24th COBEM - 2017

ABCM Associação Brasileira de Engenharia e Ciências Mecânicas

24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-0324
ROBUST CONTROL OF A NON-MINIMUM PHASE MULTIVARIABLE SYSTEM

Daniel Barbuto Rossato
Faculdade de Tecnologia SENAI "Mariano Ferraz"
Rua Jaguaré Mirim, 71 - São Paulo/SP
daniel.rossato@sp.senai.br

Karl Heinz Kienitz
Instituto Tecnológico de Aeronáutica
Praça Marechal Eduardo Gomes, 50 - São José dos Campos/SP
kienitz@ita.br

Abstract. *A non-minimum phase multivariable system is controlled by three techniques to compare performance and robustness: decoupling/PID, LQG/LTR and H_∞. The presence of a real half plane (RHP) zero entails challenges to design a robust control which frequency-based techniques (LQG/LTR and H_∞) are better suited to deal with. However, decoupling the non-minimum phase multivariable system properly yields a robust performance as well.*

Keywords: *robust control, non-minimum phase, LQG/LTR, H_∞, PID.*

1. INTRODUCTION

In addition to engineering motivations, theoretical and technological advances contributed to the development of multivariable control techniques based on state-space models. Celebrated results with engineering application were the Linear Quadratic Regular (LQR), an optimal solution via state feedback and the Kalman filter, the optimal state estimator with a duality relation to the LQR and with a strong robustness to Gaussian noise. However, once used together, a Kalman filter with LQR, the set does not guarantee robustness. Hence, other techniques were developed based on frequency specifications, among them, Linear Quadratic Gaussian with Loop Transfer Recovery (LQG/LTR) control. Notwithstanding, that technique showed some limitations, such as limited flexibility in dealing with model uncertainties and disturbances. During the 1980's, techniques based on the use of H_∞ norm were developed and until now have been further improved (Petersen and Tempo, 2014). In industry, due to the broad use of PID compensators, decoupling techniques have been used in MIMO PID applications. In this paper, these three techniques were applied to a four tank system with adjustable zero and analyzed regarding performance and robustness.

2. NOMINAL MODEL

In this section, the physical process is described as well as the nominal model based on (Johansson, 2000). The process comprises a system with four tanks connected to each other (see Fig. 1). Water levels in tank 1 and tank 2 shall be controlled by two motor-pumps. The opening position of the two three-way valves are adjusted previously. Applying mass balance equations and Bernoulli's Law to the process, the following system of equations is obtained:

$$\begin{aligned}
\frac{dh_1(t)}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1(t)} + \frac{a_3}{A_1}\sqrt{2gh_3(t)} + \frac{\gamma_1 k_1}{A_1}v_1(t) \\
\frac{dh_2(t)}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2(t)} + \frac{a_4}{A_2}\sqrt{2gh_4(t)} + \frac{\gamma_2 k_2}{A_2}v_2(t) \\
\frac{dh_3(t)}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3(t)} + \frac{(1-\gamma_2)k_2}{A_3}v_2(t) \\
\frac{dh_4(t)}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4(t)} + \frac{(1-\gamma_1)k_1}{A_4}v_1(t)
\end{aligned} \tag{1}$$

where A_i is the transversal section of the tank i, a_i is the transversal section of outlet i, h_i is the water level in tank i $(i = 1, ..., 4)$, v_j is the voltage applied to motor-pump j whose corresponding flow is given by $k_j v_j$ $(j = 1, 2)$. The openness of the three-way valves are defined by parameters $\gamma_1, \gamma_2 \in [0, 1]$. The gravity acceleration is g. The level measurement signals are $k_c h_1$ and $k_c h_2$.

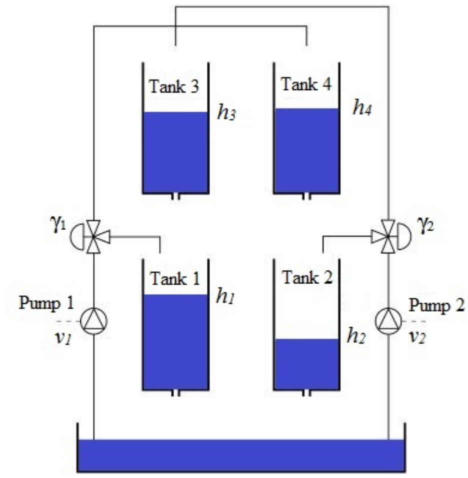

Figure 1. Four tanks system - adapted from (Johansson, 2000)

The nominal values of the parameters are given in Tab. 1.

Table 1. Nominal values.

