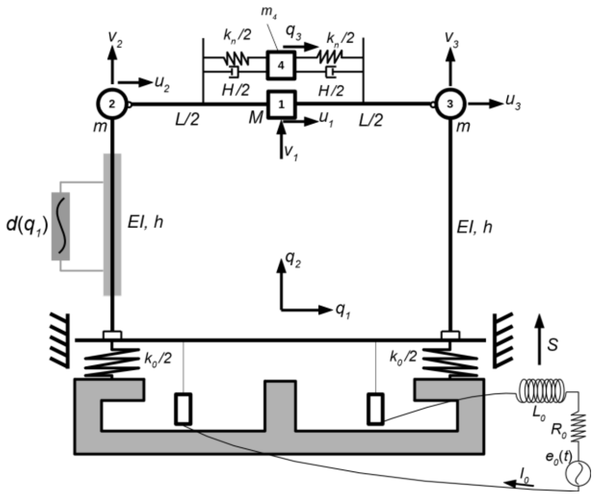

Figure 1. Portal frame platform excited by an electro-dynamical shaker with a nonlinear piezoelectric material coupled to a column and a nonlinear energy sink coupled to the mid span of the beam

2.1 Modelling of the portal frame system

Nodal displacements, shown in Fig. 1, are given by Eq. (1).

$$\begin{aligned} &u_1 = q_1 \qquad && u_2 = u_1 + \frac{B}{4} v_1^2 \qquad && u_3 = u_1 - \frac{B}{4} v_1^2 \\ &v_1 = S + q_2 \qquad && v_2 = S - \frac{A}{2} u_1^2 \qquad && v_3 = S - \frac{A}{2} u_1^2 \end{aligned} \tag{1}$$

where A = 6/5h and B = 24/5L. The stiffness of the beam and column calculated by the Rayleigh-Ritz method are, respectively, $k_b = 48EI/L_3$ and $k_c = 3EI/h_3$.

The generalized coordinates are the displacements of the concentrated mass at the mid span of the beam M. Using nodal displacements of Eq. (1), the kinetic energy is denoted by Eq. (2).

$$T = \frac{1}{2} M \left(\dot{u}_1^2 + \dot{v}_1^2 \right) + \frac{1}{2} m \left(\dot{u}_2^2 + \dot{u}_3^2 + \dot{v}_2^2 + \dot{v}_3^2 \right) + \frac{1}{2} m_0 \dot{S}^2 + \frac{1}{2} m_4 \dot{q}_3^2 + \frac{1}{2} L_0 \dot{Q}_0^2 \tag{2}$$

Introducing the generalized coordinates q_1 and q_2, the kinetic energy becomes to Eq. (3).

$$T = \frac{1}{2} M \left(\dot{q}_1^2 + \dot{q}_2^2 + 2\dot{q}_2 \dot{S} + \dot{S}^2 \right) + \frac{1}{2} m \left(2\dot{q}_1^2 + \dot{S}^2 - 4A\dot{S} q_1 \dot{q}_1 \right) + \frac{1}{2} m_0 \dot{S}^2 + \frac{1}{2} m_4 \dot{q}_3^2 + \frac{1}{2} L_0 \dot{Q}_0^2 \tag{3}$$

The potential energy of the system is given by the strain energy of the structure, the cubic nonlinear strain of NES device, the viscoelastic damping, which is considered as a complex stiffness, the work of the weight of the masses of the beam and columns and by the electrical potential part of the piezoelectric circuit with the contribution of the piezoelectric and the capacitor, resulting in Eq. (4).

$$\begin{aligned} U = {} & \frac{1}{2} k_c \left(u_2^2 + u_3^2 \right) + \frac{1}{2} k_b \left(v_1 - \frac{v_2 + v_3}{2} \right)^2 + mg \left(v_2 + v_3 \right) + Mg v_1 + \frac{1}{4} k_n \left(q_3 - u_1 \right)^4 + \\ & \frac{1}{2} iH \left(q_3 - u_1 \right)^2 + \frac{1}{2} k_0 S^2 + K \dot{Q}_s S - \frac{d\left(q_1 \right)}{C_p} Q_p \left(u_2 + v_2 \right) + \frac{1}{2} \frac{{Q_p}^2}{C_p} \end{aligned} \tag{4}$$

Substituting Eq. (1) into Eq. (4), in terms of the general coordinates q_1, q_2, q_3, Q_p and Q_s, the potential energy becomes to Eq. (5).

