24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-0623
POSTBUCKLING OPTIMIZATION OF LAMINATED COMPOSITE RECTANGULAR PLATES AND STIFFENED PANEL UNDER COMPRESSION USING FIREFLY ALGORITHM

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Abstract. *Laminated composite materials have been widely used in applications requiring high strength and stiffness and low weight. Structures made of these materials may also contain reinforcements (like stringers) which further improve their load carrying capacity. One of the criteria for designing these structures relies on the postbuckling regime, where a nonlinear stability analysis is very important for the determination of their behavior. As a way to improve their efficiency, it is also sought to optimize the stacking sequence of their plies. Within this context, this work proposes optimizing laminated composite structures in the postbuckling regime using a discrete version of the firefly algorithm (DFA). In this paper, the DFA is applied to optimize imperfect rectangular flat plates and a curved panel with five reinforcements. The purpose of the optimization is to find the optimum stacking sequence, using discrete ply angles, that maximizes the load in the postbuckling regime (purely geometric nonlinearity) constrained by a prescribed maximum transverse deflection or by a failure criterion. The implementation of the algorithm was made in Python and connected to a finite element model built in the commercial code Abaqus. The results of the optimization are presented and the performance of DFA is discussed.*

Keywords: *laminated composites, stiffened panel, postbuckling, optimization, firefly algorithm*

1. INTRODUCTION

Composite material is defined by Mendonça (2005) as a set of two or more different material combined in a macroscopic scale, to function as a unit, aiming to obtain a set of properties that none of the individual components presents. In same way Jones (1999) explain that those materials consist of layers of at least two different materials that are bonded together. More specifically, a laminate is a stack of lamina with different orientations of the principal material directions. Lamina is a single layer or ply of unidirectional composite material (Staab, 1999). Those materials are used in many industries applications as automobilist, spatial and aeronautic due to their potential advantages considering that their ultrahigh strength and stiffness fibers that can be tailored to efficiently meet design requirements.

One of the approaches of composite structures is related to postbuckling analysis due to the knowledge of critical postbuckling loads is essential for lightweight structural design. Focusing on it, many different nature-inspired heuristics have been employed to optimize stacking sequence of composite laminated structures. For example, ant colony optimization was used by Aymerich and Serra (2008) and Koide (2010); harmony search was employed in the studies of Lakshmi *et al.* (2015). A relatively new heuristic to solve optimization problems is the firefly algorithm (FA), which is inspired in the behavior of fireflies. Some basic concepts and applications of FA can be found in Yang (2010) and Koide

(2016). Considering the characteristics of fireflies' light irradiation, Yang formulated an algorithm based on the behavior of fireflies. Firefly algorithm (FA), in its discrete form (DFA) (Durkota, 2011) was applied by Koide (2016) to optimize the stacking sequence of imperfect rectangular composite plates aiming to obtain the maximum postbuckling load.

In this paper, a plate with different aspect ratios is studied and the structure is subjected to a uniaxial compression load and the transverse displacement is limited to a maximum value. Also, a stiffened curved panel is optimized for maximum postbuckling load, constrained by the Hashin failure criterion. FA was coded in Python and the response of the structures is obtained through a finite element model (FEM) built in Abaqus (ABAQUS, 2014). The postbuckling responses for the plate and curved panel are presented in "Numerical Results" section and "Conclusions" section presents a summary of the main findings.

2. FIREFLY ALGORITHM

Firefly algorithm is a nature-inspired heuristic designed to solve optimization problems. Fireflies emit certain flashes, lights and rhythmic with some intensity that are used to communicate to each other or used as a protection against predators. These characteristics of real fireflies were used in the idealization of the algorithm. The attractiveness is proportional to the brightness or light intensity and is related to the distance between the fireflies. The luminous intensity is affected or determined by the objective function corresponding value. Local or global search optimization takes place by the movement o fireflies depending on the light intensity.

2.1 Firefly behavior

Fireflies produce small and rhythmical lights (flashes). The pattern of lights is often unique to each particular species. A process of bioluminescence from a biochemical process produces the light flash. The luminescent organisms produce only slow modulated flashes or glows. However, adult fireflies, in many species, are able to control their bioluminescence to emit intense flashes and discrete (Fister *et al*., 2013). It is considered as functions of intermittent emission of light or flashes to attracting partners, and the potential prey. Additionally, it may be a protective mechanism against predators of fireflies. Collective decisions are highly connected with the behavior of the emission lights of the flashes that serve as the main basis of development of fireflies' algorithm (Fister *et al*., 2013). The algorithm considers each firefly observes its reference position when it tries to move to a higher source of strength greater than its own.