Parameter	Value	Unit
A_1, A_3	28	$[cm^2]$
A_2, A_4	32	$[cm^2]$
a_1, a_3	0.071	$[cm^2]$
a_2, a_4	0.057	$[cm^2]$
k_1	3.14	$[cm^3/Vs]$
k_2	3.29	$[cm^3/Vs]$
k_c	0.50	$[V/cm]$
g	981	$[cm/s^2]$

For linear control design purpose, the nonlinear model can be linearized using Taylor series approximation around a point given by initial values of the water levels in tank i, h_i^0 and initial values of voltage at motor-pumps j, v_j^0.
Let state variables be $x_i = h_i - h_i^0 (i = 1, ..., 4)$, input variables $u_j = v_j - v_j^0 (j = 1, 2)$ and output variables $y_l = k_c(h_l - h_l^0)(l = 1, 2)$. Space-state equations of the linearized system are:

$$\frac{dx(t)}{dt} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 & \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2 T_4} & 0 \\ 0 & 0 & -\frac{1}{T_3} & 0 & \\ 0 & 0 & 0 & -\frac{1}{T_4} & \end{bmatrix} x(t) + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} x(t) \tag{2}$$

The transfer matrix of that system is G(s):

$$\begin{bmatrix} H_1(s) \\ H_2(s) \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\gamma_1 c_1}{1+sT_1} & \frac{(1-\gamma_2)c_1}{(1+sT_3)(1+sT_1)} \\ \frac{(1-\gamma_1)c_2}{(1+sT_4)(1+sT_2)} & \frac{\gamma_2 c_2}{1+sT_2} \end{bmatrix}}_{G(s)} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} \tag{3}$$

where $c_1 = T_1 k_1 k_c / A_1$ and $c_2 = T_2 k_2 k_c / A_2$, and the time constants $T_i (i = 1, ..., 4)$ are given by:

$$T_i(s) = \frac{A_i}{a_i} \sqrt{\frac{2h_i^0}{g}} \tag{4}$$

Poles and zeros of the multivariable system were obtained using the well known algorithm from (MacFarlane and Karcanias, 1976). The system has four real stable poles $p_i(s) = -1/T_i$ and two real zeros. The system has a right half plane (RHP) zero if $0 < \gamma_1 + \gamma_2 < 1$ (Johansson, 2000). The initial conditions adopted are those given in Tab. 2.

Table 2. Initial conditions.

Parameter	Value	Unit
h_1^0	12.6	$[cm]$
h_2^0	13.0	$[cm]$
h_3^0	4.8	$[cm]$
h_4^0	4.9	$[cm]$
v_1^0	3.15	$[V]$
v_2^0	3.15	$[V]$
γ_1	0.43	—
γ_2	0.34	—

Using nominal values and initial conditions given above, the poles and zeros are:

$$p_1 = -0.0256, p_2 = -0.0158, p_3 = -0.0109, p_4 = -0.0178, z_1 = -0.0562, z_2 = 0.0128 \tag{5}$$

Thus, the process is a 4th order system, stable (all poles are real negatives) and with non-minimum phase due to the RHP zero (z_2). A non-minimum phase system is of special interest and concern because it imposes closed-loop bandwidth (BW) limitation as follows:

$$BW_{max} = z_{RHP} = 0.0128 rad/s \qquad \tau_{min} = \frac{1}{BW_{max}} = 78.12s \tag{6}$$

In order to obtain an adequate sensitivity function for robustness (Skogestad and Postlethwaite, 2005):

$$BW_{max} = \frac{z_{RHP}}{2} = 0.0064 rad/s \qquad \tau_{min} = \frac{1}{BW_{max}} = 156.25s \tag{7}$$

More details about limitations due to RHP zeros are found in (Freudberg and Looze, 1985). Besides, the system is state controllable and state observable which can be verified using a variety of tests (Kienitz, 2012).

3. CONTROL DESIGN

The four tank system with adjustable zero is a plant with non-minimum phase which entails challenges to achieve control specifications. The controlled signals are the water levels in tanks 1 and 2 h=[h1, h2], the manipulated variables are the voltages of motor-pumps $v = [v1, v2]$ and the reference signals are the desired water levels in tanks 1 and 2 $r = [r1, r2]$ as shown in the closed loop diagram at Fig. 2. For that purpose, requirements of design need to be addressed first. $Matlab^{®}$ software was used to compute the controllers parameters.

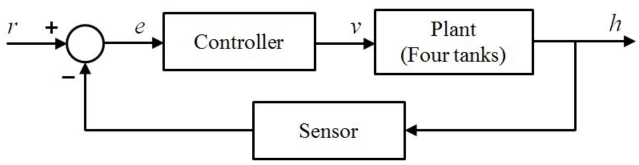

Figure 2. Closed loop diagram

3.1 Requirements

The following performance requirements are established for design purposes. After a step signal at set-point input 1 or at set-point input 2:

- Steady-state error: max. 1%;
- Overshoot/undershoot: max. 15%;
- Settling time (using 2% criterion): max. 320 s;

As the process to be controlled is of zero type, an addition of at least one integrator is necessary to have minimum steady-state error for a step signal. The robustness requirements are:

- Rejection of low frequency disturbances ($< 0.01s^{-1}$) min. 20 dB;
- Rejection of high frequency noise ($> 6000s^{-1}$) min. 40 dB;
- Robustness to parametric uncertainties as specified below;
- Robustness to unstructured uncertainties as specified below;

A 20% variation at the valve openness parameter will be taken into account as parametric uncertainty (or structured uncertainty). Hence, for robustness control design, the intervals to be considered are:

$$0.43 < \gamma_1 < 0.54 \qquad 0.34 < \gamma_2 < 0.41 \tag{8}$$

The dynamic of the motor is not included into the nominal model and will be considered as unstructured uncertainty for robustness control design. For the sake of simplicity, both motor-pumps will be modeled as first order systems:

$$G_m(s) = \frac{1}{T_m s + 1} \tag{9}$$

where T_m is a time constant of $0.1s$.

3.2 Decoupling and PID

The decoupling approach applied uses the matrix K^{pre} to mitigate plant interactions (see Fig. 3).

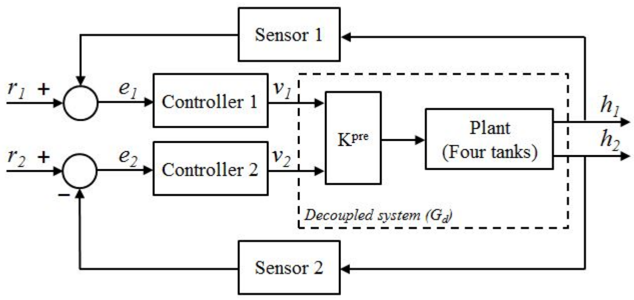

Figure 3. Block diagram of decoupling technique

The Relative Gain Array (RGA) of the transfer matrix G (Bristol, 1966) computed at the crossover frequency is:

$$RGA(G) = G \circ (G^{-1})^T = \begin{bmatrix} -0,12 & 1,12 \\ 1,12 & -0,12 \end{bmatrix} \tag{10}$$

where $\circ$ denotes the Hadamard product (entry-wise product of two matrices of the same dimensions). The RGA is anti-diagonal, so it suggests to pair h_1 to v_2 and h_2 to v_1. Large difference in values at the two diagonals implies success in decoupling the system (Bosgra *et al.*, 2007).

The desired decoupled dynamics was chosen as:

$$G_d = \begin{bmatrix} 0 & \frac{(s-0.0128)}{(s+0.01728)(s+0.02563)} \\ \frac{(s-0.0128)}{(s+0.01728)(s+0.02563)} & 0 \end{bmatrix} \tag{11}$$

That choice is consistent with the RGA analysis. Besides, the RHP zero must be considered to avoid lack of robustness. Finally, in order to have a proper, realizable system, a minimum of two out of four poles must be added.

Matrix K^{pre} was computed as a solution to $K^{pre}.G = G_d$:

$$K^{pre} = \begin{bmatrix} \frac{-2.251(s+0.01094)}{(s+0.05623)(s+0.02563)} & \frac{41.475(s+0.01582)}{(s+0.05623)} \\ \frac{57.215(s+0.01094)}{(s+0.05623)} & \frac{-1.239(s+0.01582)}{(s+0.05623)(s+0.01782)} \end{bmatrix} \tag{12}$$

Proportional+Integrative+Derivative (PID) in the standard (or ideal) realizable form were adopted for both controllers. The dynamics to be controlled by each PID, in the canonical form, based on Eq.(11) is:

$$G_{dpid} = \frac{(s - 0.0128)}{(s + 0.01728)(s + 0.02563)} = \frac{K(-\beta s + 1)}{\tau^2 s^2 + 2\tau\zeta s + 1} = \frac{-28.03(-78.12s + 1)}{2189.6s^2 + 95.14s + 1} \tag{13}$$

The PID gains were adjusted using the Internal Model Control applied to PID controllers (IMC-PID) (Chien and Fruehauf, 1990):

$$K_C = \frac{2\zeta\tau}{K(\beta + \lambda)} = -0.0265 \qquad T_I = 2\zeta\tau = 95.13 \qquad T_D = \frac{\tau}{2\zeta} = 23.01 \tag{14}$$

where λ is the closed-loop time constant (for minimum phase system). To obtain the desired performance it was chosen $\lambda = 50s$.

3.3 Linear Quadratic Gaussian with Loop Transfer Recovery (LQG/LTR)

The LQG/LTR technique is based on loop shaping approach to accomplish performance and robustness requirements. The controller uses a typical structure (in this case: LQG) adjusted with suited gains to recovery the properties of the desired loop dynamics. Although the recovery property of LQG/LTR technique is proved only for minimum phase systems, the limitations inherent to non-minimum phase zero are considered at design stage and recovery will take place in lower frequencies (Athans, 1986).