$$U=\left(k_c-mgA\right)q_1^2+\frac{1}{2}k_b\left(q_2^2+Aq_2q_1^2\right)+Mgq_2+\left(2m+M\right)gS+\frac{1}{2}k_0S^2+K\dot{Q}_sS+\frac{1}{4}k_n\left(q_3-q_1\right)^4-\frac{d\left(q_1\right)}{C_p}Q_p\left(q_1+S+\frac{B}{4}q_2^2\right)+\frac{1}{2}\frac{Q_p{}^2}{C_p} \tag{5}$$

According to (Rao and Yap, 2011), the complex term of viscoelastic damping can be represented as an equivalent real damp H/ω_1, i.e., an approximation of the complex term to harmonic excited systems whose damp term depends on the natural frequency. As the NES moves in horizontal direction, it is considered the natural frequency of the horizontal motion of the portal frame. Next, the energy dissipation of the system is considered by comprising the structural and NES damping defined by a Rayleigh function and the resistance of the electrical circuits. Then, the energy dissipation function is denoted by Eq. (6).

$$D=\frac{1}{2}c_1\dot{q}_1^2+\frac{1}{2}c_2\dot{q}_2^2+\frac{1}{2}c_0\dot{S}^2+\frac{1}{2}R_0\dot{Q}_s{}^2+\frac{1}{2}R_p\dot{Q}_p^2+\frac{1}{2}\frac{H}{\omega_1}\left(\dot{q}_3-\dot{q}_1\right)^2 \tag{6}$$

The harmonic excitation force is given by (7).

$$Q_{ext}=e_0\cos(\omega_n t) \tag{7}$$

The Lagrangean function is defined by Eq. (8). Substituting Eqs. (3) and (5) into Eq. (8), the Lagrange's function is given by Eq. (9).

$$L\left(q,\dot{q},t\right)=T-U \tag{8}$$

$$L=\frac{1}{2}M\left(\dot{q}_1^2+\dot{q}_2^2+2\dot{q}_2\dot{S}+\dot{S}^2\right)+\frac{1}{2}m\left(2\dot{q}_1^2+\dot{S}^2-4A\dot{S}q_1\dot{q}_1\right)+\frac{1}{2}m_0\dot{S}^2+\frac{1}{2}m_4\dot{q}_3^2+\frac{1}{2}L_0\dot{Q}_0^2-\left[\left(k_c-mgA\right)q_1^2+\frac{1}{2}k_b\left(q_2^2+Aq_2q_1^2\right)+Mgq_2+\left(2m+M\right)gS+\frac{1}{2}k_0S^2+K\dot{Q}_sS+\frac{1}{4}k_n\left(q_3-q_1\right)^4-\frac{d\left(q_1\right)}{C_p}Q_p\left(q_1+S+\frac{B}{4}q_2^2\right)+\frac{1}{2}\frac{Q_p{}^2}{C_p}\right] \tag{9}$$

In the following, using Euler-Lagrange, Eq. (10), the equation of motion of the system are developed and denoted by Eqs. (11)-(16).

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{q}_i}\right)-\frac{\partial L}{\partial q_i}+\frac{\partial D}{\partial\dot{q}_i}=Q_{ext}\qquad i=1,4 \tag{10}$$

$$\left(2m+M\right)\ddot{q}_1+2\left(k_c-mgA\right)q_1+Ak_bq_1q_2+c_1\dot{q}_1-2Am\dot{q}_1\dot{S}-k_n\left(q_3-q_1\right)^3-\frac{H}{\omega_1}\left(\dot{q}_3-\dot{q}_1\right)=\frac{d\left(q_1\right)}{C_p}Q_p \tag{11}$$

$$M\left(\ddot{q}_2+\ddot{S}\right)+k_bq_2+c_2\dot{q}_2+Mg+\frac{Ak_b}{2}q_1^2=\frac{d\left(q_1\right)}{C_p}\frac{B}{2}Q_pq_2 \tag{12}$$

$$\left(M+2m+m_0\right)\ddot{S}+M\ddot{q}_2+k_0S+c_0\dot{S}+\left(2m+M\right)g+KI-2Am\dot{q}_1^2=\frac{d\left(q_1\right)}{C_p}Q_p \tag{13}$$

$$m_4\ddot{q}_3+k_n\left(q_3-q_1\right)^3+\frac{H}{\omega_1}\left(\dot{q}_3-\dot{q}_1\right)=0 \tag{14}$$

$$L_0\dot{I}+R_0I-K\dot{S}=e_0\cos\omega t \tag{15}$$

$$R_p\dot{Q}_p - \frac{d\left(q_1\right)}{C_p}\left(q_1 + S + \frac{B}{4}q_2^2\right) + \frac{Q_p}{C_p} = 0 \tag{16}$$

where $I = dQ_s/dt$.

