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STOCHASTIC TORSIONAL VIBRATION ANALYSIS OF A VERTICAL DRILL-STRING IN THE TRANSITION BETWEEN TWO ROCK LAYERS

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Abstract. *Undesired vibrations in drill-strings, like stick-slip oscillations, reduces drilling performance and increases the costs. The main uncertainties in drilling operation are because of bit-rock interaction and becomes more important when variations in the rock being drilled are observed. This work is focused on the modelling of the uncertainties in bit-rock interaction when there is a transition between two rock layers and the consequences in torsional dynamics. The drill-string is modelled with one degree-of-freedom where the top speed is prescribed, a viscous friction represents the drilling mud and a non-linear model describes bit-rock interaction. Rock strength data available in literature served as inspiration to build models to describe rock layers transitions; moreover, uncertainties are included in the parameters. The stochastic dynamic response of the system is assessed and the conclusions are presented.*

Keywords: *drill-string dynamics, rock transition, stochastic, bit-rock interaction.*

1. INTRODUCTION

The drilling process is certainly one of the most important and complex in oil and gas industry. In this kind of operation, drillstrings are used to dig the rock in search of oil and their vibration is considered one of the main causes to performance reduction (Khulief *et al.*, 2007). Three types of vibration are common in drillstrings: axial, lateral and torsional. The torsional one can be considered one of the most common and destructive vibrations, in which a phenomenon called Stick-Slip can happen. This phenomenon happens when the bit stuck and then is released (Slip), with fast repeated cycles reaching high rotational speeds.

It can be found in literature a variety of models describing drill-string dynamics. The models can consider uncoupled modes, i.e. each vibration mode is studied separately, or coupled modes, i.e. vibration modes are studied in combined ways. In some cases, it can be shown that a pure torsional model is enough to describe drill-string dynamics when axial and lateral vibrations are small (Ritto *et al.*, 2017). This paper will only consider the uncoupled torsional mode.

The models used to study the uncoupled torsional vibration are usually classified into two categories: distributed parameter models and lumped parameter ones. In distributed parameter models, the results are given in function of time and location. The classical wave equation is widely used to describe the torsional vibration of drill-strings (Boussaada *et al.*, 2012; Fridman *et al.*, 2010) and the simulation is no easy task because of the nonlinearities (Saldivar *et al.*, 2014). In lumped parameter models, the results are given only in function of time. The drill-string is commonly modeled as a torsional pendulum (Kamel and Yigit, 2014; Tang *et al.*, 2015; Zhu and Tang, 2015; Depouhon and Detournay, 2014; Tang *et al.*, 2015; Zhu and Tang, 2015; Navarro-Lopez and Suarez, 2004; Navarro-Lopez and Cortes, 2007) and the simulation is simpler. Even some articles about coupled modes of vibration, specially the axial-torsional, use the torsional pendulum to model the torsional behavior of the drill-string (Zamanian *et al.*, 2007; Richard *et al.*, 2004). This work uses the torsional pendulum concept as well.

In lumped parameter models, the BHA is usually considered as a rigid body and the drill pipes are modeled inertialess and represented by a torsional spring. The rotary table is some times modeled as a inertia with a control system Lima *et al.* (2015); Brett (1992); Navarro-Lopez and Suarez (2004) or using a prescribed constant rotational speed (Richard *et al.*, 2004; Zhu and Tang, 2015; Zamanian *et al.*, 2007; Tang *et al.*, 2015). The bit-rock interaction is considered the source of the non-linearities of the problem. In this kind of model, this interaction is usually modeled as a pure dry friction (Tang *et al.*, 2015; Zhu and Tang, 2015) or using a decaying function on slip phase (Navarro-Lopez and Suarez, 2004; Hong *et al.*, 2016; Vromen *et al.*, 2015). The decaying behavior is observed in experimental data (Brett, 1992; Ritto *et al.*, 2017) and it is considered in this article.

Other models for the bit-rock interaction were also proposed in the last years. Detournay and Defourny (1992) proposed a model that combines cutting and frictional forces in coupled axial-torsional vibration and this model was used in

(Butlin and Langley, 2015; Depouhon and Detournay, 2014; Bakhtiari-Nejad and Hosseinzadeh, 2017; Kamel and Yigit, 2014). Another model was proposed by Franca (2011) for bits under rotary and percussive actions.

