24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-0243
DETERMINATION OF AN ARTIFICIAL BASE ACCELEROGRAM COMPATIBLE WITH A NATIONAL CODE FOR SEISMIC ANALYSIS OF A SHEAR BUILDING MODEL

Reyolando M. L. R. F. Brasil
Ronnie Chtcot Brito
Department of Structural Engineering, Polytechnic School, University of São Paulo
Av. Prof. Luciano Gualberto, 380, São Paulo, SP, 05508-010, Brazil
reyolando.brasil@poli.usp.br
ronnie_chtcot@hotmail.com

Abstract. *Although strong seismic events are rare in Brazil, Brazilian structural engineers are frequently involved in such analysis for neighboring Latin American countries. Here, we present a study on the seismic response of a shear building mathematical model of a tall building. Step-by-step numerical time integration via Finite Differences is implemented to solve the ordinary differential equations of motion. Description of seismic base excitation, of random nature, is not, in general, available. Usual National Building Codes for seismic resistant constructions do not provide design base accelerograms to compute the inertia forces at each pavement of the building. The standard information is the so called elastic design spectrum that provides the maximum response acceleration for a one degree of freedom linear damped system for each country. Much research is being developed in order to generate artificial base motion accelerograms compatible with these Code spectra. We presented a proposal for generating artificial base motion accelerograms compatible with the Brazilian National Code for Seismic Resistant Building. A base motion accelerogram generate according to our proposal was applied to a 10-store shear building model.*

Keywords: *structural dynamics, seismic analysis, Brazilian seismic resistant building code.*

1. INTRODUCTION

Although strong seismic events are rare in Brazil, Brazilian structural engineers are frequently involved in such analysis for neighboring Latin American countries. Here, we present a study on the seismic response of a shear building mathematical model of a tall building. Step-by-step numerical time integration via Finite Differences is implemented to solve the ordinary differential equations of motion.

Information on seismic base excitation, of random nature, is not, in general, available. Usual National Building Codes for seismic resistant constructions do not provide design base accelerograms to compute the inertia forces at each level. The standard information is the so called elastic design spectrum that provides the maximum response acceleration for a one degree of freedom linear damped system for each country. Much research is being developed in order to generate artificial base motion accelerograms compatible with these Code spectra. We presented a proposal for generating artificial base motion accelerograms compatible with the Brazilian National Code for Seismic Resistant Building.

A base motion accelerogram generate according to our proposal was applied to a 10-store shear building model

2. THE MATHEMATICAL MODEL

2.1 The structural model

We will consider the 10 stores shear building model of Fig. 1, excited by seismic base motion in just one direction. In such a model, the pavements are considered rigid and the columns do not change their original length. Thus, only horizontal motions are possible, resulting just one degree of freedom per store.

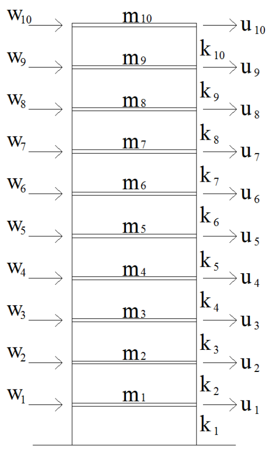

Figure 1. A 10 stores shear building model

In Fig. 1:
u_i = store i horizontal displacement, (m),
m_i = store i lumped mass, (ton),
w_i = inertia force at level i, equal to the mass mi times the base acceleration, (KN),
k_i = store i stiffness, (KN/m).

Dynamic equilibrium at level i is given by Fig. 2.

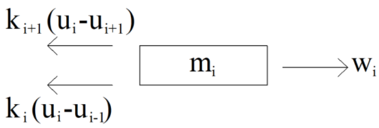

Figure 2. Dynamic equilibrium at level i

Resulting that the equation of motion at level i, is:

$$m_i\ddot{u}_i - k_i u_{i+1} + (k_i + k_{i+1})u_i - k_{i+1}u_{i+1} = w_i \tag{1}$$

If all pavements have the dame mass, the resulting (diagonal) mass matrix is

$$[M] = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m \end{bmatrix} \tag{2}$$

and the (banded) stiffness matrix is:

$$[K]=\begin{bmatrix} 2k & -k & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k & 2k & -k & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k & 2k & -k & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k & 2k & -k & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k & 2k & -k & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k & 2k & -k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k & 2k & -k & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -k & 2k & -k & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k & 2k & -k \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k & 2k \end{bmatrix} \tag{3}$$

The matrix equation of motion is

$$[m]\{\ddot{u}\}+[c]\{\dot{u}\}+[k]\{u\}=\{w\} \tag{4}$$

considering a given viscous damping matrix [c].

