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NUMERICAL ASSESSMENT OF CURVATURE EFFECTS IN THE ANALYSIS OF SHELL STRUCTURES USING GENUINELY CURVED AND FLAT FINITE ELEMENTS.

Fabio Augusto Schuh[1], Rogério José Marczak[2]

[1]Mechanical Engineering Department, UFRGS, Porto Alegre, Rio Grande do Sul, Brasil

[2] Mechanical Engineering Department, UFRGS, Porto Alegre, Rio Grande do Sul, Brasil

*Corresponding author: fabio.schuh@ufrgs.br

Abstract: The current paper compares the finite element simulation of an arbitrary shell structure using two different finite element formulations. The first one is a shell element, where the curvature of the structure in inherent to the element. The second is an assembly of plate elements in space, in order to form a similar shell structure, but the formulation is of a flat element. The paper compares the convergence of the results from the two distinct formulations and presents the resulting boundary reactions for each assembly. The curvature is usually not considered in the calculation of reactions in commercial finite element software, but it is shown in this paper to have relevancy in the obtained results.

Keywords: finite element, shell, plate

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Fabio Augusto Schuh
Mechanical Engineering Department
Universidade Federal do Rio Grande do Sul - UFRGS
fabio.schuh@ufrgs.br

Rogério José Marczak
Mechanical Engineering Department
Universidade Federal do Rio Grande do Sul - UFRGS
rato@mecanica.ufrgs.br

ABSTRACT

The current paper compares the finite element simulation of an arbitrary shell structure using two different finite element formulations. The first one is a shell element, where the curvature of the structure in inherent to the element. The second is an assembly of plate elements in space, in order to form a similar shell structure, but the formulation is of a flat element. The paper compares the convergence of the results from the two distinct formulations and presents the resulting boundary reactions for each assembly. The curvature is usually not considered in the calculation of reactions in commercial finite element software, but it is shown in this paper to have relevancy in the obtained results.

Keywords: finite element, shell, plate, curvature

1 INTRODUCTION

Finite element analysis represented a huge improvement in analysis cost and time and since its beginning, many approaches were proposed. The plate and shell finite element approaches propose to reduce an intrinsically tridimensional problem into a bi-dimensional problem, and in order to succeed, a series of restrictions and simplifications are applied to the three-dimensional solids mechanics.

In 1970, Ahmad, Irons and Zienckiewicz presented a shell element formulation, also known as the AIZ shell element formulation, which is a continuum element with several assumptions that characterizes a degenerated solid element, not based on any plate/shell theory. This formulation suffers with the locking phenomenon, thus selective numerical integration is necessary [1]. In the early 1980s, the MITC4 element was developed in order to overcome the locking problem, and is the standard shell element for many finite element codes, with the limitation of application to infinitesimal strains [2-4].

This paper focuses on implementation of a finite element program based on the shell theory proposed by Ahmad, Irons and Zienkiewicz, but it considers the effect of curvature in the calculation of external reactions to each element, as foreseen by the classical theory of and shells.

2 CLASSICAL SHELL THEORY

The theory of shells is strongly related to space curves and surface theories, because the most important characteristic of a shell is its reference surface, which determines the shell shape and behavior [5].

There are three basic sets of equations in elasticity theory: the equilibrium equations, kinematic relations (strain-displacement) and constitutive relations (Hooke's law) [6]. These equations were originally derived by Love, in 1888, and together with assumptions made, they form a theory of elastic shells commonly referred as Love's first approximation [5].

2.1 Basic assumptions

Love's first approximation to the theory of thin shells can be summarized by the following postulates:

- The shell is thin: one of the dimensions is considerably smaller than the other two.
- The shell deflections are assumed to be small, compared to its thickness.
- The transverse normal stress (direction normal to the thin dimension) is negligible.
- A line originally normal to the reference remains normal and undergo no change in length during deformation.

The first statement is the basis to the entire theory, and it makes the other postulates possible. Although no precise definition of thinness is available, it should be possible to state that the relation of the shell thickness (h) to one of the radius of curvature (R_i) is small compared to unity ($h/R_i \ll 1$). As a general rule, it is suggested that the resulting theory be applied to shells whose thickness is everywhere less than one tenth of the radius of curvature of the reference surface.

