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DYNAMIC ANALYSIS OF LAMINATE PLATES VIA CARRERA UNIFIED FORMULATION

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Abstract. *The uses of composite materials are increasing over last decades. Composite design offers the advantage of achieving favorable deformation modes through elastic tailoring. In the aerospace industry, interest in the composite structure has been pursued, with great intensity because of the high strength-to-weight and stiffness-to-weight properties of composite materials. In addition, laminated composite structures can enhance certain characteristics of the structure, such as the coupling properties, by proper arrangement of the stacking sequence of the layers. Thus, the uses of numerical methods for design composite structures has become strategic. Among the numerical methods, Finite Element Method is the most used. Regarding the FEM, high order formulation can improve plate elements performance. Therefore, this work applies the Carrera's Unified Formulation (CUF) for laminated plates, implemented as a FORTRAN subroutine linked with commercial finite element software ABAQUSTM, to simulate free vibration of composite laminate. CUF is a hierarchical formulation, which offers a procedure to obtain refined structural theories that account for variable kinematic description. Rectangular plates made of composite material, resin epoxy and carbon fiber, were submitted to experimental vibration tests. The experimental tests were carried out by using an impact hammer, which excited the structure with pulse signal, and accelerometers, which measured the output data. Typical, natural frequencies, mode shapes and Frequency Response Functions (FRFs) are presented for the specimens. The numerical results are compared with experiments showing the improvement of simulation capability using Carrera's high order finite element formulation.*

Keywords: *Carrera's Unified Formulation, finite element, composite materials, free vibration analysis.*

1. INTRODUCTION

Composite materials have been applied in many industries, particularly in automotive, sporting, aerospace and constructions, replacing old materials due to their mechanical properties, such as high stiffness, low density and resistance to corrosion. On the other hand, Finite Element Method (FEM) are very useful to solve engineering problems, when complex geometries or phenomena are involved. Carrera's Unified Formulation (CUF) is a hierarchical formulation that offers a procedure to obtain refined structural theories that account for variable kinematic description (Carrera *et al.*, 2013). Using CUF, it is possible to test almost any plate theory, with very few modifications in the formulation. Thus, much more powerful through the thickness displacements theory is possible, as shown in Fig. 1.

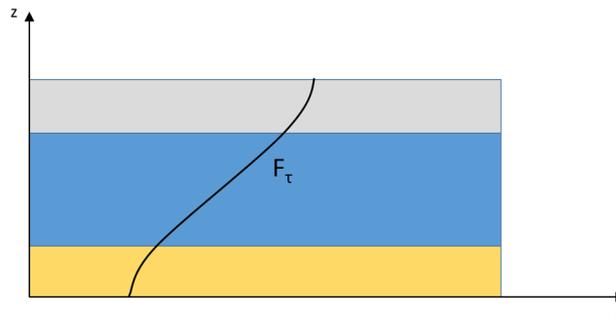


Figure 1. High order displacement field.

Carrera *et al.* (2011(a)) presented the hierarchical finite elements based on the Carrera Unified Formulation for free vibrations analysis of beam with arbitrary section geometries. The displacement components are expanded in terms of the section coordinates, (x, y) , using a set of 1-D generalized displacement variables. N-order Taylor type expansions are employed. The proposed model can detect 3-D effects on the vibration modes as well as predicting shell-type vibration modes in case of thin walled beam sections. Ferreira *et al.* (2011) combined the CUF and a radial basis function collocation technique for predicting the static deformations, free vibrations and buckling behavior of thin and thick cross-ply laminated plates. They developed by the CUF two Zig-Zag theories according to Murakami's Zig-Zag function. Both theories account for through-the-thickness deformations, allowing the analysis of thick plates. The accuracy and efficiency of this collocation technique for static, vibration, and buckling problems are demonstrated through numerical examples.

Rodrigues *et al.* (2011) proposed to use the Murakami's zig-zag theory for the static and vibration analysis of laminated plates, by local collocation with radial basis functions in a finite differences framework. The equations of motion and the boundary conditions are obtained by the CUF, and further interpolated by a local collocation with radial basis functions and finite differences. Carrera *et al.* (2013) used the CUF to perform free-vibrational analyses of rotating structures. These theories are obtained by expanding the unknown displacement variables over the beam section axes by adopting Taylor's expansions of N-order, in which N is a free parameter. Ferreira *et al.* (2014) combined the CUF Formulation and the generalized differential quadrature technique for predicting the static deformations and the free vibration behavior of thin and thick isotropic as well as cross-ply laminated plates. The proposed methodology appears to be able to deal not only with uniform boundary conditions, such as fully clamped or completely simply-supported, but also with mixed external conditions, that can be clamped, supported or free.