Given the nominal plant $G_N(s)$ with space state-matrices A,B,C,(D=0), the stationary LQG controller is:

$$K(s) = G(sI - A + BG + HC)^{-1}H, \tag{15}$$

where G and H are matrices to be computed at design stage. Choosing the cost function:

$$J = \int_0^\infty [x^T(t)C^TCx(t) + \rho u^T(t)u(t)]dt, \tag{16}$$

the loop function $G_N(s).K(s)$ converges to the transfer matrix of the Kalman-Bucy filter as $\rho \to 0$ (Doyle and Stein, 1981). It is the so-called Loop Transfer Recovery (LTR) result:

$$G_{KF} = [C(sI - A)^{-1}B][G(sI - A + BG + HC)^{-1}H \quad \overset{\rho \to 0}{\to} \quad C(sI - A)^{-1}H \tag{17}$$

Therefore, the structure of the target feedback loop (TFL) to be shaped is given by: $G_{KF}(s) = C(sI - A)^{-1}H$.

The LQG/LTR is considered a robust control technique due to the fact that uncertainties are considered at design stage. That technique is well suited to systems having multiplicative errors $E_M(s)$ (Cruz, 1996):

$$G_R(s) = G_N(s)[I + E_M(s)] \qquad E_M(s) = [G_R(s) - G_N(s)]G_N^{-1}(s). \tag{18}$$

where G_R is the actual process and G_N is the nominal model. Let $e_m(\omega) = \overline{\sigma}[E_M(j\omega)]$, the frequency response of TFL is limited by $1/e_m(\omega)$(Athans, 1986):

$$\overline{\sigma}(G_{KF}[j\omega)] < 1/e_m(\omega). \tag{19}$$

Given the nominal plant model in state-space form:

$$\dot{x_p}(t) = A_p x_p(t) + B_p u_p(t) \qquad y(t) = C_p x_p(t). \tag{20}$$

An integrator is included at process input to have minimum steady-state error with a zero type system:

$$\dot{x_i}(t) = A_i x_i(t) + B_i u(t) \qquad u_p(t) = C_i x_i(t) + D_i u(t). \tag{21}$$

where

$$A_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad B_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad C_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad D_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{22}$$

The augmented plant $G_a(s)$ becomes:

$$\begin{bmatrix} \dot{x_p}(t) \\ \dot{x_i}(t) \end{bmatrix} = \begin{bmatrix} A_p & B_p C_i \\ 0 & A_i \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_i(t) \end{bmatrix} + \begin{bmatrix} B_p D_i \\ B_i \end{bmatrix} u(t) \qquad y(t) = \begin{bmatrix} C_p & 0 \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_i(t) \end{bmatrix} + 0u(t). \tag{23}$$

Given the augmented plant $G_a(s)$ with states x_a and matrices A_a, B_a, C_a, the Kalman-Bucy filter was computed:

$$\begin{aligned} \dot{x_a}(t) &= A_a x_a(t) + B_a u(t) + Lw(t), \qquad y(t) = C_a x_a(t) + v(t) \\ E[w(t)w(t+\tau)^T] &= I\delta(t), \qquad E[v(t)v(t+\tau)^T] = \mu I \delta(t). \end{aligned} \tag{24}$$

The H matrix is obtained as follows:

$$H = \frac{1}{\mu} M C^T \tag{25}$$

where M is computed using the following matrix Riccati equation:

$$A_a M + M A_a^T - \frac{1}{\mu} M C_a^T C_a M + L L^T = 0. \tag{26}$$

The values of the L matrix and the μ constant are chosen to shape the TFL: $C_a(sI - A_a)^{-1}H$ in such a way that the requirements of performance and robustness are complied. A first suggestion is to make $L = B_a$ and increase the value of μ until obtain the desired loop shape. In this case, $\mu = 10^3$.

At a second stage, the LQR was designed to minimize the cost-function:

$$J = \int_0^\infty (x_a^T(t) C_a^T C^T x(t) + \rho u^T(t) u(t)) dt, \qquad \text{s.t.: } \dot{x_a}(t) = A_a x_a(t) + B_a u(t) \tag{27}$$

The matrix G is obtained as follows:

$$G = \frac{1}{\rho} N^{-1} B_a^T P \tag{28}$$

where P is computed using the following matrix Riccati equation:

$$A_a^T P + P A_a - \frac{1}{\rho} P B_a N^{-1} B_a^T P + C_a^T C_a = 0 \tag{29}$$

The value of the ρ constant was chosen by decreasing its value until the target loop is recovered. In this case, $\rho = 10^{-8}$.

Figure 4 shows the maximum and minimum singular values of the loop transfer function $G_a(s)K(s)$. The multiplicative error is an upper limit for the system achieve robustness and is shown in dashed line. Note that requirements of bandwidth, steady-state error, overshoot, rejection of low frequency disturbances are complied as well as robustness to uncertainties and noise rejection.

