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NUMERICAL ANALYSIS THROUGH CFD OF THE FREE CONVECTION IN A VERTICAL FLAT PLATE

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Abstract. *This paper presents a numerical investigation of heat transfer through free (natural) convection inside the thermal and hydrodynamic laminar boundary layer of an isothermal and crimped vertical flat plate. The equations that governs the process are discretized by the Finite Volume Method through the ANSYS/Fluent™ 15.0 software. Initially the paper aims to compare the results obtained in the numerical simulation for different mesh refining with the analytical model available in the literature. Afterwards, it is sought to verify how the boundary conditions and plate thickness influence at the flow, at the heat exchange and in the characteristic dimensionless parameters for this type of flow. The numerical data were compared with the analytical model available in the literature and showed good consent, regarding the variations in the boundary conditions, it was noticed that these conditions influence in the field of velocity and therefore in the transfer of heat.*

Keywords: *Vertical Isothermal Plate, Natural Convection, Finite Volume Method.*

1. INTRODUCTION

Many common heat transfer applications involve as their main mechanism the free convection, among these applications, it is worth mentioning the heat transfer in cooling coils, in the cooling of electronic equipment, in solar energy collectors, and insulation of aircraft cabins.

The free convection occurs when a body force acts on a fluid in which there are density gradients. The temperature variations cause these gradients in the fluid in the presence of a gravitational field (Bergman *et al.*, 2015). According to Güths (1998), a problem that can be modeled from this definition is the heat transfer in a vertical isothermal free plate. It has a large number of applications in engineering; however, a good collection of reliable and ready mathematical correlations is needed to provide relevant data for projects, such as the heat transfer coefficient (Dias and Milanez, 2004).

Machado (2013) comments on his work that problems involving free convection are difficult to be solved analytically, because the governing equations, with their respective boundary conditions, has non-linear terms, so obtaining trustworthy results has become increasingly laborious and time-consuming, for some cases, it is impossible.

In this context, the present work numerically analyzes the heat transfer by free convection through the thermal and hydrodynamic laminar boundary layer of an isothermal and crimped vertical flat plate. The governing equations are discretized by the Finite Volume Method and solved iteratively. When it is possible, a comparison is made between the results and the analytical solution exposed by Ostrach (1952).

The paper still seeks to provide a theoretical-numerical basis for an extension of the studies focused on a heat transfer area by free convection in isothermal flat plates.

2. MATHEMATICAL MODEL

The process of heat transfer by convection, regardless of its nature, can be expressed mathematically by Newton's law of cooling.

$$q_{CONV} = hA_s(T_s - T_\infty) \quad (1)$$

Since that, q_{CONV} [W] is the convection heat transfer rate, h [W/(m².K)] is the heat transfer coefficient by convection, A [m²] is the heat transfer area, T_s [K] is the surface temperature, and T_∞ [K] is the temperature of the fluid sufficiently far from the surface.

However the equations that govern the physical phenomenon of free convection in the laminar boundary layer of an isothermal vertical flat plate, assuming that the fluid is Newtonian and incompressible and that the flow happens in a two-dimensional and permanent regime, are the differential equations of conservation of Mass, amount of movement and energy, these are expressed by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T_s - T_\infty) \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (4)$$

Where, u [m/s] is the velocity component in the x direction, v [m/s] is the velocity component in the y direction, g [m/s²] is the acceleration of gravity, ν [m²/s] the thermal kinematic viscosity of the fluid, α [m²/s] is the thermal diffusivity of the fluid, and β is the coefficient of thermal volumetric expansion.