The small deflection assumption allows the theory to be entirely referred to the original configuration of the shell, thus we do not need to distinguish between Eulerian and Lagrangian descriptions, and along with Hooke's law, the resulting theory will be a linear, elastic one.

The third statement postulates that σ_n is much smaller than the normal in-plane stresses σ_{11} and σ_{22}. Setting down Hooke's law for an homogeneous, orthotropic elastic medium, whose three mutually perpendicular planes of elastic symmetry are associated with the mutually perpendicular α_1, α_2 and with the normal direction, we obtain the following equations:

$$\varepsilon_{11} = \frac{\sigma_{11}}{E_{11}} - \frac{v_{12}}{E_{22}}\sigma_{22} - \frac{v_{1n}}{E_n}\sigma_n \tag{1}$$

$$\varepsilon_{22} = \frac{\sigma_{22}}{E_{22}} - \frac{v_{21}}{E_{11}}\sigma_{11} - \frac{v_{2n}}{E_n}\sigma_n \tag{2}$$

$$\varepsilon_n = \frac{\sigma_n}{E_n} - \frac{v_{n1}}{E_1}\sigma_{11} - \frac{v_{n2}}{E_{22}}\sigma_{22} \tag{3}$$

$$\gamma_{12} = \frac{\tau_{12}}{G_{12}} \tag{4}$$

$$\gamma_{1n} = \frac{\tau_{1n}}{G_{1n}} \tag{5}$$

$$\gamma_{2n} = \frac{\tau_{2n}}{G_{2n}} \tag{6}$$

where σ_{11}, σ_{22} and σ_n are the normal stresses along the three mutually perpendicular directions, ε_{11}, ε_{22} and ε_n are the corresponding normal strains, γ_{12}, γ_{1n} and γ_{2n} are the shearing strains, and τ_{12}, τ_{1n} and τ_{2n} are the shearing stresses. E_{11}, E_{22}, E_n, G_{12}, G_{1n}, G_{2n} and v_{12}, v_{21}, v_{1n}, v_{2n}, v_{n1}, v_{n2} are the elastic constants (Young's modulus, shear modulus and Poisson's ratio) along the three coordinate directions.

The fourth of Love's postulates is a hypothesis that concerns the preservation of the normal element, and it is an analogy to the Euler hypothesis of plane sections remaining plane in beam theory. The assumption that the normal remains constant during deformation implies that the strain components in the direction of the normal to the reference surface vanish, therefore:

$$\varepsilon_n = \gamma_{1n} = \gamma_{2n} = \sigma_n = 0 \tag{7}$$

In conjunction with the isotropic Hooke's law, we can write:

$$\tau_{12} = G\gamma_{12} = 2G\varepsilon_{12} \tag{8}$$

and

$$\tau_{1n} = \tau_{2n} = 0 \tag{9}$$

As a consequence of third and fourth of Love's postulates, the stress-strain relation system if reduced to the following bi-dimensional constitutive law of thin elastic shells:

$$\varepsilon_{11} = \frac{\sigma_{11}}{E_{11}} - \frac{v_{12}}{E_{22}}\sigma_{22} \tag{10}$$

$$\varepsilon_{22} = \frac{\sigma_{22}}{E_{22}} - \frac{v_{21}}{E_{11}}\sigma_{11} \tag{11}$$

$$\gamma_{12} = \frac{\tau_{12}}{G_{12}} \tag{12}$$

As well as equations already described in (7) [5-7].

2.2 Shell coordinates

The statement of preservation of the normal implies that the displacements be linear thought the thickness of the shell, therefore the behavior of any point in the shell can be related to the behavior of the reference surface of the shell using a distance ζ from this surface.