Demasi *et al.* (2017) proposed a multi-theory architecture based on the Generalized Unified Formulation (GUF) to provide an accurate prediction of the displacement and stress fields in an efficient computational framework. This feature allows the user to tailor the computational accuracy and cost to the needs of the case under investigation and is inherently well suited for optimization and reliability problems. Keshava Kumar *et al.* (2017) considered the modal analysis of delaminated composite shell structures with double-curvature geometry. The finite element for shell with variable through-the-thickness kinematic is adopted for the analysis. The refined models are grouped in the Unified Formulation by Carrera (CUF) and they permit the distribution of displacements along the thickness of the multilayered shell to be accurately described.

In this context, this work applies the Carrera's Unified Formulation (CUF) for laminated plates, implemented as a FORTRAN subroutine linked with commercial finite element software ABAQUS™, to simulate free vibration of composite laminate. Rectangular plates made of composite material, resin epoxy and carbon fiber, were submitted to experimental vibration tests. The experimental tests were carried out by using an impact hammer, which excited the structure with pulse signal, and accelerometers, which measured the output data. Natural frequencies, mode shapes and Frequency Response Functions (FRFs) are presented for the specimens. The numerical results are compared with experiments showing the improvement of simulation capability using Carrera's high order finite element formulation. This formulation was implemented for the first time as ABAQUS user element subroutine, allowing to use the commercial finite element software pre-processor, solver and post processor, improving the formulation capabilities.

2. CARRERA'S UNIFIED FORMULATION

Most of the high-performance structures can be modeled using shell and/or plates, thus the importance of finite element for these applications are significant. By one hand, classical plate theory is simple and reliable, on the other hand, if there is strong anisotropy due to material mechanical properties, or if the plate is relative thick other theories, as high order shear deformation theories, must be applied to improve the model accuracy.

Regarding Hooke's law for orthotropic materials is:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \\ \sigma_{zz} \end{Bmatrix} = \begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} & \bar{C}_{16} & 0 & 0 & \bar{C}_{13} \\ \bar{C}_{21} & \bar{C}_{22} & \bar{C}_{26} & 0 & 0 & \bar{C}_{23} \\ \bar{C}_{16} & \bar{C}_{26} & \bar{C}_{66} & 0 & 0 & \bar{C}_{36} \\ 0 & 0 & 0 & \bar{C}_{55} & \bar{C}_{45} & 0 \\ 0 & 0 & 0 & \bar{C}_{45} & \bar{C}_{44} & 0 \\ \bar{C}_{13} & \bar{C}_{23} & \bar{C}_{36} & 0 & 0 & \bar{C}_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \\ \varepsilon_{xz} \\ \varepsilon_{yz} \\ \varepsilon_{zz} \end{Bmatrix} \quad (1)$$

Where C_{ij} are the stiffness components.

The strain-displacement relations for in-plane and out-of-plane are:

$$\boldsymbol{\varepsilon}_p = \mathbf{D}_p \mathbf{u} \quad (2)$$

$$\boldsymbol{\varepsilon}_n = (\mathbf{D}_{np} + \mathbf{D}_{nz}) \mathbf{u} \quad (3)$$

Where \mathbf{u} is the displacement components vector ($\mathbf{u} = [\mathbf{u}_x \ \mathbf{u}_y \ \mathbf{u}_z]$). The differential matrix for in-plane strain is:

$$\mathbf{D}_p = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \quad (4)$$

The out-of-plane differential matrices are:

$$\mathbf{D}_{np} = \begin{bmatrix} 0 & 0 & \frac{\partial}{\partial x} \\ 0 & 0 & \frac{\partial}{\partial y} \\ 0 & 0 & 0 \end{bmatrix} \quad (5)$$

$$\mathbf{D}_{nz} = \begin{bmatrix} \frac{\partial}{\partial z} & 0 & 0 \\ 0 & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \end{bmatrix} \quad (6)$$

Unified formulation [Demasi, 2009; Carrera *et al.*, 2011(b); Ferreira *et al.*, 2013; Cinefra *et al.*, 2014; Carrera *et al.*, 2014] gives the possibility to easily implement any plate theories. Any expansion can be used for the displacements. The same expansion is used for the displacements v and w . From the internal virtual strain energy, the fundamental nucleus for this formulation is presented in Eq. (2).