According to Çengel and Ghajar (2012), the equations governing free convection can be dimensionless by dividing all dependent and independent variables by appropriate constant quantities. Within the dimensionless groups, it is necessary to highlight for the free convection the number of Grashof, the number of Nusselt punctual and average and the number of Rayleigh. These parameters are expressed respectively by Eqs. (5), (6), (7) and (8) and are extremely relevant for analysis of problems involving free convection, as they describe the flow regime, as in the case of the Grashof number, and the relationship between the term of Fluctuation and viscosity, a case of Rayleigh number.

$$Gr_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \quad (5)$$

$$Nu = \frac{hL_c}{k} \quad (6)$$

$$\overline{Nu} = \int_0^L Nu(y) dy \quad (7)$$

$$Ra_L = Gr_L Pr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} Pr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu\alpha} \quad (8)$$

Since the term, $g\beta(T_s - T_\infty)$ is the fluctuation term, Pr is the Prandtl number given by $Pr = \frac{\mu C_p}{k}$, where μ [(N.s)/m²] is the dynamic viscosity of the fluid, C_p [J/(Kg.K)] is the specific heat at constant pressure, and k [W/(m.K)] is the thermal conductivity.

3. NUMERICAL MODEL

The Finite Volume Method, described in detail by Patankar (1980) and Maliska (2004) solved the governing equations numerically. This method first consists in dividing the domain of the problem into finite volumes, thus forming a computational mesh. Subsequently, the integration into an elementary volume of the differential equation of

each of the dependent variables involved is effected. Thus, the conservation of the involved properties is satisfied in each elemental volume of the mesh and, consequently, in the entire domain of solution. From this procedure, we derive a system of algebraic equations for each model variable. Starting from the resolution of the system of linear algebraic equations, we finally obtain the distribution of the domain property of the problem. The pressure-velocity coupling is solved through the SIMPLE algorithm (*Semi Implicit Linked Equations*).

The following sections provide a general background of how the numerical procedure of a problem solved through the Finite Volume Method occurs.

3.1 Moving arrangement

Before the differential equations be integrated in each control volume, it is necessary to define the arrangement of the variables in the mesh to be used. In the present work, we opted for the disjointed arrangement of variables, where the equation of Amount of movement is displaced in relation to the control volume from the other properties and the continuity and energy equations are integrated in the original control volumes. According to Filho (2013), the strong coupling between pressure and velocity makes it difficult to choose where to store the components of the velocity vector in the computational mesh. In accordance with Patankar (1980) storing velocity fields and pressure fields at the same point in the mesh implies the possibility of occurring oscillatory pressure fields, which are physically unrealistic, is what happens when a co-located arrangement is used.

One way to avoid such erroneous solutions is to locate the pressures so that it functions as a driving force of the velocity located between two pressure points, thus generating an arrangement shifted to the velocity field, which provides instability for the pressure-velocity coupling.

Figure 1 illustrates the mesh used to integrate the equations of continuity and energy, called the original mesh. For the original control volume, we have a nodal point P , the lowercase letters s , n , e and w represent the phases of the control volume, since the velocities v and w are supposedly known and stored on the faces of the control volume.

Figures 2 and 3 show the displaced control volumes in which the quantity of movement equation was integrated respectively for the horizontal and vertical directions.

It can be noted that for the horizontal direction, the velocity component v is calculated for the phases e and w , and for the vertical direction the velocity component w is obtained in phases n and s .