In order to describe the position of an arbitrary point in the shell, we define the position vector $\boldsymbol{R}(\alpha_1, \alpha_2, \zeta)$:

$$\boldsymbol{R}(\alpha_1, \alpha_2, \zeta) = \boldsymbol{r}(\alpha_1, \alpha_2) + \zeta\boldsymbol{n}(\alpha_1, \alpha_2) \tag{13}$$

where **r** is the position vector of a point in the reference surface, **n** is the unit normal vector to the reference surface at **r**, and ζ is the distance between the point and the reference surface along **n**. α_1 and α_2 are the parametric lines of the reference surface, and coincide with the orthogonal lines of principal curvature.

The magnitude of an arbitrary differential element of length in the space defined by vector $\boldsymbol{R}(\alpha_1, \alpha_2, \zeta)$ can be obtained by means of the first fundamental form of a surface located at a distance ζ from the reference surface, as indicated in the following equation.

$$(ds)^2 = d\boldsymbol{R}.d\boldsymbol{R} = (d\boldsymbol{r} + \zeta d\boldsymbol{n} + \boldsymbol{n}d\zeta).(d\boldsymbol{r} + \zeta d\boldsymbol{n} + \boldsymbol{n}d\zeta) \tag{14}$$

Proceeding with the indicated operation, and considering the orthogonality of the coordinate system, the result is:

$$(ds)^2 = {A_1}^2\left(1 + \frac{\zeta}{R_1}\right)^2 (d\alpha_1)^2 + {A_2}^2\left(1 + \frac{\zeta}{R_2}\right)^2 (d\alpha_2)^2 + (d\zeta)^2 \tag{15}$$

Once the coordinate system is stablished, a shell element of thickness $d\zeta$, at a distance ζ of the reference surface, as illustrated in Figure 1 can be defined. The lengths of the edges of this fundamental element can be defined as:

$$ds_1(\zeta) = A_1\left(1 + \frac{\zeta}{R_1}\right) d\alpha_1 \tag{16}$$

$$ds_2(\zeta) = A_2\left(1 + \frac{\zeta}{R_2}\right) d\alpha_2 \tag{17}$$

And the differential areas of the fundamental element can be defined as:

$$d\Sigma_1(\zeta) = A_1\left(1 + \frac{\zeta}{R_1}\right) d\alpha_1 d\zeta \tag{18}$$

$$d\Sigma_2(\zeta) = A_2\left(1 + \frac{\zeta}{R_2}\right) d\alpha_2 d\zeta \tag{19}$$

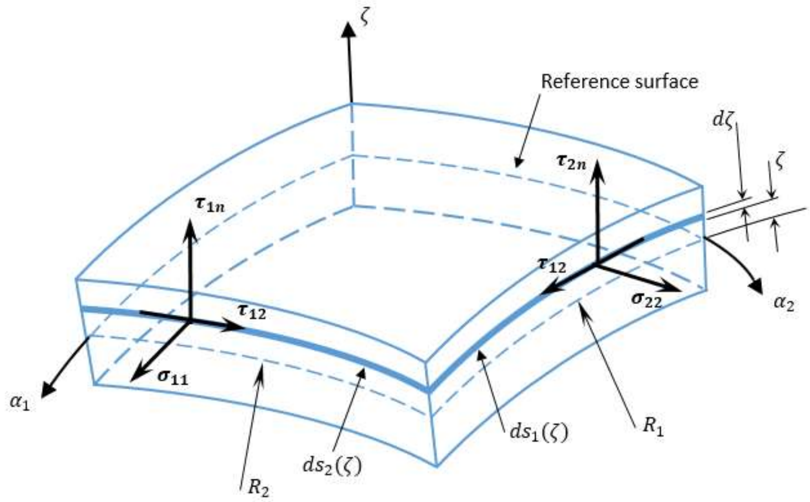

Figure 1 – Differential element of a shell

The definition of the fundamental element and the coordinate system allows the derivation of the equations of the shell theory [5-6].

2.3 Strain-displacement relations

In order to define the equations that relate displacements and strains for the shell theory, a displacement vector is firstly defined:

$$\boldsymbol{U}(\alpha_1,\alpha_2,\zeta) = U_1(\alpha_1,\alpha_2,\zeta)\boldsymbol{t_1} + U_2(\alpha_1,\alpha_2,\zeta)\boldsymbol{t_2} + W(\alpha_1,\alpha_2,\zeta)\boldsymbol{n} \tag{20}$$

where $\boldsymbol{t_1}, \boldsymbol{t_2}$, $\mathbf{n}$ are unit vector along α_1, α_2, and the normal to the reference surface, and U_1, U_2, W are the components of the displacement vector in the corresponding orthogonal coordinate directions.