For a better analysis, a dimensionless process is carried out, resulting the dimensionless equations of motion of the system given by Eqs. (17)-(22)

$$x_1'' + \mu_1 x_1' + x_1 + \alpha_1 x_1 x_2 = \beta_1 x_1' Y' + K_1\left(x_3 - x_1\right)^3 + H_1\left(x_3' - x_1'\right) + \theta\left(1 + \Theta\left|x_1\right|\right)\delta_1 V_p \tag{17}$$

$$x_2'' + Y'' + \mu_2 x_2' + \omega_2^2 x_2 + \alpha_2 x_1^2 + G_2 = \theta\left(1 + \Theta\left|x_1\right|\right)\delta_2 V_p x_2 \tag{18}$$

$$Y'' + \beta_0 x_2'' + \mu_0 Y' + \omega_0^2 Y + \gamma_0 U + G_0 = \beta_2 x_1'^2 + \theta\left(1 + \Theta\left|x_1\right|\right)\delta_3 V_p \tag{19}$$

$$x_3'' + K_3\left(x_3 - x_1\right)^3 + H_3\left(x_3' - x_1'\right) = 0 \tag{20}$$

$$U' + \gamma_1 U - \gamma_2 Y' = E_0 \cos\Omega\tau \tag{21}$$

$$V_p' - \theta\left(1 + \Theta\left|x_1\right|\right)\left(\delta_4 x_1 + \delta_5 Y + \delta_6 x_2^2\right) + \delta_4 V_p = 0 \tag{22}$$

where the dimensionless parameters are

$$x_1 = \frac{q_1}{h} \quad x_2 = \frac{q_2}{L} \quad x_3 = \frac{q_3}{h} \quad V = \frac{Q_p}{Q_0} \quad \tau = \omega_1 t \quad \omega_1 = \sqrt{\frac{2\left(k_c - mgA\right)}{2m + M}} \quad d\left(x_1\right) = \frac{h}{q_0} d\left(q_1\right)$$

$$U = \frac{I}{I_0} \quad Y = \frac{S}{L} \quad \mu_1 = \frac{c_1}{\left(2m + M\right)\omega_1} \quad \alpha_1 = \frac{Ak_b L}{\left(2m + M\right)\omega_1^2} \quad \beta_1 = \frac{2Aml}{2m + M} \quad K_1 = \frac{k_n h^2}{\left(2m + M\right)\omega_1^2}$$

$$H_1 = \frac{H}{\left(2m + M\right)\omega_1^2} \quad \mu_2 = \frac{c_2}{M\omega_1} \quad \omega_2 = \frac{1}{\omega_1}\sqrt{\frac{k_b}{M}} \quad \alpha_2 = \frac{Ak_b h^2}{2M\omega_1^2 L} \quad G_2 = \frac{g}{\omega_1^2 L} \quad \beta_2 = \frac{2Amh^2}{\left(2m + M + m_0\right)L}$$

$$H_3 = \frac{H}{m_4\omega_1^2} \quad K_3 = \frac{k_n h^2}{m_4\omega_1^2} \quad \beta_0 = \frac{M}{2m + M + m_0} \quad G_0 = \frac{\left(2m + M\right)g}{\left(2m + M + m_0\right)\omega_1^2 L} \quad \omega_0 = \frac{1}{\omega_1}\sqrt{\frac{k_0}{2m + M + m_0}}$$

$$\gamma_0 = \frac{KI_0}{\left(2m + M + m_0\right)} \quad \mu_0 = \frac{c_0}{\left(2m + M + m_0\right)\omega_1} \quad \gamma_1 = \frac{R_0}{L_0\omega_1} \quad \gamma_2 = \frac{KL}{L_0 I_0} \quad E_0 = \frac{e_0}{L_0 I_0 \omega_1}$$

$$\delta_1 = \frac{Q_0^2}{\omega_1^2 h^2\left(2m + M\right)C_p} \quad \delta_2 = \frac{BQ_0^2}{2M\omega_1^2 LC_p} \quad \delta_3 = \frac{Q_0^2}{\omega_1^2 hl\left(2m + M + m_0\right)C_p} \quad \delta_4 = \frac{1}{R_p C_p \omega_1}$$

$$\delta_5 = \frac{L}{R_p C_p \omega_1 h} \quad \delta_6 = \frac{BL^2}{4R_p C_p \omega_1 h} \quad \Omega = \frac{\omega_n}{\omega_1} \tag{23}$$

To calculate the harvested power of the considered system, Eqs. (24) and (25) are given as dimensional and dimensionless harvested power, respectively.

$$P = R\dot{Q}^2 \tag{24}$$

$$P = R_0 V'^2 \tag{25}$$

where $R_0 = R(\omega_1 q_0)^2$.