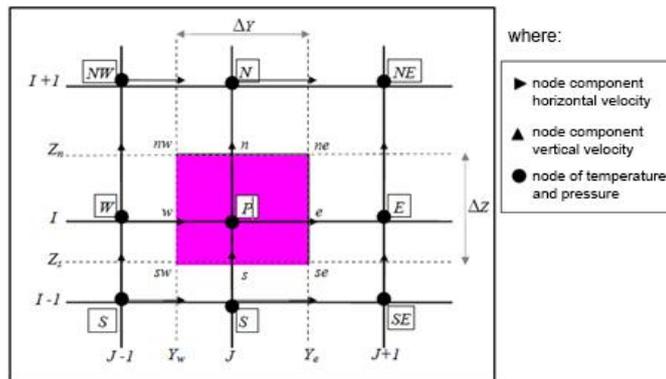


Figure 1. Original control volume

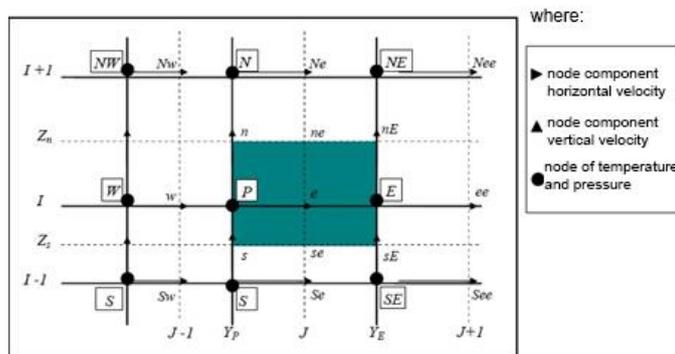


Figure 2. Displacement control volume used to integrate the horizontal component of the velocity into the equation of the momentum

230 μm for the region inside of the boundary layer up to 1850000 with a size of 2 cm for regions distant from the plate, was chosen for the simulation.

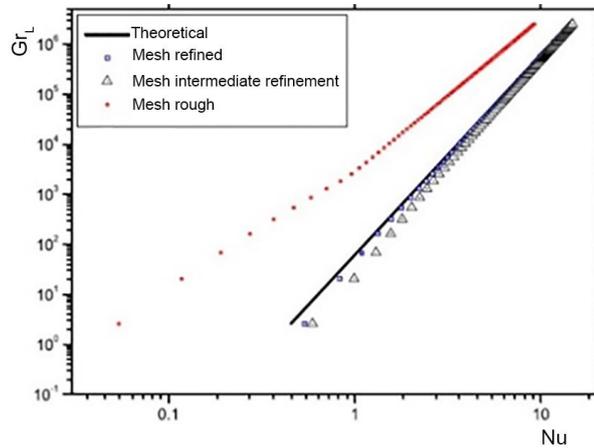


Figure 4. Nusselt for different mesh refinements

Figure 5 explains the basic form used for numerical analysis of the problem, the following boundary conditions were considered: *prescribed pressure* of 1 atm and *prescribed temperature* of 25 °C on the upper, right and lower sides; *Adiabatic wall* and *non-slip condition* in the above and below the crimped plate; Wall with a *prescribed temperature* of 45 °C and *adhesion* to the plate.

It was also adopted a thickness of 1 mm in the computational domain to simulate two-dimensional modeling. The quiescent fluid chosen was dry air with a temperature of 25 °C at atmospheric pressure, the gravitational force being the driving potential. The Rayleigh number and the Reynolds number, as expressed by Eq. (11), were respectively of order 10^6 and 375, characterizing the flow as being laminar.

$$\text{Re} = \frac{\rho VL}{\mu} \quad (11)$$

Afterwards, the boundary conditions and the thickness of the plate varied, to verify how these situations influence the flow on the plate. The boundary conditions compared to these situations were: Prescribed pressure and symmetry.

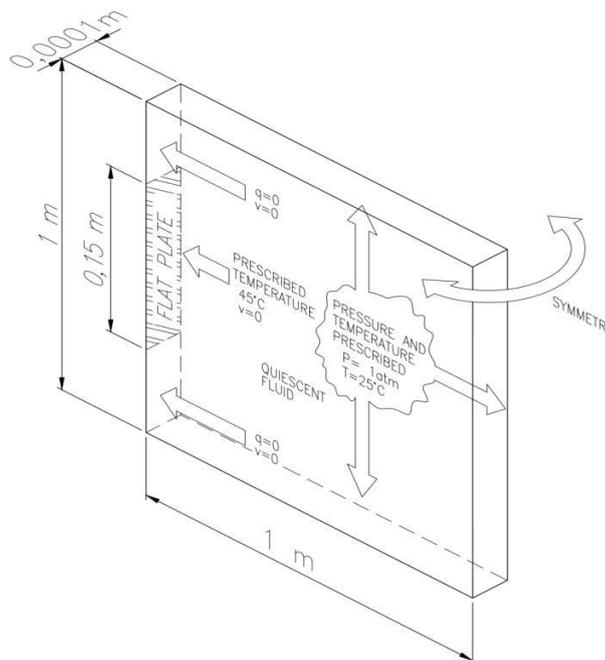


Figure 5. Model of the problem

5. RESULTS AND DISCUSSION

Based on the methodology presented previously, in this part of the paper will be presented and discussed the results obtained.