The normal and shearing components of strain in an orthogonal curvilinear coordinate system, are [8]:

$$\varepsilon_i = \frac{\partial}{\partial\alpha_i}\left(\frac{u_i}{\sqrt{g_i}}\right) + \frac{1}{2g_i}\sum_{k=1}^{3}\frac{\partial g_i}{\partial\alpha_k}\frac{u_k}{\sqrt{g_k}}, \qquad i = 1, 2, 3 \tag{21}$$

$$\gamma_{ij} = \frac{1}{\sqrt{g_i g_j}}\left[g_i\frac{\partial}{\partial\alpha_j}\left(\frac{u_i}{\sqrt{g_i}}\right) + g_j\frac{\partial}{\partial\alpha_i}\left(\frac{u_j}{\sqrt{g_j}}\right)\right], \qquad i = 1, 2, 3 \qquad i \neq j \tag{22}$$

where, for the application to shells:

$$\begin{aligned}
\alpha_1 &= \alpha_1\\
\alpha_2 &= \alpha_2\\
\alpha_3 &= \zeta\\
u_1 &= U_1\\
u_2 &= U_2\\
u_3 &= \mathrm{W}\\
g_1 &= {A_1}^2\left(1+\frac{\zeta}{R_1}\right)^2\\
g_2 &= {A_2}^2\left(1+\frac{\zeta}{R_2}\right)^2\\
g_3 &= 1
\end{aligned} \tag{23}$$

The following strain-displacement equations are then obtained:

$$\varepsilon_1 = \frac{1}{A_1\left(1+\frac{\zeta}{R_1}\right)}\left(\frac{\partial U_1}{\partial\alpha_1} + \frac{U_2}{A_2}\frac{\partial A_1}{\partial\alpha_2} + \frac{A_1 W}{R_1}\right) \tag{24}$$

$$\varepsilon_2 = \frac{1}{A_2\left(1+\frac{\zeta}{R_2}\right)}\left(\frac{\partial U_2}{\partial\alpha_2} + \frac{U_1}{A_1}\frac{\partial A_2}{\partial\alpha_1} + \frac{A_2 W}{R_2}\right) \tag{25}$$

$$\varepsilon_n = \frac{\partial W}{\partial\zeta} \tag{26}$$

$$\gamma_{1n} = \frac{1}{A_1\left(1+\frac{\zeta}{R_1}\right)}\frac{\partial W}{\partial\alpha_1} + A_1\left(1+\frac{\zeta}{R_1}\right)\frac{\partial}{\partial\zeta}\left[\frac{U_1}{A_1\left(1+\frac{\zeta}{R_1}\right)}\right] \tag{27}$$

$$\gamma_{2n} = \frac{1}{A_1\left(1+\frac{\zeta}{R_2}\right)}\frac{\partial W}{\partial\alpha_2} + A_1\left(1+\frac{\zeta}{R_2}\right)\frac{\partial}{\partial\zeta}\left[\frac{U_2}{A_2\left(1+\frac{\zeta}{R_2}\right)}\right] \tag{28}$$

$$\gamma_{12} = \frac{A_2\left(1+\frac{\zeta}{R_2}\right)}{A_1\left(1+\frac{\zeta}{R_1}\right)}\frac{\partial}{\partial\alpha_1}\left[\frac{U_2}{A_2\left(1+\frac{\zeta}{R_2}\right)}\right] + \frac{A_1\left(1+\frac{\zeta}{R_1}\right)}{A_2\left(1+\frac{\zeta}{R_2}\right)}\frac{\partial}{\partial\alpha_2}\left[\frac{U_1}{A_1\left(1+\frac{\zeta}{R_1}\right)}\right] \quad (29)$$