The average power of the system can be calculated by Eq. (26), as in (Triplett and Quinn, 2009; Iliuk, et al. 2013).

$$P_{avg} = \frac{1}{T}\int_0^T P(\tau)\,d\tau \tag{26}$$

Next, section 3 will show some numerical simulations with and without the NES device, considering the nonlinear piezoelectric contribution fixed in $\Theta = 0$.

3. NUMERICAL SIMULATIONS

The numerical simulations were carried out using the 4th order Runge-Kutta method. The parameters are described in Tab. 1

Table 1 - Dimensional System Parameters (Rocha, 2016)

Parameters	Values	Means			
g [m/s^2]	9.81	Gravity aceleration	k_n [N/m^3]	74	Nonlinear NES Stiffness
M [kg]	2.00	Beam Mass	k_0 [kg/m]	86176	Base Stiffness
m [kg]	0.50	Column Mass	L [m]	0.52	Beam Length
m_4 [kg]	Vary	NES Mass	h [m]	0.36	Column Length
c_1[Ns/m]	1.55	Column Damping	R_p [$k\Omega$]	100	Piezoelectric Resistance
c_2[Ns/m]	3.14	Beam Damping	C_p[μF]	1	Piezoelectric Capacitance
H[Ns/m]	14.7	NES Damping	ω_n [rad/s]	148	Frequency of the shaker
c_0[Ns/m]	534	Base Damping	θ	0.3	Linear Piezoelectric Coefficient
EI [Nm^2]	128	Linear Stiffness	Θ	0	Nonlinear Piezoelectric Coefficient
L_0	[mH]	Inductance of the shaker	e_0[V]	40	External Excitation Amplitude
			R_0 [Ω]	0.3	Electric resistance of the shaker
			K[N/A]	130	Electromagnetic force of the shaker

The parameters of Tab. 1 will be considered as default values, except the of the NES's mass, which will be varied. Next section will show the numerical simulations.

4. NUMERICAL SIMULATIONS AND DISCUSSIONS

In this section, the numerical simulations will be presented analyzing only the portal frame motions and the harvested power from the piezoelectric material. First, the dynamics and energy harvesting of the portal frame without the NES device will be presented to be compared when the NES is coupled. Such dynamical analyses will be through time histories of displacements, Poincaré maps and phase planes.

Afterwards, NES will be coupled and the dynamics and energy harvesting of the portal frame is again analyzed by using parametric variation of the NES, bifurcation diagram and Lyapunov's exponent.

4.1 Dynamics and energy harvesting of the portal frame without NES

In this subsection, the dynamical behaviour of the system without NES will be presented.

Figures 2a and 2b show the time histories of horizontal (in red) and vertical (in black) displacements and harvested power, respectively, without the NES coupling. Due to the resonant conditions, saturation occurred in vertical displacement, partial-transferring its vibration energy and the exceeding vibration due to external excitation to the horizontal displacement. As the amplitude of vibration of horizontal motion became higher than the vertical one, the energy harvesting became possible. It is noted in Fig. 2b that the average harvested power in this configuration is 80.99 amount of power.

Figures 2c and 2d shows the phase planes (in black) and Poincaré map (in red) of the horizontal and vertical motions, respectively. The behaviour of the system showed to be periodic, i.e., the best behaviour to harvest energy using a piezoelectric material.