From the Figs. 6 and 7 it can be noted that the adopted numerical model obtained a good agreement when compared to the analytical model, even with a small discrepancy in the Nusselt number between the two models, it can be stated that the numerical solution to the problem of free convection in crimped flat plates is satisfactory.

The Fig. 6 shows that the average Nusselt number has an increasing linear relationship with its height on the flat plate. As for the coefficient of heat transfer by convection, it can be noticed from Fig. 7 a concordance between the values and the decay of the coefficient in relation to the height of the plate.

Analyzing the data obtained it can be stated that for an isothermal flat plate with the contour conditions mentioned above, the Nusselt number is directly proportional to the plate height, and this in turn is inversely proportional to the heat transfer coefficient by convection, which results in a relation of the type: $Nu / h = y$.

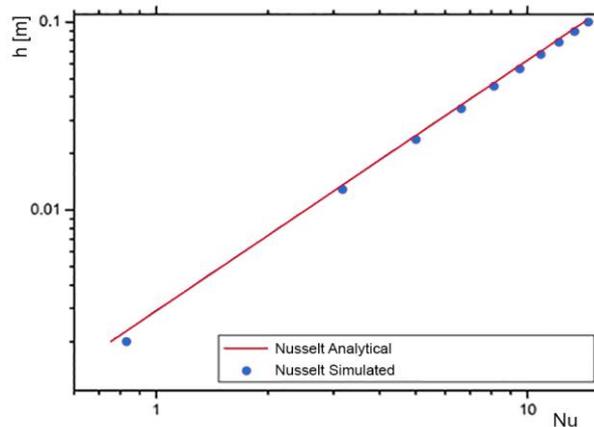


Figure 6. Variation of the Average Nusselt number related to the plate height

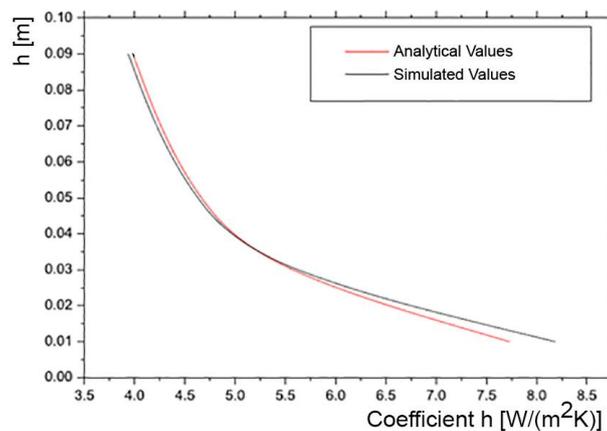


Figure 7. Variation of convective coefficient related to the plate height

In order to analyze how the boundary conditions influence the flow and the dimensionless parameters, it was modified on the open surfaces the prescribed pressure condition for the symmetry condition.

It is noteworthy from Fig. 8 that a small variation occurs in the analytical coefficient of heat exchange by convection when compared to the condition of symmetry. The Figs. 9 and 10 show that such variation is a consequence of the modification in the velocity fields.