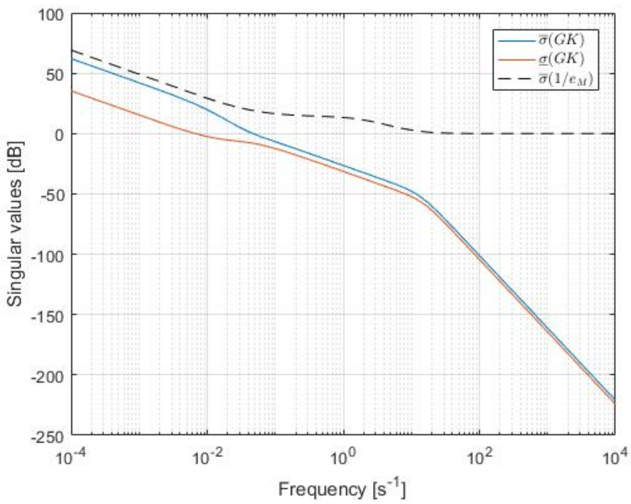

Figure 4. Singular values of the loop transfer function

3.4 Mixed-sensitivity H_∞

The mixed-sensitivity H_∞ controller design is based on the optimization of the H_∞-norm of a set of sensitivity functions designed for compliance with robustness and performance requirements. For robust control design of the four tanks plant, the modeling error was considered as multiplicative uncertainty at the process input:

$$G_R(s) = G_N(s)[I + W_I(s).\Delta_I] \tag{30}$$

Taking into account the actuators dynamics and variation of +20% at parameters γ_1 and γ_2 which are multiplicative factors in the diagonal of the transfer matrix of the process, it yields:

$$\Delta_I = \begin{bmatrix} \frac{-T_m s+0,2}{T_m s+1} & 0 \\ 0 & \frac{-T_m s+0,2}{T_m s+1} \end{bmatrix} \tag{31}$$

Given the performance specifications, the target sensibility function $S(s)$ and the target complementary sensitivity function $T(s)$ for each pair of inputs-outputs:

$$S(s) = \frac{s}{s+0,013}, \qquad T(s) = \frac{0,013}{s+0,013}. \tag{32}$$

The mixed-sensitivity H_∞ technique is based on an algorithm which computes a stable controller K that minimizes the H_∞ norm of $S(s)$ and $T(s)$ with weights W_1, W_2 and W_3 at a suboptimal form:

$$\left\| \begin{matrix} W_1 S \\ W_2 K S \\ W_3 T \end{matrix} \right\|_\infty \leq \gamma. \tag{33}$$

where $S = (I + GK)^{-1}$ and $T = GK(I + GK)^{-1}$. As a first attempt, one can choose $W_1 = S^{-1}$ e $W_3 = T^{-1}$ and modify them to get proper (realizable) functions. After some adjustments based on (Skogestad and Postlethwaite, 2005), the weights were obtained (for each input-output pair):

$$W_1(s) = \frac{s+0,04}{s+0,00004}, \qquad W_2(s) = 1, \qquad W_3(s) = \frac{100000(s+0,04)}{s+4000}. \tag{34}$$

The controller dynamics obtained by minimization of the H_∞ norm is of order 10. The singular values of the Hankel matrix show that it is possible to reduce the controller to order 7 without having major changes in performance. Thus, the controller was reduced using Balanced Stochastic model Truncation (BST) via Schur method yielding the following matrices in state-space form:

$$\begin{aligned}
A &= \begin{bmatrix} -0,187 & 0,828 & -0,877 & 0,083 & -0,300 & -1,309 & 0,425 \\ 0,30 & -1,717 & 1,789 & -0,146 & 0,472 & 2,622 & -1,707 \\ -0,277 & 1,718 & -1,976 & 0,166 & -0,503 & -2,948 & 0,893 \\ 0,046 & -0,242 & 0,269 & -0,024 & 0,078 & 0,404 & -0,049 \\ -0,277 & 1,267 & -1,403 & 0,134 & -0,468 & -2,115 & 0,316 \\ -0,399 & 2,524 & -2,708 & 0,227 & -0,665 & -4,140 & 1,282 \\ 0,570 & -1,768 & 1,740 & -0,124 & 0,798 & 1,791 & -10,90 \end{bmatrix} \\
B &= \begin{bmatrix} -0,514 & 0,758 \\ 1,572 & -2,035 \\ -1,930 & 2,095 \\ 0,147 & -0,208 \\ -0,983 & 1,101 \\ -2,692 & 3,213 \\ 0,307 & -0,365 \end{bmatrix} \\
C &= \begin{bmatrix} 0,621 & -1,845 & 1,902 & -0,160 & 0,893 & 2,869 & -0,688 \\ -0,519 & 1,699 & -2,122 & 0,177 & -1,026 & -3,121 & -0,463 \end{bmatrix} \\
D &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
\end{aligned} \tag{35}$$

The suboptimal minimization can be considered to have reached a good result in terms of optimization as the value of γ obtained is not large ($\gamma = 5,314$). Figure 5 shows that the controller K satisfies the complementary sensibility function T for all frequencies and the condition reached for sensibility function S.