$$\begin{aligned}
K_{xx}^{k\tau sij} &= Z_{pp11}^{k\tau s} \langle N_{i,x} N_{j,x} \rangle_{\Omega} + Z_{pp16}^{k\tau s} \langle N_{i,y} N_{j,x} \rangle_{\Omega} + Z_{pp16}^{k\tau s} \langle N_{i,x} N_{j,y} \rangle_{\Omega} + \\
&\quad + Z_{pp66}^{k\tau s} \langle N_{i,y} N_{j,y} \rangle_{\Omega} + Z_{nn55}^{k\tau s, z} \langle N_i N_j \rangle_{\Omega} \\
K_{xy}^{k\tau sij} &= Z_{pp12}^{k\tau s} \langle N_{i,x} N_{j,y} \rangle_{\Omega} + Z_{pp26}^{k\tau s} \langle N_{i,y} N_{j,y} \rangle_{\Omega} + Z_{pp16}^{k\tau s} \langle N_{i,x} N_{j,x} \rangle_{\Omega} + \\
&\quad + Z_{pp66}^{k\tau s} \langle N_{i,y} N_{j,x} \rangle_{\Omega} + Z_{nn45}^{k\tau s, z} \langle N_i N_j \rangle_{\Omega} \\
K_{xz}^{k\tau sij} &= Z_{pn13}^{k\tau s, z} \langle N_{i,x} N_j \rangle_{\Omega} + Z_{pn36}^{k\tau s, z} \langle N_{i,y} N_j \rangle_{\Omega} + Z_{nn55}^{k\tau s} \langle N_i N_{j,x} \rangle_{\Omega} + \\
&\quad + Z_{nn45}^{k\tau s} \langle N_i N_{j,y} \rangle_{\Omega} \\
K_{yx}^{k\tau sij} &= Z_{pp12}^{k\tau s} \langle N_{i,y} N_{j,x} \rangle_{\Omega} + Z_{pp26}^{k\tau s} \langle N_{i,x} N_{j,x} \rangle_{\Omega} + Z_{pp26}^{k\tau s} \langle N_{i,y} N_{j,y} \rangle_{\Omega} + \\
&\quad + Z_{pp66}^{k\tau s} \langle N_{i,x} N_{j,y} \rangle_{\Omega} + Z_{nn45}^{k\tau s, z} \langle N_i N_j \rangle_{\Omega} \\
K_{yy}^{k\tau sij} &= Z_{pp22}^{k\tau s} \langle N_{i,y} N_{j,y} \rangle_{\Omega} + Z_{pp26}^{k\tau s} \langle N_{i,x} N_{j,y} \rangle_{\Omega} + Z_{pp26}^{k\tau s} \langle N_{i,y} N_{j,x} \rangle_{\Omega} + \\
&\quad + Z_{pp66}^{k\tau s} \langle N_{i,x} N_{j,x} \rangle_{\Omega} + Z_{nn44}^{k\tau s, z} \langle N_i N_j \rangle_{\Omega} \\
K_{yz}^{k\tau sij} &= Z_{pn23}^{k\tau s, z} \langle N_{i,y} N_j \rangle_{\Omega} + Z_{pn36}^{k\tau s, z} \langle N_{i,x} N_j \rangle_{\Omega} + Z_{nn45}^{k\tau s} \langle N_i N_{j,x} \rangle_{\Omega} + \\
&\quad + Z_{nn44}^{k\tau s} \langle N_i N_{j,y} \rangle_{\Omega} \\
K_{zx}^{k\tau sij} &= Z_{nn55}^{k\tau s, z} \langle N_{i,x} N_j \rangle_{\Omega} + Z_{nn45}^{k\tau s, z} \langle N_{i,y} N_j \rangle_{\Omega} + Z_{np13}^{k\tau s} \langle N_i N_{j,x} \rangle_{\Omega} + \\
&\quad + Z_{np36}^{k\tau s} \langle N_i N_{j,y} \rangle_{\Omega} \\
K_{zy}^{k\tau sij} &= Z_{nn45}^{k\tau s, z} \langle N_{i,x} N_j \rangle_{\Omega} + Z_{nn44}^{k\tau s, z} \langle N_{i,y} N_j \rangle_{\Omega} + Z_{np23}^{k\tau s} \langle N_i N_{j,y} \rangle_{\Omega} + \\
&\quad + Z_{np36}^{k\tau s} \langle N_i N_{j,x} \rangle_{\Omega} \\
K_{zz}^{k\tau sij} &= Z_{nn55}^{k\tau s} \langle N_{i,x} N_{j,x} \rangle_{\Omega} + Z_{nn45}^{k\tau s} \langle N_{i,y} N_{j,x} \rangle_{\Omega} + Z_{nn45}^{k\tau s} \langle N_{i,x} N_{j,y} \rangle_{\Omega} + \\
&\quad + Z_{nn44}^{k\tau s} \langle N_{i,y} N_{j,y} \rangle_{\Omega} + Z_{nn33}^{k\tau s, z} \langle N_i N_j \rangle_{\Omega}
\end{aligned} \tag{7}$$