2.2 Firefly algorithm formulation

Light intensity, I, is related to the inverse of the square of the distance r, i.e., $I \propto 1/r^2$ ratio. This means that light intensity decreases with increasing distance between two fireflies. Due to the absorption of light by air, the intensity also weakens with increasing distance. Considering the characteristics of fireflies' light irradiation, Yang formulated an algorithm based on the behavior of fireflies (Yang, 2009). For the idealization of the algorithm, it is assumed some simplifications and the algorithm inspired by fireflies is based on three rules:

- The attractiveness is proportional to the brightness and both decrease with distance. For two fireflies, any of the lower brightness will move towards greater brilliance.
- If a particular firefly does not shine or not irradiate flashes, his move to shall be random.
- The brightness of a firefly is affected or determined by evaluating the objective function, i.e., the brightness is directly proportional to the objective function.

Based on these three rules, the basic procedures of a pseudocode of FA are shown in Figure 1. The parameter *nv* represents the number of design variables and *nr_fly* the amount of fireflies.

Reviews about firefly algorithm can be found in Yang (2009), Yang (2010), Fister *et al*., (2013) and Koide (2016).

```
Input: f(x), x = (x_1, x_2, ..., x_nv);                    // objective function
       nr_fly, I_0, γ, α;                                   // defined initial constants
Output: x^min;                                  // objective function optimization value

For i ← 1 to nr_fly do
    x^i ← Initial_Solution();
End
While criterion not satisfied do;
    min ← arg_{i∈{1,...,m}} min (f(x^i));
    For i ← 1 to nr_fly do
        For j ← 1 to nr_fly do
            If f(x^i) < f(x^j) then                         // move x^i → x^j
                r_{i,j} ← Distance(x^i, x^j);
                β ← Attractiveness(I_0, γ, r_{i,j});
                x^i ← (1 − β)x^i + βx^j + α(Random() − 1/2);     // movement
            End
        End
    End
    x^min ← x^min + α(Random() − 1/2);            //best firefly moves randomly
End
```

Figure 1. Firefly algorithm pseudocode

2.3 Discrete firefly algorithm (DFA) applied to laminated composites

Originally, FA has also been developed for continuous variables. Firefly algorithm in the discrete form, called Discrete Firefly Algorithm (DFA), was first developed by Durkota, particularly for the QAP problem (Durkota, 2011). Whereupon, the search space S_{np} all possible permutations (1,2,…,*np*) were considered. The equation of motion due to the attractiveness can be written as

$$x_i \leftarrow (1-\beta)x_i\beta x_j + \alpha(random() - 0.5), \quad (1)$$

which was redefined by

$$x_i \leftarrow attractiveness(x_i, x_j, \alpha, \beta). \quad (2)$$

The definition of attractiveness function for discrete variables requires solutions represented by permutations and its operations. The set of initial population of fireflies, S_{np} are obtained randomly with permutations (1,2,…,*np*). The distance measurement can be evaluated with the Hamming distance. Hamming distance between two permutations is the number of mismatched elements in a sequence (Durkotta, 2011).

The distance measurement can be evaluated with the Hamming distance or a certain number of exchanges of the first solution to obtain the second solution. Hamming distance between two permutations is the number of mismatched elements in a sequence (Durkota 2011). For permutations π_1, π_2, $\pi_3 \in S_{np}$,

$$\begin{aligned} \pi_1 &= [1\ 2\ 3\ 4\ 5\ 6] \\ \pi_2 &= [1\ 2\ 4\ 3\ 6\ 5] \\ \pi_3 &= [1\ 2\ 4\ 5\ 6\ 3] \end{aligned} \quad (3)$$

It is noted in Eq. (3) that the distance_Hamming(π_1,π_2) between π_1 and π_2 is 4, due to only the first two elements are the same and the others four elements are mismatched.

The attractiveness, as defined by Eq. (1) for continuous variables and by Eq. (2) for discrete variables, is dependent on β and α. This dependence can be rewritten first computing the movement in function of β and then only in function of α. This order must be respected to prevent the influence of the value of α, resulting in the very near or far from the compared firefly movement. Equation (1) can be reformulated as

$$\begin{aligned} x_i &\leftarrow (1-\beta)x_i\beta x_j \\ x_i &\leftarrow x_i + \alpha(random() - 0{,}5). \end{aligned} \quad (4)$$

During the iterations of the algorithm, the distances of fireflies in relation to other fireflies tend to be lower, i.e., succeeding distances are reduced. As the Hamming distance for the case of discrete variables is used, the amount of equal elements must be increased to obtain the smallest distance between two fireflies. The step-β, begins with the check of the common elements between the permutations.