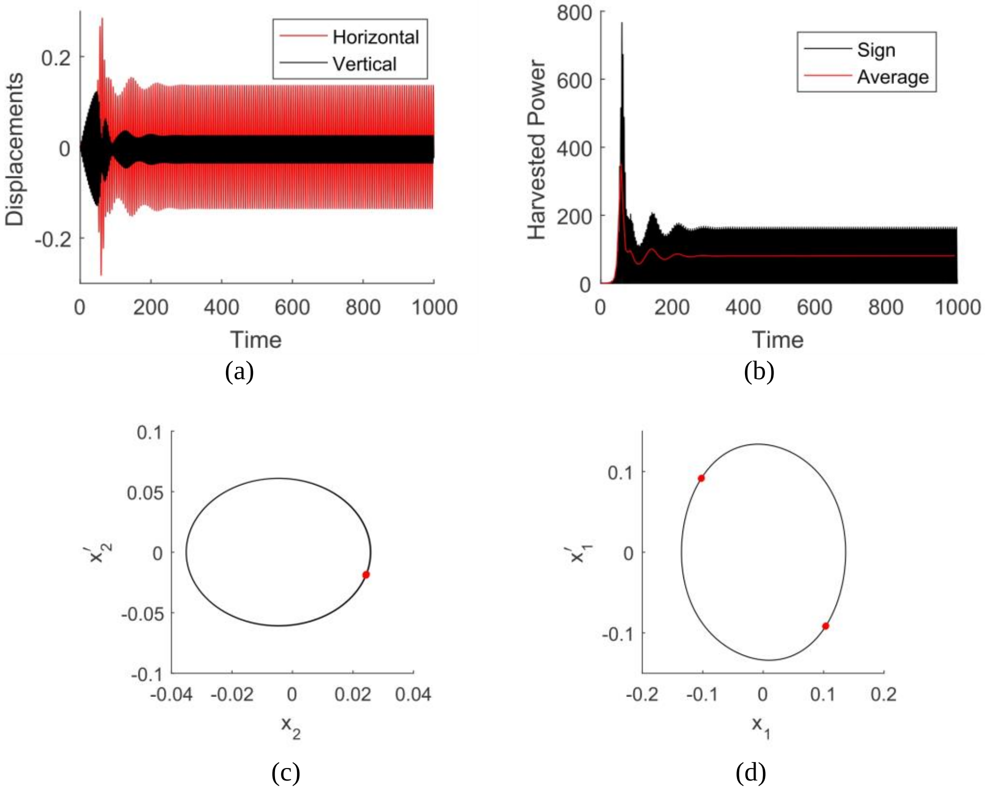

Figure 2. Time histories of (a) horizontal (in red) and vertical (in black) displacements and (b) harvested power; Phase planes (in black) and Poincare maps (red dots) of (c) horizontal motion and (d) vertical motion; without NES

Next, the analysis of the system considering NES will be carried out in the next subsection.

4.2 Dynamics and energy harvesting of the portal frame with NES

Considering the mass of the NES as a powerful tuner due to its direct influence in the behaviour of the system some variations considering the interval of $0 \leq m_4 \leq 1.0$ [kg] were carried out.

Figures 3a and 3b show a parametric variation and a bifurcation diagram of m_4 considering the interval $0 \leq m_4 \leq 1.0$ [kg]. It is observed that, with the hysteretic damping the system had irregular motions that may be quasiperiodic or periodic behaviours. However, even with irregular motion, the harvested power almost doubled beginning from 80.99 up to 161.57 (m_4 = 0.502kg).

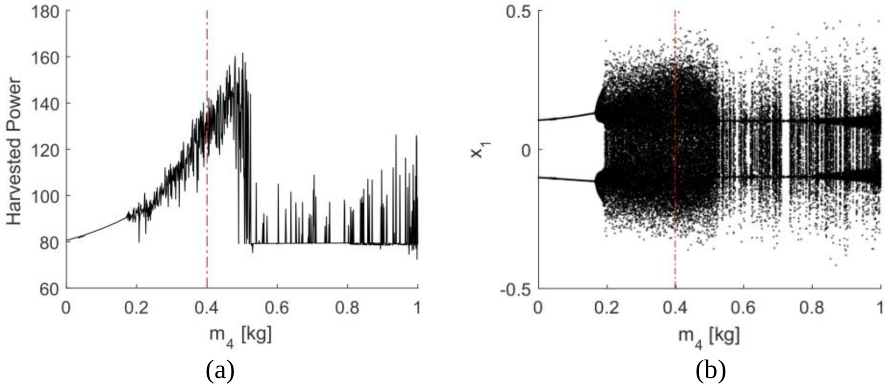

Figure 3 – (a) Parametrical variations of m_4 related to the average harvested power; (b) Bifurcation diagram of m_4

Due to the high amount of power in the irregular, it is important to study what kind of behaviour is the irregular motion. Therefore, choosing the mass of the NES as m_4 = 0.4kg, which is a value in the middle of the irregular motions, a deep analysis of the behaviour of the system is performed.

Figure 4a show the time history of horizontal (in red) and vertical (in black) displacements. It is possible to observe the irregular motion. Even with the irregular motion the average harvested power is 132.00 (Fig. 4b), approximately, whose value is higher than the value without the NES (80.99). Figures 4c, 4d, and 4e show the Lyapunov's exponent analysis, and Poincaré maps of horizontal (Fig. 4d) and vertical (Fig. 4e) motions, respectively. As Lyapunov's exponents showed two positives values, and the Poincaré maps showed infinite cycles, the behaviour of the system showed to be chaotic.