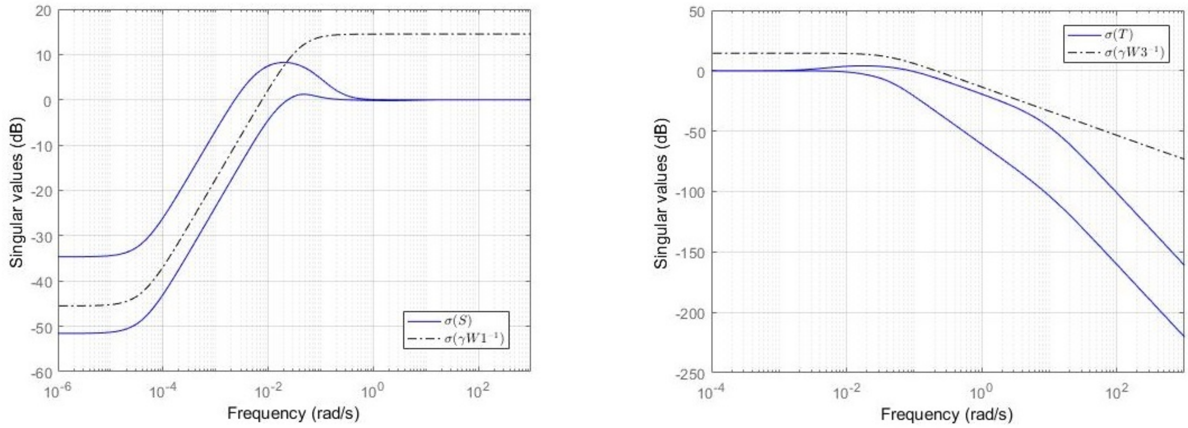

Figure 5. Singular values of the sensibility functions

4. PERFORMANCE ANALYSIS

Simulations of the closed loop system using the nominal model were made for each controller. Figure 6 shows the water level in tank 1 (h_1) and the water level in tank 2 (h_2) as responses to a unitary step at the first reference input (r_1). Figure 7 shows the motor-pump voltage 1 (v_1) and the motor-pump voltage 2 (v_2).