The equations above do not yet reflect Love's postulates, once no simplifications at all were applied, in the context of infinitesimal strain. Considering de assumption of the preservation of the normal, the displacements are linearly distributed through the thickness of the shell. We may then assume the following relations represent the displacement components:

$$U_1(\alpha_1,\alpha_2,\zeta) = u_1(\alpha_1,\alpha_2) + \zeta u_1{}'(\alpha_1,\alpha_2,0) \quad (30)$$

$$U_2(\alpha_1,\alpha_2,\zeta) = u_2(\alpha_1,\alpha_2) + \zeta u_2{}'(\alpha_1,\alpha_2,0) \quad (31)$$

$$W(\alpha_1,\alpha_2,\zeta) = w(\alpha_1,\alpha_2) \quad (32)$$

where a prime denotes the derivative with respect to ζ. u_1, u_2 and w represent the components of the displacement vector of a point on the reference surface, and $u_1{}'$and $u_2{}'$ represent the rotations of tangents to the surface oriented along the parametric lines α_1 and α_2, respectively. These rotations are from now denoted β_1 and β_2, respectively, and can be determined from the hypothesis $\gamma_{1n} = \gamma_{2n} = 0$, and substituting U_1, U_2 and W, the following equations are obtained:

$$\beta_1 = \frac{u_1}{R_1} - \frac{1}{A_1}\frac{\partial w}{\partial\alpha_1} \quad (33)$$

$$\beta_2 = \frac{u_2}{R_2} - \frac{1}{A_2}\frac{\partial w}{\partial\alpha_2} \quad (34)$$

The substitution of the equations 30-32 into the exact strain-displacement relations yields:

$$\varepsilon_{11} = \frac{1}{\left(1+\frac{\zeta}{R_1}\right)}(\varepsilon_1^0 + \zeta\kappa_1) \quad (35)$$

$$\varepsilon_{22} = \frac{1}{\left(1+\frac{\zeta}{R_2}\right)}(\varepsilon_2^0 + \zeta\kappa_2) \quad (36)$$

$$\gamma_{12} = \frac{1}{\left(1+\frac{\zeta}{R_1}\right)}(\omega_1 + \zeta\tau_1) + \frac{1}{\left(1+\frac{\zeta}{R_2}\right)}(\omega_2 + \zeta\tau_2) \quad (37)$$

$$\varepsilon_n = \gamma_{1n} = \gamma_{2n} = 0 \quad (38)$$

where:

$$\varepsilon_1^0 = \frac{1}{A_1}\frac{\partial u_1}{\partial\alpha_1} + \frac{u_2}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2} + \frac{w}{R_1} \quad (39)$$

$$\varepsilon_2^0 = \frac{1}{A_2}\frac{\partial u_2}{\partial \alpha_2} + \frac{u_1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1} + \frac{w}{R_2} \tag{40}$$

$$\kappa_1 = \frac{1}{A_1}\frac{\partial \beta_1}{\partial \alpha_1} + \frac{\beta_2}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} \tag{41}$$

$$\kappa_2 = \frac{1}{A_2}\frac{\partial \beta_2}{\partial \alpha_2} - \frac{\beta_1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1} \tag{42}$$

$$\omega_1 = \frac{1}{A_1}\frac{\partial u_2}{\partial \alpha_1} - \frac{u_1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} \tag{43}$$

$$\omega_2 = \frac{1}{A_2}\frac{\partial u_1}{\partial \alpha_2} - \frac{u_2}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1} \tag{44}$$

$$\tau_1 = \frac{1}{A_1}\frac{\partial \beta_2}{\partial \alpha_1} - \frac{\beta_1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} \tag{45}$$

$$\tau_2 = \frac{1}{A_2}\frac{\partial \beta_1}{\partial \alpha_2} - \frac{\beta_2}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1} \tag{46}$$

The quantities ε_1^0, ε_1^0, ω_1 and ω_2 represent the normal and shearing strains of the reference surface as can be confirmed if it is set $\zeta = 0$. The quantities κ_1 and κ_2 represent the change in the curvature, and the quantities τ_1 and τ_2 represent the torsion of the reference surface during deformation [5,6].