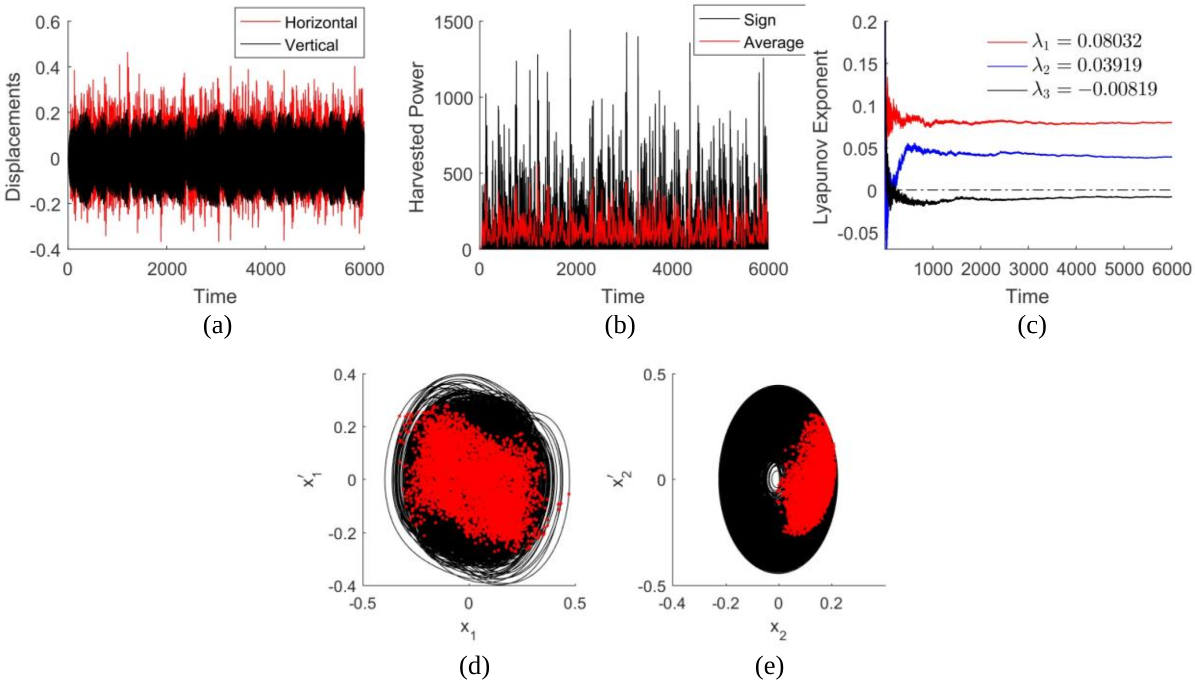

Figure 4. Time histories of (a) horizontal (in red) and vertical (in black) displacements and (b) harvested power; (c) Lyapunov's exponent; phase planes (in black) and Poincaré maps (red dots) of (d) horizontal motion and (e) vertical motion; considering m_4 = 0.4kg

5. CONCLUSIONS

In this work, the dynamics and energy harvesting of a portal frame platform with a NES device, considering a hysteretic damping, through a piezoelectric material, were analyzed.

Due to the hysteretic damping of the NES, the system showed chaotic behaviours with a higher amount of power. Moreover, the use of these kind of passive control strategies eliminates the use of active or semi-active controllers which is necessary to spend some energy, what is unfeasible to energy harvesting systems.

The piezoelectric energy harvesting is usually maintained with periodic behaviour due to the complexity of electric components to process the signal of chaotic behaviour. However, as the NES is a control strategy and improved the energy harvesting, it changed the behaviour of the system, and showed that it is worth to study the energy harvesting from chaotic behaviours.

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7. REFERENCES

Avanço, R. H., Navarro, H. A., Brasil, R. M., Balthazar, J. M., Bueno, A. M., and Tusset, A. M. (2015). Statements on nonlinear dynamics behavior of a pendulum, excited by a crank-shaft-slider mechanism. Meccanica, pages 1-20.

Balthazar, J. M., Felix, J. L. P., Brasil, R. M. L. R. d. (2003). On nonlinear dynamics and control of a particular portal frame foundation model, excited by a non-ideal motor. In Materials science forum, volume 440, pages 371{380. Trans Tech Publ.

Balthazar, J. M., Rocha, R. T., Brasil, R. M. F. L., Tusset, A. M., de Pontes, B. R., & Silveira, M. (2014, August). Mode Saturation, Mode Coupling and Energy Harvesting From Ambient Vibration in a Portal Frame Structure. InASME

2014 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference (pp. V008T11A044-V008T11A044). American Society of Mechanical Engineers.