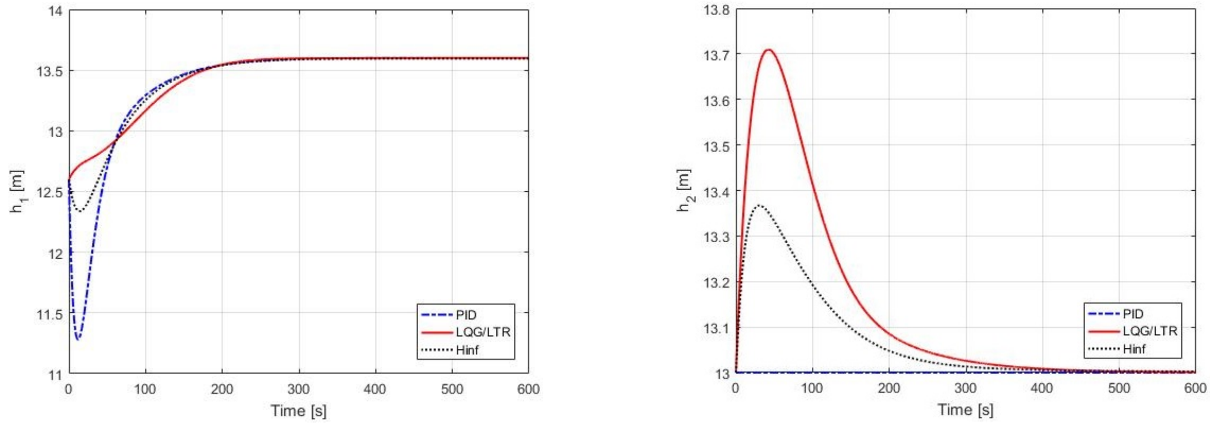

Figure 6. Nominal Performance - Water levels in tank 1 and 2 - Step at input 1

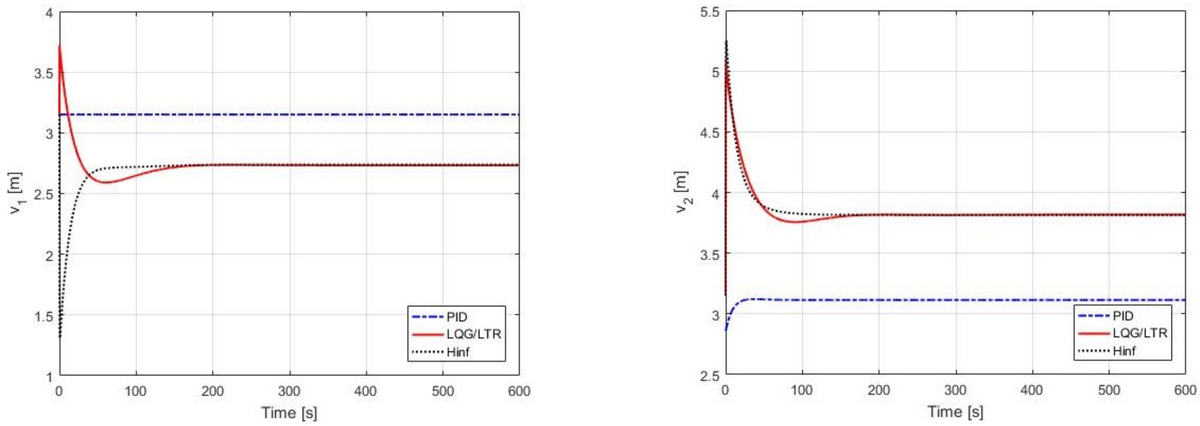

Figure 7. Nominal Performance - Motor-pump voltages - Step at input 1

Performance parameters are compared at Tab. 3 including percentage of overshoot (PO), settling time (t_S), percentage of undershoot (PU) and total energy consumed by motor-pumps (u).

None of the controllers resulted in overshoot in the process variable due to conservative gain adjustment motivated by

Table 3. Nominal Performance to a unitary step at r_1.

Technique.	$PO[\%]$	$t_S[s]$	$PU[\%]$	$u[J]$
Decoup.+PID	0	256	10.5	1.2
LQG/LTR	0	236	0	418.5
Mixed-S H_∞	0	267	2.3	447.4

RHP zero. The same can be said for the settling time which is almost the same for all controllers and complies with the criterion. Due to exact decoupling, the water level in tank 2 remained unchanged while the LQG/LTR controller resulted in the worst variation. Regarding energy consumption, the application of decoupling with PID technique resulted in a much more economical system.

5. ROBUSTNESS ANALYSIS

Next, the system was modified to test robustness. To simulate the actual process, actuator dynamics was added to the nominal model and parameters $\gamma1$ and $\gamma2$ were modified by $+20\%$, worsening the limitation stemming from the RHP zero. Figure 8 shows responses to a step signal at the first input. The control signal for this case is shown in Fig. 9.