where, $(Z_{pp}^{k\tau s}, Z_{pn}^{k\tau s}, Z_{np}^{k\tau s}, Z_{nn}^{k\tau s}) = (C_{pp}^k, C_{pn}^k, C_{np}^k, C_{nn}^k) E_{\tau s}$,

$(Z_{pn}^{k\tau s, z}, Z_{nn}^{k\tau s, z}, Z_{np}^{k\tau s, z}, Z_{nn}^{k\tau s, z}, Z_{nn}^{k\tau s, z, z}) = (C_{pn}^k E_{\tau s, z}, C_{nn}^k E_{\tau s, z}, C_{np}^k E_{\tau, z, s}, C_{nn}^k E_{\tau, z, s}, C_{nn}^k E_{\tau, z, s, z})$,

$(E_{\tau s}, E_{\tau s, z}, E_{\tau, z, s}, E_{\tau, z, s, z}) = \int_{A_k} (F_{\tau} F_s, F_{\tau} F_{s, z}, F_{\tau, z} F_s, F_{\tau, z} F_{s, z}) dz$, $C_{pp}, C_{pn}, C_{np}, C_{nn}$ are the stiffness coefficients, and F_{τ}

and F_s are the expansions used for high order plate theory.

The symbol $\langle \rangle$ means an integral over the finite element domain.

To proceed with the dynamic analysis, it is necessary to define the mass matrix also in CUF framework. Starting with the virtual variation of the inertial work, using the CUF's notation yields:

$$\delta W_{inertia} = \delta \mathbf{u} \left(\int_V \mathbf{N}^T \mathbf{I} \rho \mathbf{N} dV \right) \ddot{\mathbf{u}} \tag{8}$$

The fundamental nucleus for mass matrix is defined as:

$$m^{ij} = \begin{bmatrix} m_{xx}^{ij} & 0 & 0 \\ 0 & m_{yy}^{ij} & 0 \\ 0 & 0 & m_{zz}^{ij} \end{bmatrix} \tag{9}$$

The diagonal mass matrix has components:

$$m_{xx}^{ij} = \int_z \int_V N_j \rho F_\tau F_s N_i dAdz \quad (10)$$

$$m_{yy}^{ij} = \int_z \int_V N_j \rho F_\tau F_s N_i dAdz \quad (11)$$

$$m_{zz}^{ij} = \int_z \int_V N_j \rho F_\tau F_s N_i dAdz \quad (12)$$

Where, ρ is the composite density, N are the usual Lagrange Finite Element shape functions, F_τ and F_s are the expansion terms.

3. EXPERIMENTAL ANALISE

The vibration experiments were designed to simulate free edge conditions for rectangular plate specimens. Such conditions lead to accurate measurement of natural frequencies; for this reason, they have been widely adopted in the past despite the absence of an explicit solution to the corresponding theoretical problem. The computational analysis presented previously is compared by using data from experiments performed in plates made of CFRP (Carbon Fiber Reinforced Plastic).

The natural frequencies and FRFs were obtained using accelerometers. Figure 2 shows all data acquisition set-ups used in the experiments. The specimen is suspended by using elastomeric wires to simulate “free-free” boundary conditions. The accelerometers and the hammer were connected to a LMS SCADAS Mobile equipment, which was controlled by the Test.Lab software (LMS Test.Lab). The LMS SCADAS Mobile is plug and play equipment, and it has multifunction analogy, digital and timing I/O board for USB bus computers. The excitation for both sets of vibration tests was applied by using an impulse signal through an impact hammer PCB Model 0860C3 (Piezotronics).