Crawley, E. F., & Anderson, E. H. (1990). Detailed models of piezoceramic actuation of beams. Journal of Intelligent Material Systems and Structures,1(1), 4-25.

Daqaq, M. F., Masana, R., Erturk, A., & Quinn, D. D. (2014). On the role of nonlinearities in vibratory energy harvesting: a critical review and discussion. Applied Mechanics Reviews, 66(4), 040801.

DuToit, N. E., & Wardle, B. L. (2007). Experimental verification of models for microfabricated piezoelectric vibration energy harvesters. AIAA journal, 45(5), 1126-1137.

Felix, J. L. P., Balthazar, J. M., and Brasil, R. M. L. R. F. (2005). On saturation control of a non-ideal vibrating portal frame foundation type shear-building. Journal of Vibration and Control, 11(1):121-136.

Felix, J., Silva, E., Balthazar, J., Tusset, A., Bueno, A., Brasil, R., and Belhaq, M. (2014). On nonlinear dynamics and control of a robotic arm with chaos. In Csndd 2014-International Conference On Structural Nonlinear Dynamics And Diagnosis, pages 1-6. EDP Sciences.

Felix, J. L., Bianchin, R. P., Almeida, A., Balthazar, J. M., Rocha, R. T., & Brasil, R. M. (2016). On energy Transfer between Vibration Modes under Frequency-Varying Excitations for Energy Harvesting. Applied Mechanics & Materials, 849.

Gendelman, O., Manevitch, L. I., Vakakis, A. F., & M'closkey, R. (2001). Energy pumping in nonlinear mechanical oscillators: Part I—Dynamics of the underlying Hamiltonian systems. *Journal of Applied Mechanics*, *68*(1), 34-41.

Golnaraghi, M. F. (1991). Vibration suppression of exible structures using internal resonance. Mechanics Research Communications, 18(2-3):135-143.

Iliuk, I., Balthazar, J. M., Tusset, A. M., Piqueira, J. R. C., de Pontes, B. R., Felix, J. L. P., & Bueno, Á. M. (2013). A non-ideal portal frame energy harvester controlled using a pendulum. The European physical journal. Special topics, 222(7), 1575-1586.

Iliuk, I., Balthazar, J. M., Tusset, A. M., Piqueira, J. R., de Pontes, B. R., Felix, J. L., & Bueno, Á. M. (2013). Application of passive control to energy harvester efficiency using a nonideal portal frame structural support system. Journal of Intelligent Material Systems and Structures, 1045389X13500570.

Iliuk, I., Brasil, R. M. L. R. F., Balthazar, J. M., Tusset, A. M., Piccirillo, V., & Piqueira, J. R. C. (2014). Potential Application in Energy Harvesting of Intermodal Energy Exchange in a Frame: FEM Analysis. International Journal of Structural Stability and Dynamics.

Jalili, N. (2009). Piezoelectric-based vibration control: from macro to micro/nano scale systems. Springer Science & Business Media.

Lee, Y., Frank Pai, P., and Feng, Z. (2008). Nonlinear complex response of a parametrically excited tuning fork. Mechanical Systems and Signal Processing, 22(5):1146-1156.

Mankala, R. and Quinn, D. D. (2004). Resonant dynamics and saturation in a coupled system with quadratic nonlinearities.

Mook, D. T., Plaut, R. H., and HaQuang, N. (1985). The influence of an internal resonance on non-linear structural vibrations under subharmonic resonance conditions. Journal of Sound and Vibration, 102(4):473-492.

Nayfeh, A. H. (2000). Nonlinear interactions. Wiley.

Nayfeh, A. H. and Mook, D. T. (2008). Nonlinear oscillations. John Wiley & Sons.

Oueini, S. S., Nayfeh, A. H., and Golnaraghi, M. F. (1997). A theoretical and experimental implementation of a control method based on saturation. Nonlinear Dynamics, 13(2):189-202.

Oueini, S. S. (1999). Techniques for controlling structural vibrations.

Pai, P. F.,Wen, B., Naser, A. S., and Schulz, M. J. (1998). Structural vibration control using pzt patches and non-linear phenomena. Journal of Sound and vibration, 215(2):273-296.

Pai, P. F. and Schulz, M. J. (2000). A refined nonlinear vibration absorber. International Journal of Mechanical Sciences, 42(3):537-560.

Preumont, A. (2006). Mechatronics: dynamics of electromechanical and piezoelectric systems (Vol. 136). Springer Science & Business Media.