2.2 Time step-by-step numerical integration

Ordinary differential Eq. (4) will be numerically integrated in time using the following Central Finite Difference approximations for the velocity and acceleration vectors, at a time t_i, with step h (in seconds:

$$\{\dot{u}_i\}=\frac{\{u_{i+1}\}-\{u_{i-1}\}}{2h} \tag{5}$$

$$\{\ddot{u}_i\}=\frac{\{u_{i+1}\}-2\{u_i\}+\{u_{i-1}\}}{h^2} \tag{6}$$

The resulting recurrence formula for each time step is

$$[\hat{k}]\{u_{i+1}\}=\{\hat{p}_i\} \tag{7}$$

where

$$[\hat{k}]=\frac{1}{h^2}[m]+\frac{1}{2h}[c] \tag{8}$$

$$\{\hat{p}_i\}=\{w_i\}-\left([k]-\frac{2}{h^2}[m]\right)\{u_i\}-\left(\frac{1}{h^2}[m]-\frac{1}{2h}[c]\right)\{u_{i-1}\} \tag{9}$$

Algorithm of Eqs. (7) to (9) is not self-starting, as in the origin (initial time) one does not have the displacements in the previous step. A possible starting scheme is to compute the initial acceleration from the given initial displacements and velocities and consider it to remain constant along the first time step, resulting a uniformly accelerated motion.

3. DETERMINATION OF AN ARTIFICIAL BASE ACCELEROGRAM

Usual National Building Codes for seismic resistant constructions do not provide design base accelerograms to compute the inertia forces w_i. The standard information is the so called elastic design spectrum that provides the maximum response acceleration for a one degree of freedom linear damped system for each country. Much research is being developed in order to generate artificial base motion accelerograms compatible with these Code spectra. We propose one possible solution in the following.

The method is based on the fact that any periodic function can be expressed as a superposition of sine waves modulated by an envelope function defining the temporal shape of the ground acceleration.

$$a(t)=F(t)\sum_{i=1}^{n}A_i\sin(\omega_i t+\theta_i) \tag{10}$$

where

a(t) is the artificial accelerogram sought.
n = it is a given number that increasing improves compatibility of the spectrum to give more "wealth" in the signal,
F(t) = function of predefined deterministic intensity envelope,
θ_i = phase angles values between 0 and 2π,
ω_i = the frequencies ω_i are chosen at regular intervals within a specified range,
A_i = the artificial signal *a(t)* it is compatible with the response spectrum because the A_i are calculated from the stationary function of power spectral density *GÜg(ω)* obtained, in turn, from response spectrum *Sa(T).*

3.1 Calculating amplitudes

It is known that in a given random function with a stationary zero mean process, the variance of the function is equal to the total power spectral density function (Barbat et al.,1994)

$$\sigma_{\ddot{U}g}^2 = \int_0^\infty G_{\ddot{U}g}(\omega)d\omega \tag{11}$$

Moreover, the variance of a sinusoidal function, given by

$$\ddot{y}(t) = A\sin(t) \tag{12}$$

is equal to

$$\sigma_{\ddot{y}}^2 = \frac{1}{2\pi}\int_0^{2\pi} A^2 \sin^2(t)dt = \frac{A^2}{2} \tag{13}$$

Consequently, the total power of the process defined by equation Eq. (10) according to Eqs. (11) and (13) is:

$$\int_0^\infty G_{\ddot{U}g}(\omega)d\omega = \sigma_{\ddot{U}g}^2 = \sum_{i=n}^{n} \frac{A_i^2}{2} \tag{14}$$

approximating the total power as the integral of the area under the curve $G_{\ddot{U}g}(\omega)$,

$$\sum_{i=1}^{n} G_{\ddot{U}g}(\omega_i)\Delta\omega_i = \sum_{i=n}^{n} \frac{A_i^2}{2} \tag{15}$$

this expression that is valid only when the number of sinusoids *n* in the function that defines the process *a(t)* is large. Since the power spectral density represents the relative contribution of each frequency ω_i, one can accept the hypothesis of equal addends in Eq. (15):