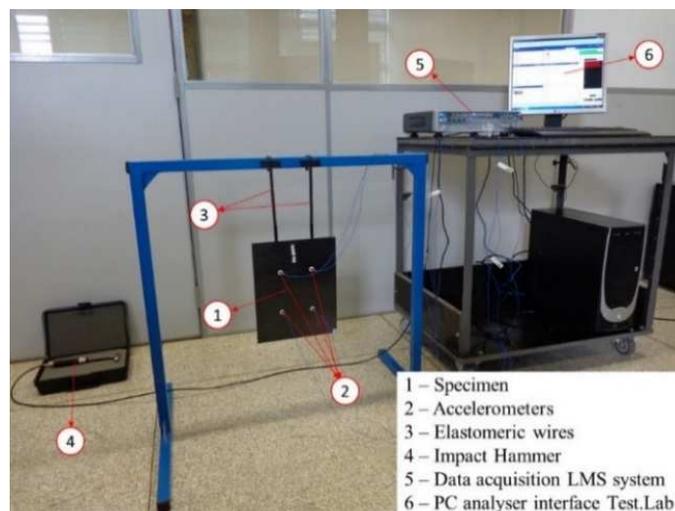


Figure 2. Schematic experimental set for vibration based identification in carbon fiber composite plate with accelerometers (De Medeiros *et al.*, 2017)

In this work, each signal consists of 2048 points and sampling occurred from 0 Hz to 512Hz. It is selected the frequency band of 512Hz. The number of averaging individual time records was selected to be five to reduce the variation effects. This analysis can be evaluated comparing not only the FRFs, but also the coherence values.

4. FINITE ELEMENT MODEL

To verify the unified formulation, a finite element model of free-free boundary condition, and unidirectional eight layers carbon fiber composite plate dynamic response was implemented. The same model is simulated with ABAQUS™ shell element to compare the results (Fig. 3(a)). The carbon fiber composite plate dimension is 304.9 mm × 245.67 mm × 2.28 mm thick and was slightly curved (radius 3036 mm). Table 1 shows the materials properties used for these simulations.

Table 1. Composite plate material properties

	Value	Unit
Young's Modulus longitudinal direction (E_{11})	140.0	GPa
Young's Modulus transversally direction ($E_{22} = E_{33}$)	10.0	GPa
Shear Modulus in plane 1-2 and 1-3 ($G_{12} = G_{13}$)	5.4	GPa
Shear Modulus in plane 2-3 (G_{23})	3.05	GPa
Poisson in plane 1-2 (ν_{12})	0.24	-
Density	1630.0	kg/m ³