Priya, S., & Inman, D. J. (Eds.). (2009). Energy harvesting technologies (Vol. 21). New York: Springer.

Quinn, D. D. (2007). Resonant dynamics in strongly nonlinear systems. Nonlinear Dynamics, 49(3):361-373.

Rao, S. S., & Yap, F. F. (2011). *Mechanical vibrations* (Vol. 4). Upper Saddle River: Prentice Hall.

Rocha, R.T. (2016). On saturation phenomenon in energy harvesting based on nonlinear piezoelectric materials coupled to a portal frame foundation with ideal and non-ideal excitations.

Rocha, R. T., Balthazar, J. M., Quinn, D. D., Tusset, A. M., & Felix, J. L. (2016, August). Non-Ideal System With Quadratic Nonlinearities Containing a Two-to-One Internal Resonance. In ASME 2016 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference (pp. V008T10A015-V008T10A015). American Society of Mechanical Engineers.

Rocha, R. T., Balthazar, J. M., Tusset, A. M., Piccirillo, V., & Felix, J. L. P. (2016). Comments on energy harvesting on a 2: 1 internal resonance portal frame support structure, using a nonlinear-energy sink as a passive controller. *International Review of Mechanical Engineering (IREME)*, *10*(3), 147-156.

Rocha, R. T., Balthazar, J. M., Tusset, A. M., Piccirillo, V., & Felix, J. L. P. (2016). Using saturation phenomenon to improve energy harvesting in a portal frame platform with passive control by a pendulum. In *Dynamical Systems: Theoretical and Experimental Analysis* (pp. 319-329). Springer International Publishing.

Rocha, R. T., Balthazar, J. M., Tusset, A. M., Piccirillo, V., & Felix, J. L. (2017). Nonlinear piezoelectric vibration energy harvesting from a portal frame with two-to-one internal resonance. *Meccanica*, *52*(11-12), 2583-2602.

Rocha, R. T., Balthazar, J. M., Tusset, A. M., & Piccirillo, V. (2017). Using passive control by a pendulum in a portal frame platform with piezoelectric energy harvesting. *Journal of Vibration and Control*, 1077546317709387.

Stanton, S. C., McGehee, C. C., & Mann, B. P. (2010). Nonlinear dynamics for broadband energy harvesting: Investigation of a bistable piezoelectric inertial generator. Physica D: Nonlinear Phenomena, 239(10), 640-653.

Stephen, N. G. (2006). On energy harvesting from ambient vibration. Journal of Sound and Vibration, 293(1), 409-425.

Syta, A., Bowen, C. R., Kim, H. A., Rysak, A., & Litak, G. (2015). Experimental analysis of the dynamical response of energy harvesting devices based on bistable laminated plates. Meccanica, 50(8), 1961-1970.

Triplett, A., & Quinn, D. D. (2009). The effect of non-linear piezoelectric coupling on vibration-based energy harvesting. Journal of Intelligent Material Systems and Structures, 20(16), 1959-1967.

Tusset, A. M., Piccirillo, V., Bueno, A. M., Balthazar, J. M., Sado, D., Felix, J. L. P., da Fonseca, R. M. L. R.,. (2015). Chaos control and sensitivity analysis of a double pendulum arm excited by an rlc circuit based nonlinear shaker. Journal of Vibration and Control, page 1077546314564782.

Tusset, A. M., Janzen, F. C., Piccirillo, V., Rocha, R. T., Balthazar, J. M., & Litak, G. (2017). On nonlinear dynamics of a parametrically excited pendulum using both active control and passive rotational (MR) damper. *Journal of Vibration and Control*, 1077546317714882

Twiefel, J., Richter, B., Sattel, T., & Wallaschek, J. (2008). Power output estimation and experimental validation for piezoelectric energy harvesting systems. Journal of Electroceramics, 20(3-4), 203-208.

Vakakis, A. F., Gendelman, O. V., Bergman, L. A., McFarland, D. M., Kerschen, G., & Lee, Y. S. (2008). *Nonlinear targeted energy transfer in mechanical and structural systems* (Vol. 156). Springer Science & Business Media.

Xu, X., Wiercigroch, M., and Cartmell, M. (2005). Rotating orbits of a parametrically excited pendulum. Chaos, Solitons & Fractals, 23(5):1537-1548.

Xu, X., Pavlovskaia, E., Wiercigroch, M., Romeo, F., and Lenci, S. (2007). Dynamic interactions between parametric pendulum and electro-dynamical shaker. ZAMMJournal of Applied Mathematics and Mechanics/Zeitschrift fur Angewandte Mathematik und Mechanik, 87(2):172-186.

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