2.4 Stress resultants and stress couples

The strain, and therefore the stresses, are linearly distributed along the thickness of a thin elastic shell. In order to have an entirely bi-dimensional theory, it is convenient to integrate the stresses through the thickness, this way eliminating the variations along ζ and obtaining statically equivalent stress resultants (N_{ij}) and stress couples (M_{ij}).

The stresses have to be first calculated from the strains, using the constitutive relations, which for this application can be written as:

$$\sigma_{11} = E_{11}^* \varepsilon_{11} + v_{21} E_{22}^* \varepsilon_{22} \tag{47}$$

$$\sigma_{22} = E_{22}^* \varepsilon_{22} + v_{12} E_{11}^* \varepsilon_{11} \tag{48}$$

$$\tau_{12} = G_{12} \gamma_{12} \tag{49}$$

where:

$$E_i^* = \frac{E_i}{\left(\frac{1}{v_{12} v_{21}}\right)}, \quad \mathrm{i} = 1{,}2 \tag{50}$$

Proceeding with the integration of the stress distributions across the thickness of the shell, the stress resultants and stress couples obtained are defined per unit of arc length on the reference surface.

The resultants and couples of the stress σ_{11} distributed over an α_1 = constant face of the fundamental element of the shell are given by:

$$N_{11} = \int_\zeta \frac{\sigma_1 d\Sigma_2(\zeta)}{ds_2(0)} = \int_\zeta \sigma_1 \frac{A_2\left(1+\frac{\zeta}{R_2}\right)d\alpha_2 d\zeta}{A_2 d\alpha_2} = \int_\zeta \sigma_1 \left(1+\frac{\zeta}{R_2}\right) d\zeta \tag{51}$$

$$M_{11} = \int_\zeta \frac{\zeta\sigma_1 d\Sigma_2(\zeta)}{ds_2(0)} = \int_\zeta \sigma_1 \frac{A_2\left(1+\frac{\zeta}{R_2}\right)d\alpha_2 \zeta d\zeta}{A_2 d\alpha_2} = \int_\zeta \sigma_1 \left(1+\frac{\zeta}{R_2}\right) \zeta d\zeta \tag{52}$$

In a similar way all the reactions can be determined:

$$N_{12} = \int_\zeta \tau_{12} \left(1+\frac{\zeta}{R_2}\right) d\zeta \tag{53}$$

$$Q_{1n} = \int_\zeta \tau_{1n} \left(1+\frac{\zeta}{R_2}\right) d\zeta \tag{54}$$

$$N_{22} = \int_\zeta \sigma_{22} \left(1+\frac{\zeta}{R_1}\right) d\zeta \tag{55}$$

$$N_{21} = \int_\zeta \tau_{21} \left(1+\frac{\zeta}{R_1}\right) d\zeta \tag{56}$$

$$Q_{2n} = \int_\zeta \tau_{2n} \left(1+\frac{\zeta}{R_1}\right) d\zeta \tag{57}$$

$$M_{12} = \int_\zeta \tau_{12} \left(1+\frac{\zeta}{R_2}\right) \zeta d\zeta \tag{58}$$

$$M_{22} = \int_\zeta \sigma_{22} \left(1+\frac{\zeta}{R_1}\right) \zeta d\zeta \tag{59}$$

$$M_{21} = \int_\zeta \tau_{21} \left(1+\frac{\zeta}{R_1}\right) \zeta d\zeta \tag{60}$$

It is important to notice that the symmetry of the stress tensor ($\tau_{12} = \tau_{21}$) does not necessarily imply that N_{12} and N_{21}, as well as M_{12} and M_{21} are equal, except when both radius R_1 and R_2 are the same, such as in a spherical or flat shell [5,6].

3 SHELL FINITE ELEMENT

Solving a three-dimensional shell problem presents difficulties due to the high number of degrees of freedom involved and numerical issues when shell thickness become small compared with the other dimensions in the element.