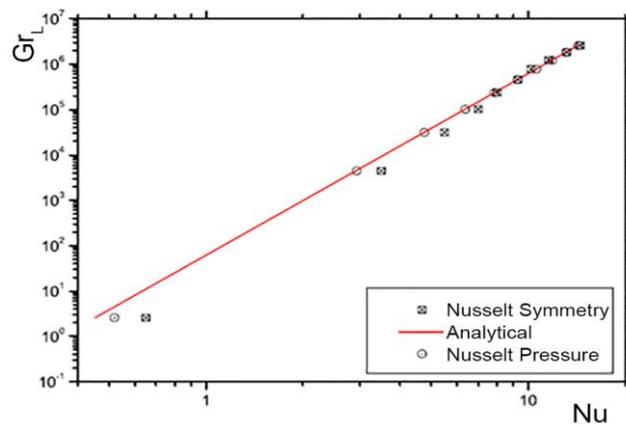


Figure 8. Variation of the Nusselt number for different contour conditions

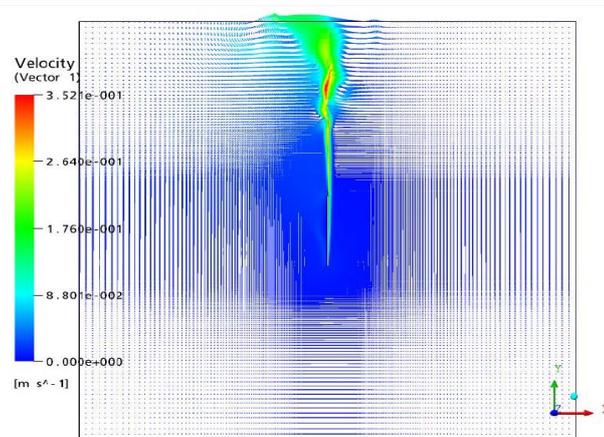


Figure 9. Velocity field for the prescribed pressure condition

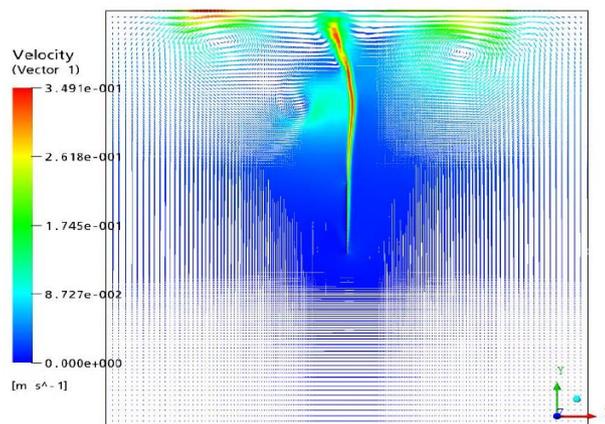


Figure 10. Velocity field for symmetry condition

Concerning the thickness of the plate, three different situations were simulated, with thicknesses of 1 mm, 2.5 mm and 5 mm. For these simulations, the prescribed pressure condition was adopted for environments exposed to the atmosphere. The initial conditions were maintained from previous simulations.

The Fig. 11 shows that the thickness of the plate almost does not influence the Nusselt number; this allows us to conclude that the two-dimensional model presented by Ostrach (1952) is satisfactory.

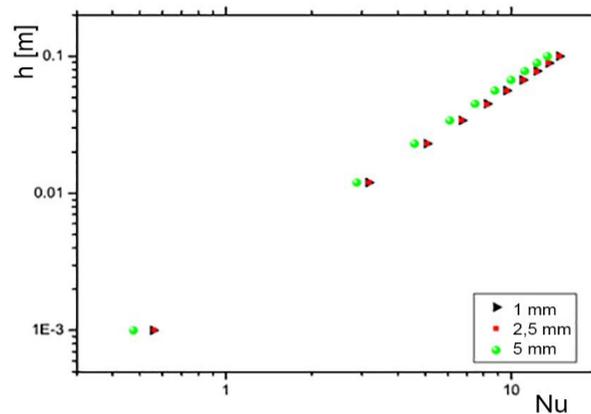


Figure 11. Variation of Nusselt number for different thicknesses

6. CONCLUSION

The numerical results showed a satisfactory agreement, presenting minimal errors when compared to the analytical model, such results indicate that the interpretation and modeling chosen to describe the physical phenomenon presented by Ostrach (1952) was elaborated correctly.

It can be noticed that the variation of the contour conditions directly influence the measurement of the characteristic dimensionless coefficients of this type of flow, as the thickness of the plate it can be concluded that the field of velocity is not influenced for this, and consequently the convection coefficient does not undergo major changes.

7. REFERENCES

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