$$G_{\ddot{U}g}(\omega_i)\Delta\omega_i \approx \frac{A_i^2}{2} \tag{16}$$

Therefore, the said function can be calculated, amplitudes defined in Eq. (10) as

$$A_i^2 \approx \sqrt{2G_{\ddot{U}g}(\omega_i)\Delta\omega_i} \tag{17}$$

3.2 Standard elastic response spectrum

The Brazilian standard NBR 15421:2006 defines the seismic actions providing maximum expected response of a linear system of one degree of freedom (SDOF) in terms of the response spectrum of pseudo-acceleration. The response spectrum represents the largest absolute value of a selected response parameter (displacement, velocity or acceleration) that a system of one degree of freedom reaches during the earthquake design, varying the natural frequencies and damping ratios.

The Brazilian Code NBR 15421:2006 defines the criteria for obtaining design response spectrum for horizontal accelerations, for a fraction of critical damping of 5% from the horizontal characteristic seismic acceleration and the site soil class. The response spectrum is then defined numerically in three ranges of periods, by the expressions:

$$S_a(T) = \begin{cases} a_{gs0}(18.75\dfrac{C_a}{C_v}+1), & 0 \le T < 0.08\dfrac{C_a}{C_v} \\ 2.5a_{gs0}, & 0.08\dfrac{C_a}{C_v} \le T < 0.4\dfrac{C_a}{C_v} \\ \dfrac{a_{gs1}}{T}, & T \ge 0.4\dfrac{C_a}{C_v} \end{cases} \tag{18}$$

where:
T = natural period of vibration, associated with each of the modes of vibration of the structure,
$S_a(T)$ = is the response spectrum pseudo-accelerations,
C_a = is the seismic soil amplification factor, for the period T= 0.0s,
C_v = is the seismic soil amplification factor, for the period T= 1.0s,
a_{gs0} = is spectral acceleration for the period T=0.0s,
a_{gs1} = is spectral acceleration for the period T=1.0s.

The latter quantities are calculated by:

$$a_{gs0} = C_a a_g \tag{19}$$

$$a_{gs1} = C_v a_g \tag{20}$$

where:
a_g = it is the horizontal seismic acceleration feature for a region, normalized to land Class B (rock), obtained from the national seismic map.

Figure 3 shows the type of design response spectrum, normalized by the acceleration of zero periods (S_a/a_{gs0}) according to the period:

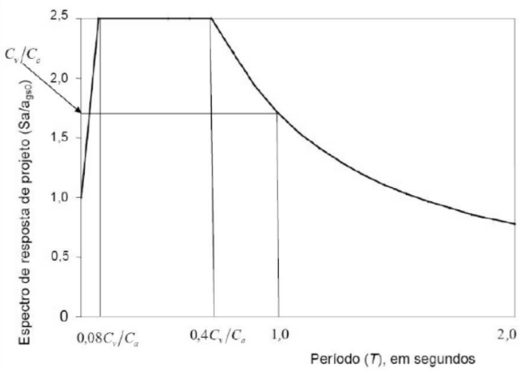

Figure 3. Design response spectrum (ABNT NBR 15421:2006)

The seismic soil amplification factors can be obtained, depending on the terrain in Table 1, linear interpolation used to obtain intermediate values between 0.10g e 0.15g. The categorization of the site soil class is associated with the propagation velocity of shear waves (V_s) in the ground 30 meters higher, from Table 2. It is also allowed the classification, in some cases, from the average results of the SPT.

Table 1. Seismic soil amplification factor (ABNT NBR 15421:2006).

Classe do terreno	*Ca*		*Cv*	
	ag ≤ 0.10g	ag ≤ 0.15g	ag ≤ 0.10g	ag ≤ 0.15g
A	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0
C	1.2	1.2	1.7	1.7
D	1.6	1.5	2.4	2.2
E	2.5	2.1	3.5	3.4

Table 2. Classification of soil (ABNT NBR 15421:2006).