Using the formulation proposed by [8], the three-dimensional problem can be reduced to a two-dimensional problem. In this formulation, the constraint of straight ‘normals’ is applied, while the energy corresponding to stresses perpendicular to the reference surface is ignored. The statement of a normal to the reference surface remaining normal after deformation, found in the theory of thin shells, has been deliberately omitted, which is important because it allows thick shells to experience shear deformations.

3.1 Element geometry and displacement

Considering a shell element as shown in Figure 2, the external faces of the element are curved, while the sections across the thickness are straight lines, and the reference surface is defined as the middle surface. A vector $\boldsymbol{V}_{3i}$, normal to the reference surface, can be defined as:

$$\boldsymbol{V}_{3i} = \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{top} - \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{bottom} \tag{61}$$

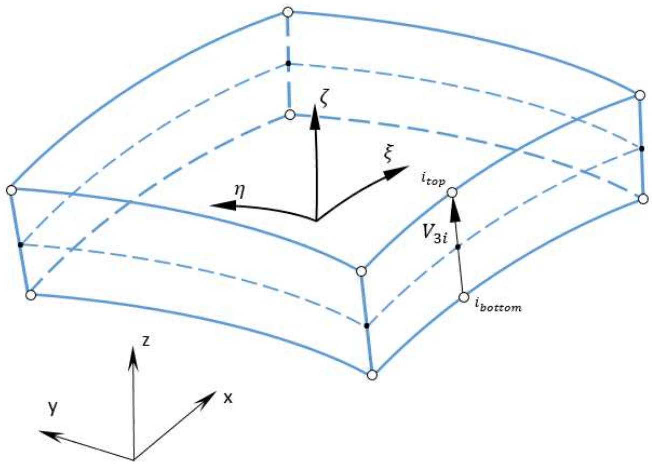

Figure 2 – Curved shell element

Defining ξ and η as two curvilinear coordinates in the reference surface of the shell and ζ a coordinate in the direction of a normal to this surface, and assuming that ξ, η and ζ vary between -1 and 1, it is possible to define a relation between the Cartesian and curvilinear coordinates, as:

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \sum N_i \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{ref} + \sum N_i \frac{\zeta}{2} \boldsymbol{V}_{3i} \tag{62}$$

where $N_i(\xi, \eta)$ are the 'shape functions', which take a value of unity at the node *i* and zero at all the other nodes.

The displacements are defined by the three Cartesian components of the mid-surface node and two rotations of the nodal vector $\boldsymbol{V}_{3i}$ about orthogonal directions normal to it. Considering that these two directions are defined by unity vectors $\boldsymbol{v}_{2i}$ and $\boldsymbol{v}_{1i}$, with corresponding rotations α_i and β_i, it is possible to write:

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum N_i \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix}_{ref} + \sum N_i \, \zeta \, \frac{t_i}{2} [\boldsymbol{v}_{1i} - \boldsymbol{v}_{2i}] \begin{Bmatrix} \alpha_i \\ \beta_i \end{Bmatrix} \tag{63}$$

Or, as a matrix:

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{i=1}^{n_{nos}} \begin{bmatrix} N_i & 0 & 0 & \frac{a_i}{2}\zeta N_i \boldsymbol{v}_{1x}{}^{i} & \frac{a_i}{2}\zeta N_i \boldsymbol{v}_{2x}{}^{i} \\ 0 & N_i & 0 & \frac{a_i}{2}\zeta N_i \boldsymbol{v}_{1y}{}^{i} & \frac{a_i}{2}\zeta N_i \boldsymbol{v}_{2y}{}^{i} \\ 0 & 0 & N_i & \frac{a_i}{2}\zeta N_i \boldsymbol{v}_{1z}{}^{i} & \frac{a_i}{2}\zeta N_i \boldsymbol{v}_{2z}{}^{i} \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \alpha_i \\ \beta_i \end{Bmatrix} \tag{64}$$

where u, v and w are the displacements in the directions of the global x, y and z axes.