Firefly metaheuristic can be applied to composite material considering that the stacking sequence (layup) determination is a combinatorial optimization problem. The sequence of orientations of each ply determines the stacking sequence of the composite laminate. This is said symmetric if presents symmetrical geometry and symmetry properties relative to its axis of symmetry. The laminate is balanced to each slide with guidance $-\theta$ has a corresponding blade with similar orientation $+\theta$. The optimum sequence is the best stacking solution found for the specified objective function. This sequence is discrete, due to the variables are discrete and usually the plies have the following orientations: 0°, ±15, ±30, ±45, 90°. Figure 2 shows a schematic diagram of the firefly algorithm applied to laminated composite structures. The objective function is the postbuckling load and the procedures of algorithm are described and related with an application for layup optimization.

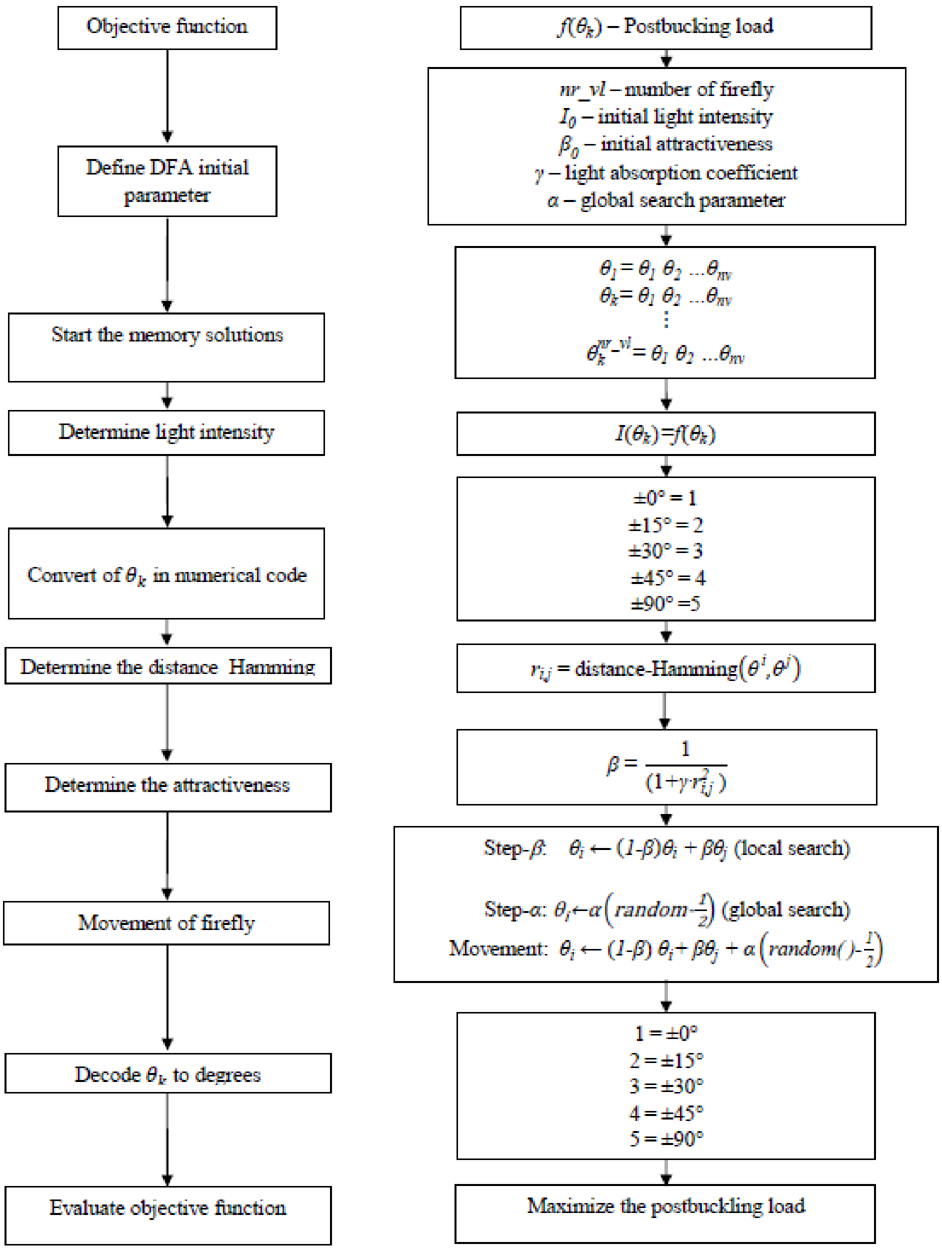


Figure 2. Flowchart of firefly algorithm applied to optimum layup of laminated composite structures

First, the initial parameters of the algorithm (*nr_fly*, I_0, β_0, γ and α) are defined. These parameters represent: the number of initial solutions (*nr_fly*); the initial light intensity (I_0), that decreases with the increasing distance between two fireflies; the initial attractiveness (β_0); the light absorption coefficient (γ) and the parameter (α), which aids in local and global searches.

The initial set of solutions is randomly generated. The determination of the light intensity for each of these solutions is equivalent to calculating the objective function for the corresponding stacking sequences. The Hamming distance, the step-β and step-α is difficult to implement considering the discrete variables $\{0_2, \pm15, \pm30, \pm45, \ldots, 90_2\}$. With the Hamming distance, it is possible to calculate the attractiveness of the fireflies. The attractiveness index enables to perform a local search and determines the firefly movement in according to step-β. Global search is computed with the step-α applying a random disturbance. The sums of these two procedures complete the movement for each iteration of the algorithm. The best solution founded is stored and reviewed at each algorithm cycle until a stopping criterion, usually a given number of iterations or number of objective function evaluations, is reached.