The drilling operation have uncertainties in everywhere we look at, but one of the main sources of this uncertainty is certainly the bit-rock interaction. It motivated several works that modeled and quantified the effects of these uncertainties in vertical drill-strings (Spanos *et al.*, 2002; Kotsonis and Spanos, 1997; Ritto *et al.*, 2009, 2010, 2013; Ritto and Sampaio, 2013) and horizontal drill-strings (Cunha *et al.*, 2015; Ritto *et al.*, 2013). Moreover, the rock layers drilled by the drill-string is not homogeneous but composed of several rock types with different characteristics as it can be seen in experimental data given by Hareland and Nygaard (2007); Onyia (1988); Rampersad *et al.* (1994) and it can influence the drill-string response as shown in (Lobo *et al.*, 2017).

Considering this, the goal of this work is to analyze different shapes for the transition between rock layers while drilling and quantify how uncertainties in the parameters the model this transition can change the response of the system. For this purpose, a torsional 1-DOF model is used with prescribed top rotational speed. Two functions are then proposed for rock transition and uncertainties are included in the parameters of such functions. The stochastic response is then assessed and the concluding remarks are made in the last section.

2. MATHEMATICAL MODEL

The drill-string is a slender structure used to drill rock layers in order to reach petroleum reservoir. This structure can achieve 8 kilometers of length and it consists mostly of drill pipes that are attached to each other. At the last 100-300 meters on the bottom, there is an assembly called BHA that is responsible for having expensive measurement units, the drill bit and heavy pipes to apply weight on bit. At the very beginning of the structure, a rotary system is responsible for imposing rotational speed to the drill-string and it also injects drilling mud inside the pipes to refrigerate and lubricate (among other functions) the drill bit.

The model used to describe the drill-string is shown in Fig. 1. The rotary table and BHA is represented by concentrated inertias. The drill pipes are modeled by a linear spring and a damper is used to model the frictions among pipes, connections and so on. A damper is also included at the BHA to represent the interaction with drilling mud. The torque on bit due to the drilling is modeled as an external torque that will be described later.
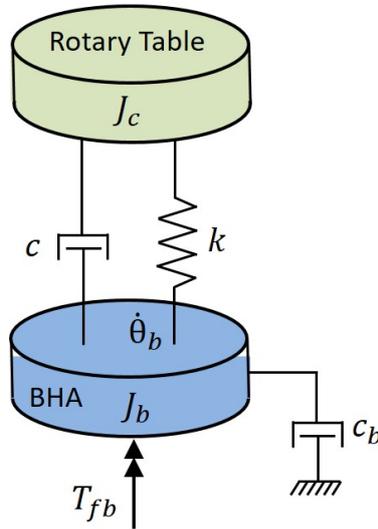

Figure 1: Drill-string model.

Some hypothesis are made in order to construct this model: (i) the borehole and drill-string are perfectly in vertical position; (ii) axial and lateral motions can be neglected; (iii) the top rotational speed is constant. The equation of motion is then written as:

$$J_b\ddot{\theta}_b + c(\dot{\theta}_b - \Omega) + k(\theta_b - \Omega t) = -c_b\dot{\theta}_b - T_{fb}(\dot{\theta}_b) \qquad (1)$$