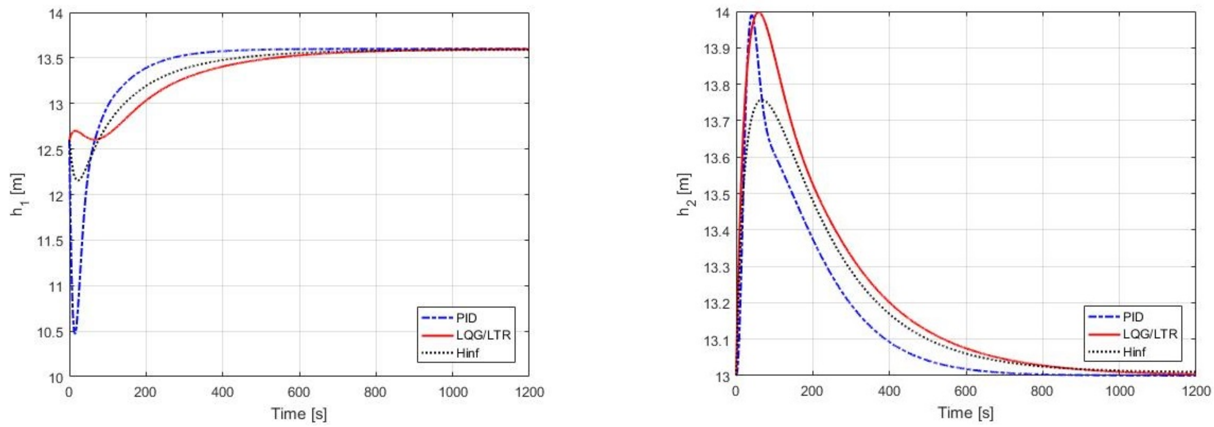

Figure 8. Robust Performance - Water level in tanks 1 and 2 - Step at input 1

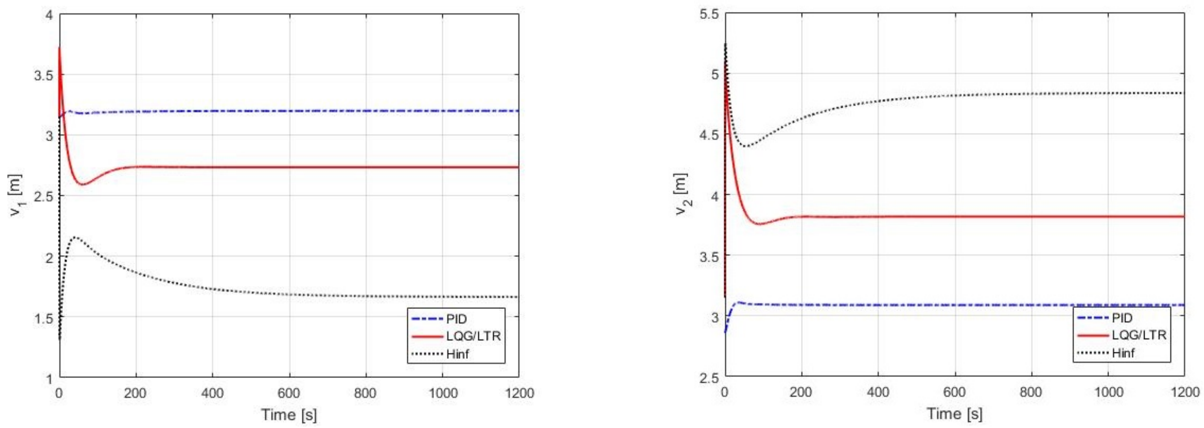

Figure 9. Robust Performance - Motor-pump voltages - Step at input 1

Tab. 4 can be used to compare the performance parameters PO, t_S, PU and u.

None of the controlled systems has reached the setpoint within the time specified (320s). It is a consequence of the modification of the RHP zero leading the actual plant to have a more limited bandwidth. Even so, all controllers maintained system stability. The LQG/LTR technique resulted in a slower settling time, but it was the only that did not yield undershoot (either with the nominal model or with the actual plant). As decoupling did not match anymore, the water level in tank 2 changed its value with the same peak shown by LQG/LTR, only that the response was faster. Again, the decoupling/PID led to the worst undershoot but, on the other hand, saved more energy.

Table 4. Robust Performance to a unitary step at r_1.

Technique.	$PO[\%]$	$t_S[s]$	$PU[\%]$	$u[J]$
Decoup.+PID	0	423	16.9	7.3
LQG/LTR	0	856	0	790.1
Mixed-S H_∞	0	820	3.6	5492.9

6. CONCLUSION

Three techniques were applied to a multivariable non-minimum phase system: decoupling/PID, LQG/LTR and mixed-sensitivity H_∞. The nominal model of the four tanks plant was considered at design stage. In order to compare robust performance, non-modeled dynamics and changes in the RHP zero value were applied. As a result, the decoupling technique using a PID in each loop showed faster responses and economical energy expenditure. But, as a drawback, it showed large undershoot values. The LQG/LTR technique presented zero undershoot but spent more energy to achieve it. Therefore, a tradeoff needs to be taken into account at each application. Finally, the mixed-sensitivity H_∞ technique led the system to the worst performance regarding energy expenditure. Besides, it presented undershoot on its responses. As a future improvement, it is suggested to use the structured H_∞ technique in which the structure of the controller can be chosen (e.g. decoupling/PID) but the design concepts of robust control remain the same.

7. REFERENCES

Athans, M., 1986. “A tutorial on the LQG/LTR method”. *Proceedings of the American Control Conference*, pp. 1289–1296.

Bosgra, O.H., Kwakernaak, H. and Meinsma, G., 2007. *Design Methods for Control Systems*. Dutch Institute of Systems and Control. Lecture notes.

Bristol, E.H., 1966. “On a new measure of interaction for multivariable process control”. *IEEE Transactions on Automatic Control*, Vol. AC-11, pp. 133–134.

Chien, I.L. and Fruehauf, P.S., 1990. “Consider IMC tuning to improve controller performance”. *Chemistry Engineering Progress*, Vol. 86, No. 10, pp. 33–41.

Cruz, J.J., 1996. *Controle Robusto Multivariável*. EDUSP, São Paulo.

Doyle, J.C. and Stein, G., 1981. “Multivariable feedback design: Concepts for a classical/modern synthesis”. *IEEE Transactions on Automatic Control*, Vol. AC-26, pp. 4–16.

Freudberg, J.S. and Looze, D.P., 1985. “Right half plane poles and zeros and design tradeoffs in feedback systems”. *IEEE Trans. on Automatic Control*, Vol. AC-30, pp. 555–561.

Johansson, K.H., 2000. “The quadruple-tank process: A multivariable laboratory with an adjustable zero”. *IEEE Transaction on Control Systems Technology*, Vol. 3, No. 8, pp. 456–465.

Kienitz, K.H., 2012. “Controllability and observability of linear dynamic systems revisited”. *Proc. 20th Mediterranean Conference on Control and Automation (MED 2012)*.

MacFarlane, A.G.J. and Karcanias, N., 1976. “Poles and zeros of linear multivariable systems: A survey of the algebraic, geometric and complex variable theory”. *International Journal of Control*, Vol. 24, No. 1, pp. 33–74.

Petersen, I.R. and Tempo, R., 2014. “Robust control of uncertain systems: Classical results and recent developments”. *Automatica*, Vol. 50, No. 5, pp. 1315–1335.

Skogestad, S. and Postlethwaite, I., 2005. *Multivariable Feedback Control: Analysis and Design*. Wiley, New York, 2nd edition.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.