3. NUMERICAL RESULTS

In this subsection, numerical results are presented for a rectangular plate with different aspect ratios and a stiffned composite panel.

3.1 Rectangular plate analysis

The laminated plate geometry, boundary conditions and the coordinate system are shown in Figure 3, where the length and the width of the plate are indicated as a and b, respectively. For a specific ply, θ represents the angle between the fibers and the x_1 axis.

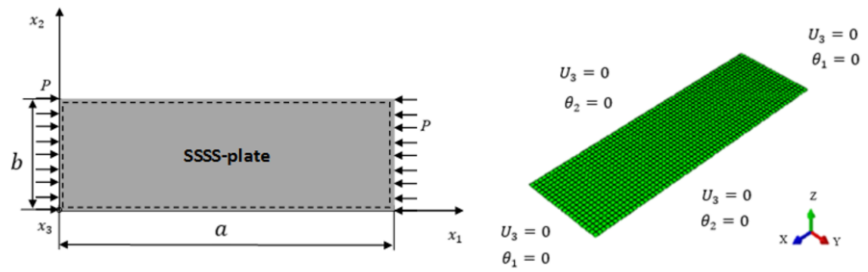


Figure 3. Plate geometry, loads, boundary conditions and finite element mesh of a rectangular plate

The finite element models were built in Abaqus using four-node shell type elements (S4R) with six degrees of freedom per node. The element length was kept constant and equal to 10 mm and all edges are simply supported (SSSS-plate under uniaxial compression).

The first step of the analysis is to predict the buckling mode shapes with a simple linear perturbation "buckle" analysis. The resulting mode shapes from this step were used to introduce imperfections into the model and its amplitudes were taken as 10% of the plate thickness for all analysis carried out in this paper. In order to perform the nonlinear analyses (including the postbuckling behavior), a static general step with stabilization coefficient was used. In this second step, prescribed displacements were applied in the horizontal edges and the corresponding loads P_{11} are obtained through the finite element solution.

The optimization problem is formulated as follows

Find: θ_k, $\theta_k \in \{0_2, \pm15, \pm30, \pm45, 90_2\}$ $k = 1,\ldots, n$
Maximize: P_{11} (postbuckling load) (5)
Subject to: transversal displacement $\leq$ 2 mm,

where $\boldsymbol{\theta_k}$ denotes the angle orientation of two contiguous plies, k is the index of the stacking sequence, n is the half number of total layers, $n = 16$. The criterion of 2 mm of maximum transversal displacement was chosen to assure the load would be taken in the postbuckling region.