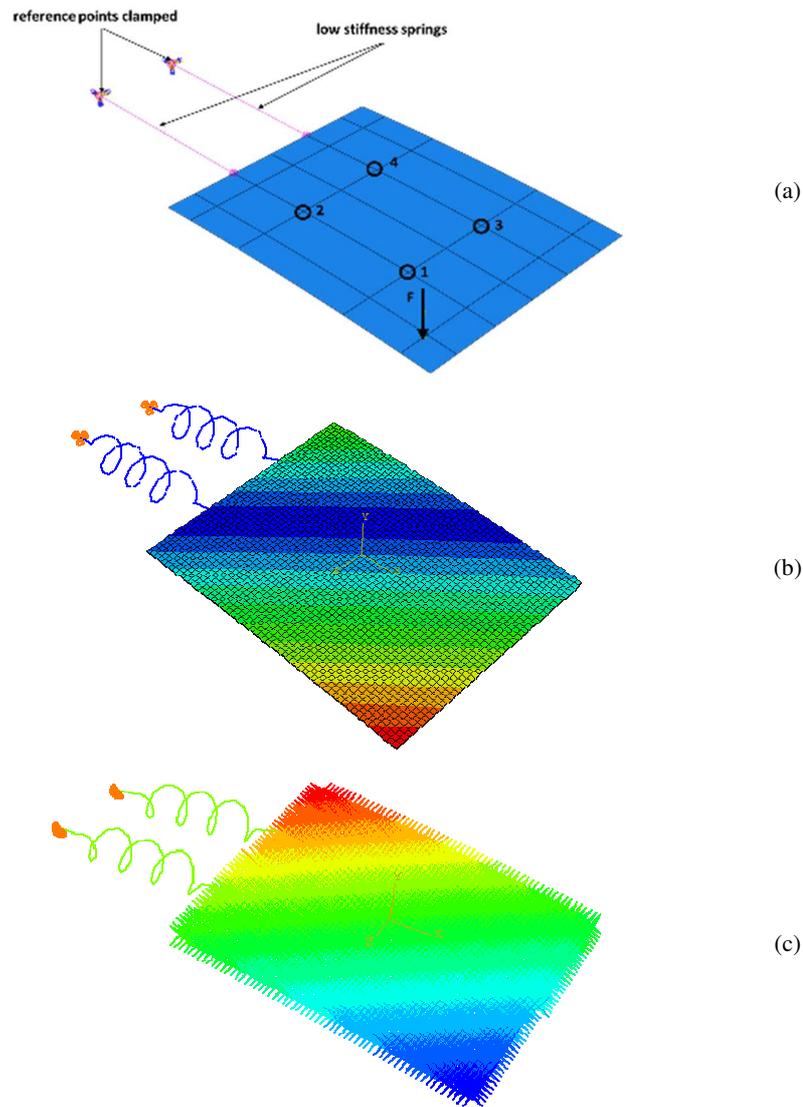


Figure 3. Finite Element Model. (a) Geometry and Boundary Conditions; (b) Out of plane displacement field for Abaqus S8 shell element; (c) Out of plane displacement field for CUF plate element.

Two very low stiffness springs are used to fix the model, allowing the Finite Element Model simulation and to reproducing how experiment would be performed. ABAQUSTM model uses a homogeneous equivalent single layer shell with 8 nodes and 6 degrees of freedom per node (Fig. 3(b) and 3(c)).

The simulations were performed using the ABAQUS implicit dynamic Hilber-Hughes-Taylor time integration algorithm with parameter specified in Hilber *et al.* (1997), ($\alpha = -0.1$, $\beta = 0.3025$ and $\gamma = 0.6$), in order to ensure an adequate dissipation for high frequency modes and, at the same time, not affecting strongly lower frequency modes. On the other hand, ABAQUSTM model simulation uses the default parameters ($\alpha = -0.05$, $\beta = 0.275625$ and $\gamma = 0.55$). The α parameter have a strongly effect on higher frequencies, for example, setting $\alpha = 0.0$, Newmark's integrations schema, the FRF after 350 Hz shows a significantly extra mode, when compared affect the higher frequencies (Fig. 4). To have an

acceptable point, to making the FRF, a fixed step time of 0.0004883 s for a total time of 1 second, resulting 2048 points. The time response was then transformed into frequency response using a Matlab code. For both models, the out of plane acceleration was taken in four different points.