where θ_b is the bit angle; J_b is bottom inertia representing BHA; Ω is the rotational speed imposed at the top; k and c are the stiffness and damping of torsional spring, respectively; c_b is the viscous parameter for drilling mud interaction; and T_{fb} is the torque on bit that is shown in Fig. 2 and modeled as a dry friction with a decaying function on slip phase. The torque on bit is modeled by the Eq. 2.
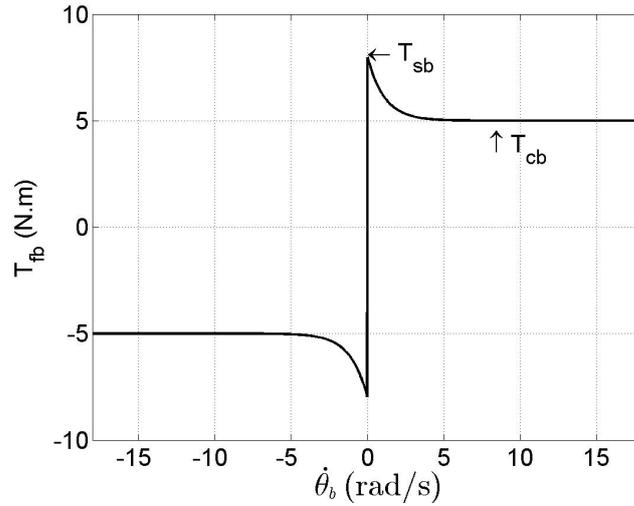
Figure 2: Torque on bit model.

$$T_{fb} = \begin{cases} T_{eb}(t, \dot{\theta}_b, \theta_b) & , \dot{\theta}_b < D_v \text{ and } |T_{eb}| < T_{sb} \\ T_{sb} \cdot sign(T_{eb}) & , \dot{\theta}_b < D_v \text{ and } |T_{eb}| > T_{sb} \\ T_b \cdot sign(\dot{\theta}_b) & , \dot{\theta}_b > D_v \end{cases} \tag{2}$$

where T_{eb} is the reaction torque equilibrium defined by Eq. 4; T_b is the torque on bit in slip phase and is defined by Eq. 3; T_{sb} and T_{cb} are the static and static friction torques, respectively; γ_b is a positive constant; and D_v is a zero-velocity band (Karnopp's model).

$$T_b = T_{cb} + (T_{sb} - T_{cb})e^{-\gamma_b \dot{\theta}_b} \tag{3}$$

$$T_{eb}(t, \dot{\theta}_b, \theta_b) = c(\Omega - \dot{\theta}_b) + k(\Omega t - \theta_b) - c_b \dot{\theta}_b \tag{4}$$

Due to its physical meaning, T_{sb} and T_{cb} were chosen to model the transition between two rock layers and the experimental data in (Hareland and Nygaard, 2007) motivated the choice of functions with step and linear shapes as in Fig. 3.
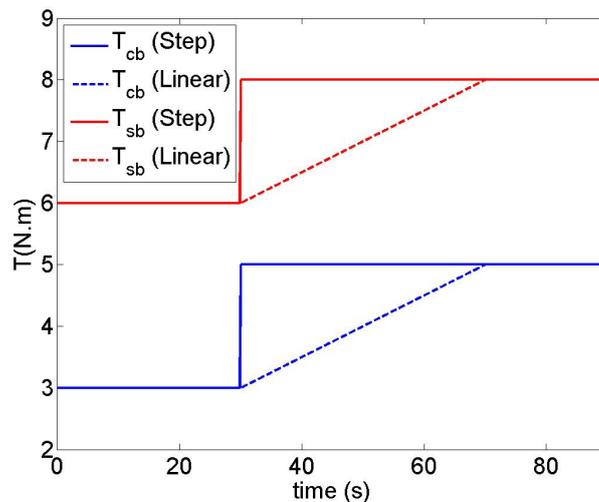

Figure 3: Rock transition model.

The values used in the simulations were taken from (Navarro-Lopez and Suarez, 2004) and shown here: $J_b = 0.0318$ $kg.m^2$; $c = 0.0001$ $N.m.s/rad$; $c_b = 0.03$ $N.m.s/rad$; $k = 0.073$ $N.m/rad$; $D_v = 10^-6$ rad/s; $\gamma_b = 0.9$; $\Omega = 20$ rad/s.

Until now, the deterministic model was presented. The stochastic model considered the initial and final values for time and torques during transition as random variables. In order to assure physical consistency, the support was set accordingly

to Tab. 1. Considering this support and that mean and variance are known, the chosen distribution is normal truncated accordingly to maximum entropy principle (Udwadia, 1987). The parameters for the random variables are set according to Tab.1 and the other variables are considered still deterministic and the values are already defined in the last paragraph.

Table 1: Support, Mean and Std. Dev. of the random variable.