To ascertain the accuracy of the FEM models to predict the postbuckling load, they were validated comparing with the results presented in Mittelstedt *et al.,* (2010) as shown in Figure 4. The validation models are symmetric and balanced laminates with the stacking sequence given by $[0°/90°/90°/0°/90°]_s$ for the SS-plate and $[0°/90°/0°/90°]_s$ for the SSSS-plate. The elastic properties of the plies are the following: E_1=157 GPa, E_2=8.5 GPa, G_{12}=4.2 GPa and ν_{12}=0.35. The plate dimensions used for the validation, in the both cases, are a=640 mm and b=200 mm.

In Figure 4, it can be noted that the results for SS-plate case exhibit a satisfactory concordance between the curves up to 2 mm of transversal displacement.

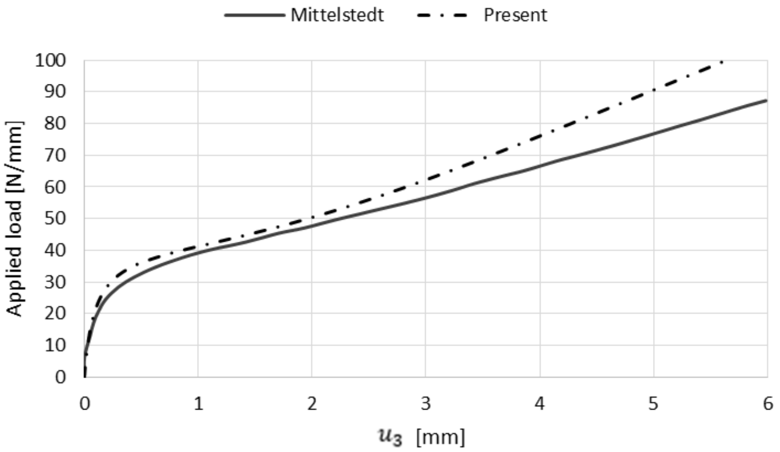


Figure 4. Postbuckling curves (load vs. displacement) for the validation of SS-plate case

In order to set the initial FA algorithm parameters, many tests were performed. The literature recommends that the number of fireflies must be between 20 and 50 (Yang 2009). Here, a satisfactory convergence was found using 40 fireflies. Others parameters, as initial β=0.8 and α=1 were considered. The tests were conducted to check whether the FA algorithm converges to an optimal solution when the number of objective function evaluations n_e increases, using the model with aspect ratio *a/b=3.4* and 32 plies.

The initial tests were carried out to determine the initial parameters, as the number of fireflies and the objective function evaluation. Here, a fixed value of 10000 function evaluations (n_e) and 40 fireflies were adopted for all analyses. Based on these initial results, optimizations of three plates with aspect ratios (*a/b*) of 2.2, 3.4, 5.2 for each load configuration case were performed and the results are summarized in Table 1. For comparison purposes, Table 2 shows the postbuckling load for balanced unidirectional laminates.

The optimization results for the different aspect ratios show, as well known, how the applied load increases as the relation *a/b* decreases. The orientation 90° appears in few cases, demonstrating how its influence in the results is much less significant than the other orientations. This orientation tends to reduce the ultimate load. On the other hand, higher postbuckling loads can be found using ± 30° and ± 45°.

For the compressive load configuration of unidirectional laminates, the orientation at 45° showed the best results, independent of the aspect ratio of the plate. The worse performance was observed for 0°. The optimization procedures found a stacking sequence combination between the orientations ± 45° and ±15°, for the smaller aspect ratios. The stacking sequence found for the 5.2 plate combined the orientations ± 45° and 90°, where these plies are disposed internally, covered by the ± 45°.

Table 1. Postbuckling optimization results using FA (rectangular plate).

Aspect ratio	Load [kN]	Stacking sequence
2.2	268	$[\pm45°_6\ \pm15°_4]_s$
3.4	265	$[\pm45°_4\ , \pm30°_2, \pm15°_2]$
5.2	264	$[\pm45°_6\ \pm90°_4]_s$

Table 2. Postbuckling load for unidirectional laminates (kN).