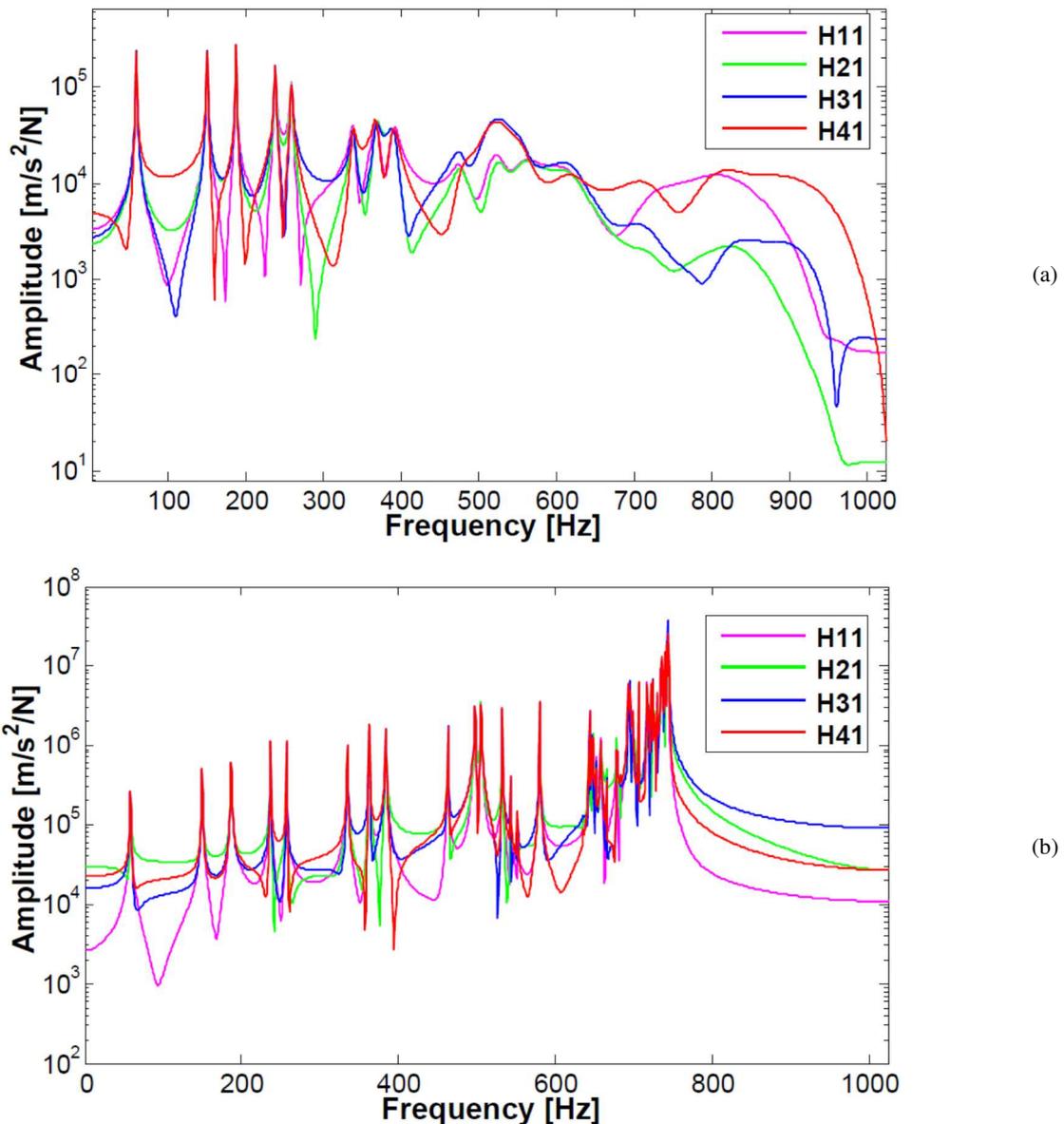


Figure 4. Effect of α parameter on FRF. (a) Using Abaqus parameters; (b) Sting $\alpha=0$ (Newmark).

Three mesh densities were tested, one with approximate element size of 5, 3 and 1. Despite the increase the simulation time, there were no significant differences in the results. Thus, the coarse mesh was used for the analysis. The final model has 2930 elements. A 4-node equivalent single layer fully integrated plate element with 9 and 12 degrees of freedom per node was implemented as a UEL (User Element) Fortran code linked to ABAQUS™. The effect of increasing the expansion was also tested, two expansions was tested, $(1 z z^3)$ and a complete cubic polynomial expansion $(1 z z^2 z^3)$. As expect, the addition of z^2 monomial did not improve the results. The expansion $1 z z^3$ were chosen to avoid shear locking for the out of plane shear components.

5. RESULTS AND DISCUSSION

The dynamic finite element simulation results are show in Fig. 5. Figure 5(a) show the FRF for CUF and Fig. 5(b) shows the FRF for Abaqus S4 element.