Variable	Support	Mean	Std. Dev.
T_{cb1} [N.m]	[2.4 , 2.6]	3.0	0.15
T_{cb2} [N.m]	[4.0 , 6.0]	5.0	0.25
T_{sb1} [N.m]	[4.8 , 7.2]	6.0	0.30
T_{sb2} [N.m]	[6.4 , 9.6]	8.0	0.40
t_1 [sec.]	[24 , 36]	30	1.5
t_2 [sec.]	[56 , 84]	70	3.5

3. RESULTS AND DISCUSSION

It was used Monte Carlo simulation for generating the following results. The first step of the analysis was to evaluate the convergence of the method. The mean square convergence is shown in Fig. 4 for step transition, for example. The linear case resulted in a similar graph, so it can be concluded that 10,000 simulations is enough for convergence.
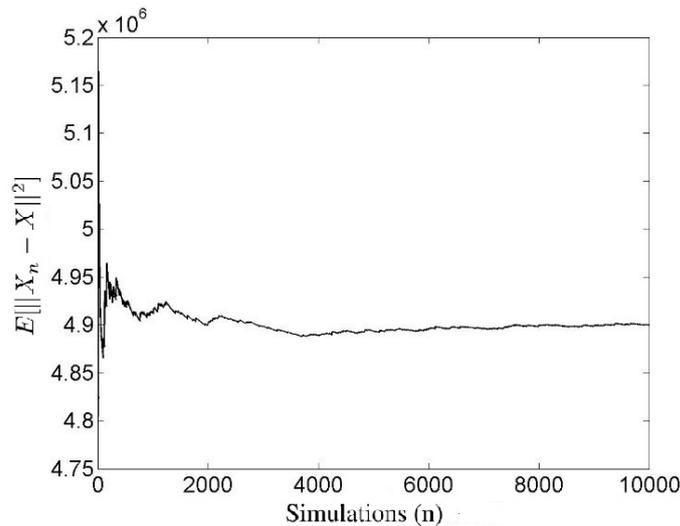

Figure 4: Mean square convergence.

The results presented in Fig. 5 showed that, for some simulations, the system presented stick-slip oscillations before the transition because of high T_{sb}/T_{cb} ratio. Otherwise, after the transition, these oscillations can vanish when the transition is smoother (linear) but proceeds in sharp transition (step).

Before the transition, the stick-slip vibration is all synchronized and results in the behavior seen in Fig. 5. After the transition, the vibration is no longer synchronized and tt occurs because the uncertainty in the initial and end times of transition.

Some simulations of the linear case also shown stick-slip vibration after the transition but it cannot be seen in envelope because it is outside the 99% confidence interval. It can also be noticed that stick-slip vibration don't vanish immediately in linear transition, but it decreases with time until it vanish after transition (t>70s).

The envelope has a minimum of zero velocity in both cases because the drill bit only rotates in one direction using this model. The maximum value for the envelope is more than 3 times the desired rotational speed, which confirms the severity of stick-slip vibrations.