Classe do terreno	Designação da classe do terreno	Propriedades médias para os 30 m superiores do terreno	
		$\bar{V}_s$	$\bar{N}$
A	Rocha sã	$\bar{V}_s \geq 1.500\ m/s$	(não aplicável)
B	Rocha	$1.500\ m/s \geq \bar{V}_s \geq 760\ m/s$	(não aplicável)
C	Rocha alterada ou solo muito rígido	$760\ m/s \geq \bar{V}_s \geq 370\ m/s$	$\bar{N} \geq 50$
D	Solo rígido	$370\ m/s \geq \bar{V}_s \geq 180\ m/s$	$\bar{N} \geq 15$
E	Solo mole	$\bar{V}_s \leq 180\ m/s$	$\bar{N} \leq 15$
	-	Qualquer perfil, incluindo camada com mais de 3 m de argila mole	
F	-	Solo exigindo avaliação especifica, como: 1. Solos vulneráveis à ação sísmica, como solos liquefazíveis, argilas muito sensíveis e solos colapsíveis fracamente cimentados 2. Turfa ou argilas muito orgânicas 3. Argilas muito plásticas; 4. Extratos muito espessos ($\geq$ 35 m) de argila mole ou média	

NBR 15421:2006 allows the engineer to represent the seismic ground motion by artificial accelerograms. However, the code does not define the PSD, requiring it to be compatible with the response spectrum specified and providing conditions for this compatibility is achieved. In particular, a PSD function $G\ddot{U}g(\omega)$, of the ground acceleration is considered compatible with an assigned acceleration RS, *Sa (T),* if a SDOF system with an assigned damping ratio, subjected to accelerogram samples generated from $G\ddot{U}g(\omega)$, experiences an absolute maximum acceleration *Sa (T)* for each value of the natural period T into a time window of the nominal duration of the pseudo-stationary part *Ts* of the earthquake (Baroni *et al*., 2015).

The spectral acceleration can also be expressed as:

$$S_a(\omega,\xi) = \omega^2 \eta_U(\omega,\xi)\sigma_U(\omega,\xi) \tag{21}$$

The peak factor η_U (Vanmarcke, 1972) is defined

$$\eta_U(\omega,\xi) = \sqrt{2\ln\{2N_U(\omega)[1-\exp(-\delta_U^{1,2}(\xi)\sqrt{\pi\ln(2N_U(\omega))})]\}} \tag{22}$$

Despite the system response is not previously known, the parameter $N_U(\omega)$ and the factor of spreading $\delta_U(\xi)$ can be expressed by the following approximate expressions (Kiureghian, 1980):

$$N_U(\omega) = -\frac{T_s}{2\pi}\frac{\omega}{\ln(p)} \tag{23}$$

$$\delta_U(\xi) = \sqrt{1-\frac{1}{1-\xi^2}\left[1-\frac{2}{\pi}\arctan\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)\right]^2} \tag{24}$$

The parameter ξ assumes the value 0.5 when the average value of peak values can be approximated by half the maximum distribution. For ξ=0.05, $\delta_U(\xi)$=0.24561. Once known PSD, the response spectrum can be easily found. However, the inverse problem is not easy because of the high nonlinearity of equation $S_a(\omega,\xi)$. To overcome this problem, an approximate expression for the response variance can be used (Vanmarcke, 1977) to determine the PSD:

$$G_{\ddot{U}g}(\omega) = \frac{\gamma}{\omega_C}\left[\left(\frac{S_a(\omega,\xi)}{\eta_U(\omega,\xi)}\right)^2 - \int_0^{\omega} G_{\ddot{U}g}(\omega)d\omega\right] \tag{25}$$

$$\gamma = \frac{4\xi}{(\pi-4\xi)} \tag{26}$$

3.3 Generation of function of power spectral density (PSD) compatible with the response spectrum standard

Determining the function of power spectral density from a response spectrum has been a considerable effort. The objective is to calculate a function for an unknown signal *a(t)* from the horizontal seismic acceleration. In this paper it is used an approximate analytical method proposed by Barone et al. (2015). This method is compatible with fairly generic forms of response spectrum and can be used for spectral response of various international standards of earthquakes.

In order to define an analytical PSD function, extensive numerical campaign was carried out varying the intensity and shape of designated RS and evaluating the corresponding PSD. It was observed that the method always returned PSD's with the format in Fig. 4 (Barone et al., 2015):