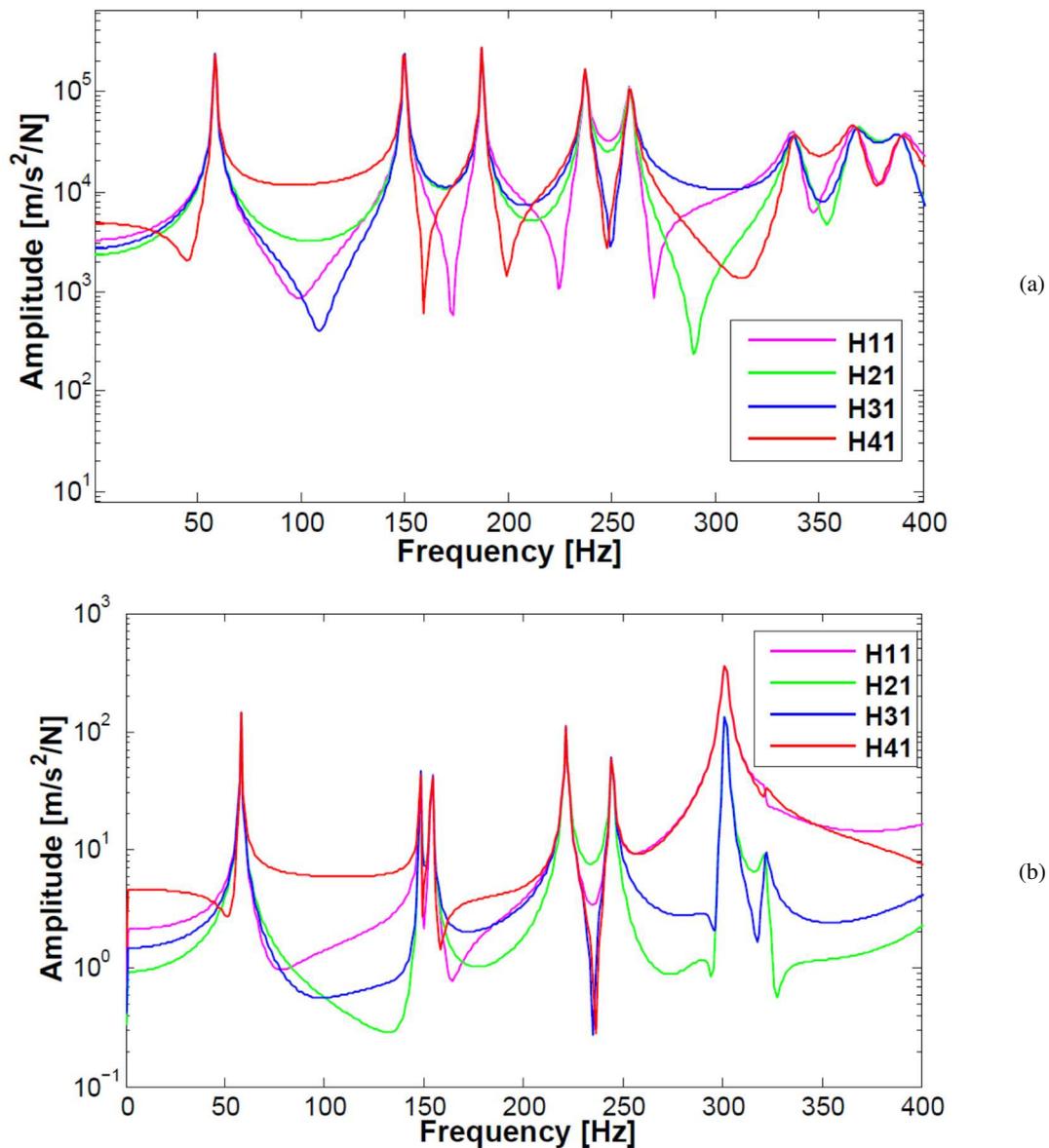


Figure 4. Results. (a) CUF; (b) Abaqus.

Table 2. Comparison between Abaqus S4 and CUF elements.

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
ABAQUS S4	58.06 Hz	148.1 Hz	154.2 Hz	221.2 Hz	244.2 Hz
CUF	58.06 Hz	149.1 Hz	187.2 Hz	237.2 Hz	258.3 Hz
ΔF	0.0 Hz	1.0 Hz	33.0 Hz	16.0 Hz	12.1 Hz
$\frac{(\text{ABAQUS} - \text{CUF})}{(\text{ABAQUS})}$	0.0 %	-0.7 %	-21.4 %	-7.2 %	-5.8 %

Regarding only the first 5 modes, the first mode frequency is the same, what is expected as this frequency are governed by mostly by the mass matrix. Abaqus native element results in a strong couple between mode 2 and mode 3, and CUF response did not result in this strong couple. It may be caused by the differences between finite element formulations, CUF is a plate element, and Abaqus is a shell element.

The differences between mode 4 and 5 are about 7% and 6% respectively. Above those frequencies, the abaqus response differs from CUF response as can be observed in Figure 4 (a) and (b). A more concise analysis must be done comparing the numerical results with experiments.

6. CONCLUSIONS

Carrera's Unified Formulation was implemented as an ABAQUS™ UEL subroutine for the first time. This approach allows changing the plate theory easily. Thus, it is possible to test the most feasible plate theory according to the user needs. Furthermore, as a UEL, it is possible to employ several features of the ABAQUS™ commercial software as, for example, contact algorithms.

The results show small differences between ABAQUS and CUF. These results are significant to continue the development of Structural Health Monitoring systems assisted by computational simulations allowing improving the contemporary methods.

7. ACKNOWLEDGEMENTS

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