Aspect ratio	$[0°]_{32}$	$[\pm15°_8]_s$	$[\pm30°_8]_s$	$[\pm45°_8]_s$	$[90°]_{32}$
2.2	83	123	216	**261**	99
3.4	82	123	215	**259**	97
5.2	80	122	215	**258**	95

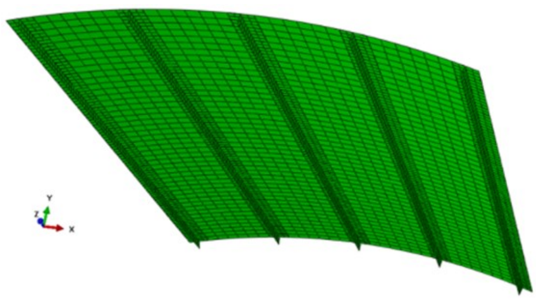
The use of FA algorithm provided the maximization of the postbuckling load in all cases investigated. For the SS-plate (uniaxial compression) the load increase achieved was much smaller, just by around 2.5% for all aspect ratios tested.

The literature recommends do not use more than four contiguous plies with the same orientation, in order to avoid delamination and constructive problem. However, in this work, the main propose is to test a methodology for the application of FA algorithm to layup design of laminated composite plates and its performance. Therefore, no restriction was considered in the optimization problem formulation to avoid these possibilities.

3.2 Stiffened panel analysis

The DFA is applied to find the ply orientations of the skin and stringers of a curved panel under a compressive axial load aiming to optimize the postbuckling maximum load. The laminate is symmetric and balanced for the skin and stringers. The panel is modeled in Abaqus, using four-node shell type elements (S4R) and its characteristics and mesh are shown in Table 3. The Hashin failure criterion was considered as a constraint in the optimization problem. Also, the Chang-Lessard damage (Chang and Lessard, 1991) is considered in the analysis. The work of Araico et al. (2010) was used to obtain the characteristics of the panel and to validate the model.

Table 3. Characteristics of the reinforced curved panel.

Panel characteristic	Dimensions or other data	Finite element mesh (Abaqus)
Panel length (skin)	780 mm	
Panel width (arc length)	560 mm	
Outer radius of the panel	1000 mm	
Numbers of stringers	5	
Stringers to stringer distance	132 mm	
Number of skin plies	8 plies	
Number of stringers plies	24 plies	
Material	Carbon/epoxy	
Ply thickness	0.125 mm	
Stringer height	14 mm	
Stringer base width	32 mm	
Skin-stringer joint	Bonded joint	
Adhesive	FM 300	
Adhesive thickness	0.26 mm	

The optimization problem can be written as

Find: θ_k, $\theta_k \in \{0, \pm 45, 90\}$, $k = 1, \ldots, n_p$ *(Skin)*
θ_l, $\theta_l \in \{0, \pm 45, 90\}$, $l = 1, \ldots, n_r$ *(Stringers)*
Maximize: λ_{pf} *(postbuckling load) with damage analysis* (6)
Subject to: max 4 contiguous plies with the same orientation;
Hashin failure criterion

In order to consider the damage, the Abaqus USDFLD subroutine, coded in Fortran, was used. The main purpose of the subroutine is to apply a degradation (reduction of up to 10%) in the elastic properties of a failed ply. The failure of the ply is evaluated with the Hashin criterion and, in the occurrence of the failure, the method of Chang and Lessard (*apud* Araico et al. (2010) is used to reduce the properties of the material. With the degradation of the elastic properties of the ply the overall stiffness of the structure is reevaluated and, for the corresponding loading step, the failure of the other plies is re-evaluated and, if no other fails, the next loading step is analyzed and so on. The complete lamination failure occurs when all plies fail.

Table 4 presents the optimization results, where n_e is the numbers of evaluation function. The stopping criterion used was 50 analyzes without modification of the best value of the objective function.

Table 4. DFA optimization for the curved panel subject to Hashin and damage criterion.

λ_{pf} (kN)	u_3 (mm)	Stacking sequence		n_e
		Skin	Stringers	
119.79	2.44	$[0 \pm 45\ \ 45]_S$	$[0_2 \pm 45_2\ 0_4\ 90_2]_S$	1000

4. CONCLUSIONS

In this work, the main goal was to apply and test a methodology for the application of DFA (discrete firefly algorithm) to optimize the layup configuration of laminated composite plates and a curved panel with stringers aiming to maximize the postbuckling capacity load. To analyze the postbuckling behavior, finite element models were created in the Abaqus code, and connected to the optimization algorithm coded in Python. Convergence and validation tests were carried out. The optimization results confirm the DFA as an alternative in the optimization of laminated composite structures.

5. ACKNOWLEDGEMENTS

The authors are grateful to Post Graduation Program in Mechanical and Materials Engineering (PPGEM-UTFPR) and the first author also acknowledges the FAE Centro Universitário and CAPES funding agency.

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