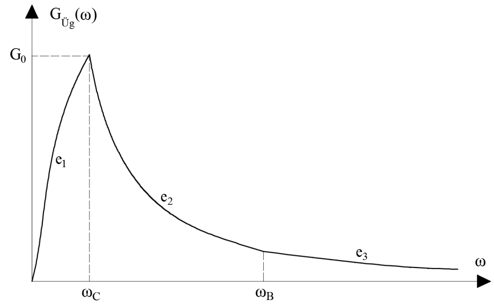

Figure 4. PSD compatible with response spectrum

Therefore, one may describe the PSD as a function with three intervals and a simple mathematical structure and completely defined by only a few parameters determined from the exact equation $G_{\ddot{U}g}(\omega)$, where G_0 pick value of the PSD to $\omega = \omega_C$:

$$G_{\ddot{U}g}(\omega) = \begin{cases} G_0\left(\dfrac{\omega}{\omega_C}\right)^{e1} & 0 \le \omega \le \omega_C \\ G_0\left(\dfrac{\omega}{\omega_C}\right)^{e2} & \omega_C < \omega \le \omega_B \\ G_0\left(\dfrac{\omega_B}{\omega_C}\right)^{e2}\left(\dfrac{\omega}{\omega_B}\right)^{e3} & \omega > \omega_B \end{cases} \tag{27}$$

To determine the exponent e_1, the Eq. (25) is, at first, rewritten for the frequency $\omega=\omega_C$

$$G_0\frac{\omega_C^{e1+1}}{\omega_B^{e1}} = \frac{\gamma}{\omega_C}\left[\left(\frac{2.5a_{gs0}\left(\dfrac{\omega_C}{\omega_B}\right)}{\eta_U(\omega_C)}\right)^2 - \int_0^{\omega_C} G_{\ddot{U}g}(\omega)d\omega\right] \tag{28}$$

and substituting the Eq. (27) into the integral term:

$$G_0\omega_C\left(\frac{\omega_C}{\omega_B}\right)^{e_1+1}\frac{\gamma+1}{\gamma} = \left(\frac{2.5a_{gs0}\left(\dfrac{\omega_C}{\omega_B}\right)}{\eta_U(\omega_C)}\right)^2 \tag{29}$$

Then, following the same reasoning, but considering a new frequency $\omega = \omega_C/\rho(\rho>1)$, the following expression is obtained:

$$G_0\omega_C\left(\frac{\omega_C}{\omega_B}\right)^{e_1+1}\frac{\gamma+1}{\gamma}=\left(\frac{2.5a_{gs0}\left(\frac{\omega_C}{\omega_B}\right)}{\eta_U\left(\frac{\omega_C}{\rho}\right)}\right)^2\rho^{-1} \tag{30}$$

Comparison of Eqs. (29) and (30) and considering the limit ρ=1, it can be demonstrated that the exponent e_1 can be expressed in closed-form as:

$$e_1=2-L(\omega_D) \tag{31}$$

where the function L(ω) is defined as:

$$L(\omega)=2\omega\frac{d(\log(\eta_U(\omega)))}{d\omega} \tag{32}$$

The evaluation of the closed-form expressions for the other parameters is based on the same concepts, but considering points on the other three branches of the PSD. After some algebra, the following set of parameters is obtained:

$$e_1=1-L(\omega_C) \tag{33}$$

$$e_2=-1-\gamma-\beta_2L(\omega_C) \tag{34}$$

$$e_3=-1-\gamma-\beta_3(L(\omega_B)+1.2) \tag{35}$$

$$G_0=\frac{\gamma}{\beta_2\omega_C}\left(\frac{2.5a_{gs0}}{\eta_U^2(\omega_C)}\right)^2 \tag{36}$$

with the following positions:

$$\beta_2=\frac{\gamma+e_1+1}{e_1+1} \tag{37}$$

$$\beta_3=\left(\frac{\omega_C}{\omega_B}\right)^{e_2+1}\beta_2+\left(1-\left(\frac{\omega_C}{\omega_B}\right)^{e_2+1}\right)\frac{\gamma+e_2+1}{e_2+1} \tag{38}$$

The resulting artificial accelerogram is presented in Fig.5,

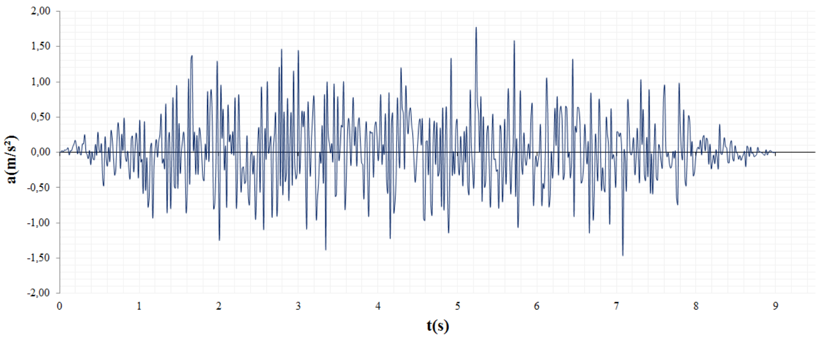

Figure 5. Artificial accelerogram

4. SIMULATION RESULTS AND DISCUSSION

Next, using a MATLAB program based on Finite Difference approximations, we simulate the response horizontal displacements of the top store of the building as function of time, in Fig,6.