The envelope for torque on bit is shown in Fig. 6. Before the transition, the stick-slip vibration results in a envelope that switch from static and coulomb friction very fast and it is synchronized for all simulations due to the absence of uncertainties in time parameters, as discussed before. After the transition, Fig. 6a shows a wide envelope, confirming the behavior discussed in the last paragraph. Otherwise, Fig. 6b shows that envelope reduces after the transition and the envelope becomes the interpretation of the pure uncertainty of coulomb friction torque after transition.
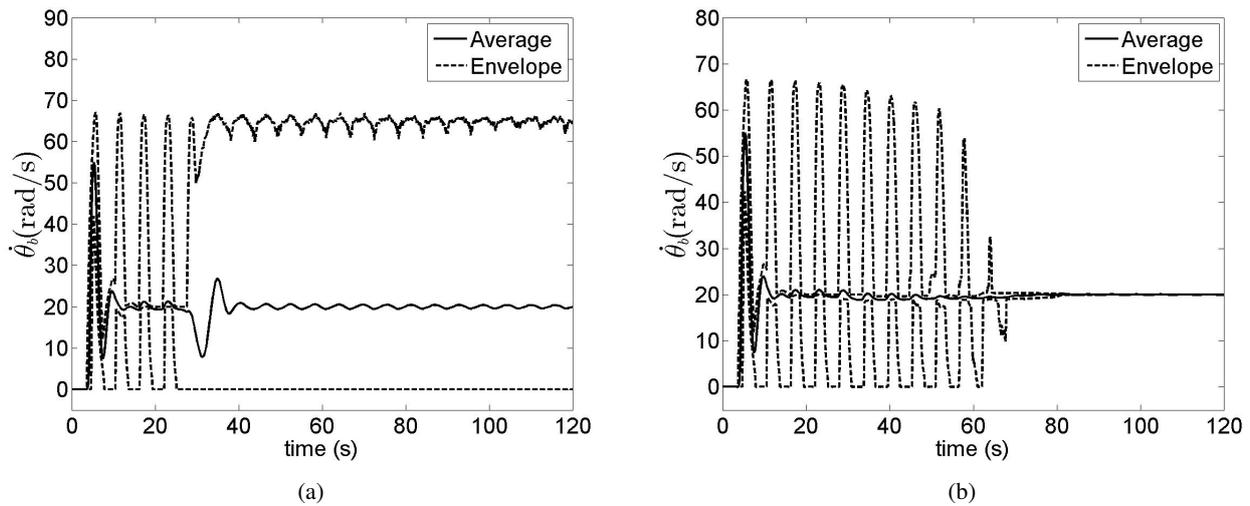
Figure 5: 99% envelope for bit speed. (a) Step transition; (b) Linear transition.
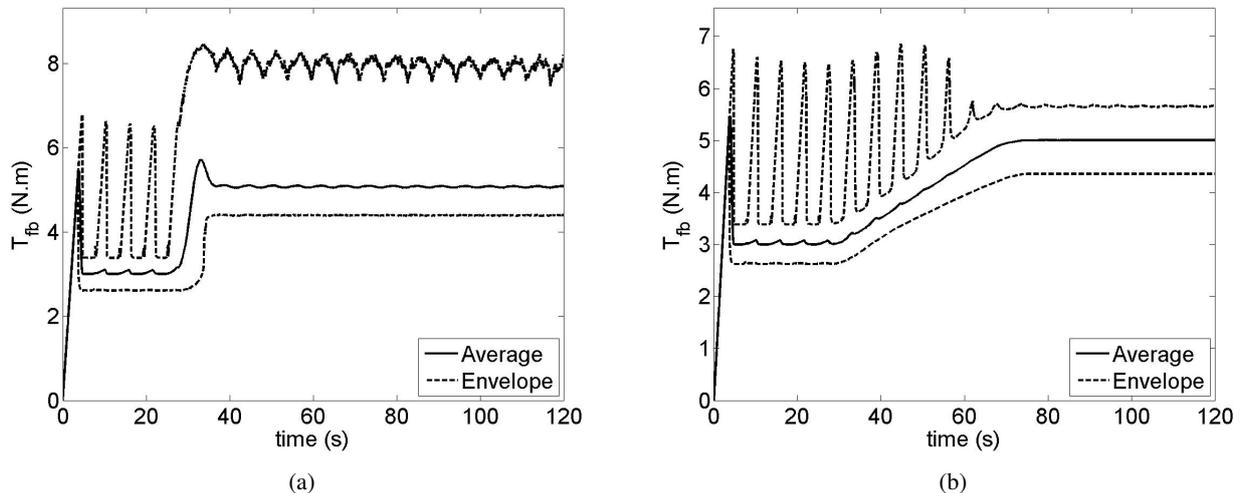

Figure 6: 99% envelope for torque on bit. (a) Linear transition; (b) Step transition.

4. CONCLUDING REMARKS

It was concluded that scenarios with sharp transitions are bad for the drill-string vibration. Also, the differences in dynamic response according to the transition showed that it is important to do this analysis in real drill-string operations with real data obtained about the rock conditions.

The uncertainties also showed that, although is possible to stick-slip before transitions, it decreases when the transition to a better scenario is smooth but increases if it is sharp.

5. ACKNOWLEDGEMENTS

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