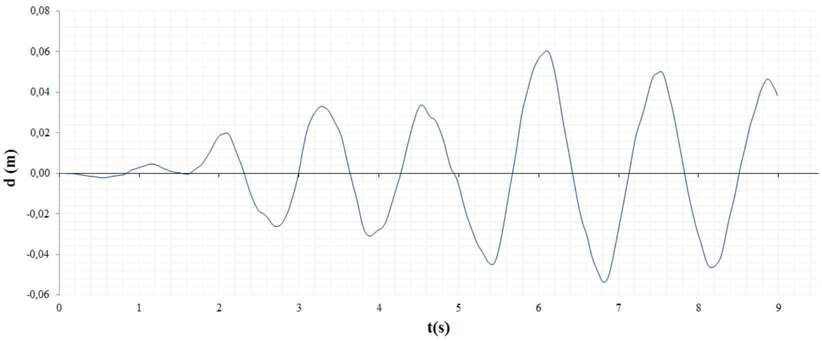

Figure 6. Top store displacements time history

Top pavement maximum horizontal displacement was found to be 6 cm with an approximated 1.5 s period of vibrations, corresponding to a frequency of 4.2 rad/s. Thus, maximum acceleration at top pavement can be estimated to be around 0.7 m/s².

5. CONCLUSIONS

We presented a proposal for generating artificial base motion accelerograms compatible with the Brazilian National Code for Seismic Resistant Building.

A base motion accelerogram generated according to our proposal was applied to a 10-store shear building model and the resulting equations of motion were step-by-step time integrated using a Central Finite Difference algorithm.

6. ACKNOWLEDGEMENTS

The authors acknowledge support by CNPq, CAPES and FAPESP, all Brazilian research funding agencies.

7. REFERENCES

ABNT, NBR 15421:2006 – Projeto de estruturas resistentes a sismos – Procedimento. Rio de Janeiro. 2006

Barbat, A. H., Orosco, L.,Hurtado, J. E. y Galindo, M.,1994. Definición de la acción sísmica, Centro internacional de Métodos Numéricos en Ingeniería, CIMNE, monografía IS-10, Barcelona,1994.

Baroni, G., Iacono, F., Navarra, G., Palmeri, A.,2015. A novel analytical model of power spectral density function coherent with earthquake response spectra. In: Papadrakakis, M., Papadopoulos, V. and Stefanou, G. (eds). Proceedings of UNCECOMP 2015: the 1st ECCOMAS Thematic Conference on Uncertainty Quantification in Computational Sciences and Engineering, Crete - Greece.

Brasil, R. M. L. R. F, 1995. O Fenômeno de Localização de Modos em Dinâmica e Estabilidade de Estruturas Modulares de Comportamento Linear ou Não-Linear, Tese de Livre Docência, Escola Politécnica da USP, São Paulo, 1996.

Brasil, R. M. L. R. F. e Da Silva, M. A, 2015. Introdução à dinâmica das Estruturas. 2ª edição. Ed. Blucher, São Paulo

Filho, F. V. 1975. Análise Matricial de Estruturas. Ed. A. Neves, Rio de Janeiro, 1975.

Kiureghian, A.,1980. A Structural response to stationary oscillation. Journal of Engineering Mechanics, 106(6), pp.1195-1213.

Ray W. Clough, J. P, 1995. Dynamics of Structures - Computers & Structures, New York.

Singiresu S. Rao, 2003. Mechanical Vibrations - 4ª edição. Ed. Pierson, São Paulo.

Vanmarcke. E.H.,1972. Properties of spectral moments with applications to random vibration. Journal of Engineering Mechanics, 98(2), pp. 425-446.

Vanmarcke, E.H.,1977. Structural response to earthquakes, C. Lominitz and E. Rosenblueth, eds., Seismic risk and engineering decisions, pp.287-337, Elsevier, New York.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.