3.2 Stress and strain

If at a point in a surface of ζ = constant, a normal z' is created, and two other axis x' and y', orthogonal to z' and tangent to this surface are defined, as shown in Figure 3, the strain components of interest are:

$$\{\boldsymbol{\varepsilon}'\} = \begin{Bmatrix} \varepsilon_{x'x'} \\ \varepsilon_{y'y'} \\ \gamma_{x'y'} \\ \gamma_{x'z'} \\ \gamma_{y'z'} \end{Bmatrix} = \begin{Bmatrix} \dfrac{du'}{dx'} \\ \dfrac{dv'}{dy'} \\ \dfrac{du'}{dy'} + \dfrac{dv'}{dx'} \\ \dfrac{dw'}{dx'} + \dfrac{du'}{dz'} \\ \dfrac{dw'}{dy'} + \dfrac{dv'}{dz'} \end{Bmatrix} \tag{65}$$

It is important to notice that the strain $\varepsilon_{z'z'}$, in the direction of z' is neglected, according to the shell assumption of a normal to the reference surface keeping undeformed.

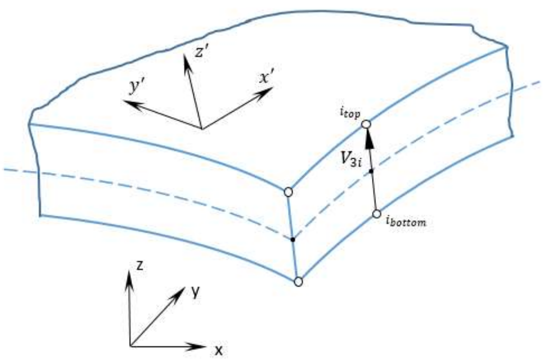

Figure 3 – Local coordinates

In order to obtain the vector of local stresses $\{\boldsymbol{\sigma}'\}$, the local elasticity matrix $[\boldsymbol{D}']$ is multiplied by the already obtained vector of local strains $\{\boldsymbol{\varepsilon}'\}$:

$$\{\boldsymbol{\sigma}'\} = [\boldsymbol{D}']\,\{\boldsymbol{\varepsilon}'\} \tag{66}$$

And the local elasticity matrix $[\boldsymbol{D}']$ is defined as:

$$[\boldsymbol{D}'] = \frac{E}{(1-v^2)}\begin{bmatrix} 1 & v & 0 & 0 & 0 \\ v & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-v}{v} & 0 & 0 \\ 0 & 0 & 0 & \frac{k(1-v)}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{k(1-v)}{2} \end{bmatrix} \quad (67)$$

where E and v are Young's modulus and Poisson's ratio. A factor k is included in the last two shear terms and it is defined as 5/6, and its purpose is to improve the shear displacement approximations. The displacement definition defines that the shear distribution is approximately constant through the thickness, while in reality the shear distribution for elastic behavior is approximately parabolic [1,10].

3.3 Stiffness matrix

The stiffness matrix $[\boldsymbol{K}]$ is obtained by the integral over the volume of the element as shown in the following equation:

$$[\boldsymbol{K}] = \int_V [\boldsymbol{B}]^T[\boldsymbol{D}][\boldsymbol{B}]\, dxdydz \quad (68)$$

where $[\boldsymbol{B}]$ is the matrix that related the strains with the nodal displacements in the element, defined as:

$$\{\boldsymbol{\varepsilon}\} = [\boldsymbol{B}]\{\boldsymbol{\delta}\}^e \quad (69)$$

The internal equations in the integral that the defines the stiffness matrix can be defined as a matrix $[\boldsymbol{S}]$, according to the following equation:

$$[\boldsymbol{S}] = [\boldsymbol{B}]^T[\boldsymbol{D}][\boldsymbol{B}] \quad (70)$$

As the integral that defines the stiffness matrix is over the volume of the element, thus in the global coordinate system, it is necessary to express $[\boldsymbol{S}]$ as a function of the curvilinear coordinates, as well as transform the infinitesimal volume dxdydz, in order to apply a numerical integration. The global displacements u, v and w are related to the curvilinear coordinates ξ, η and ζ by equation 63.

The matrix relation between the derivatives of these displacements with respect to the global x, y and z coordinates and the derivatives of these displacements with the curvilinear coordinates is:

$$\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial z} \end{bmatrix} = [J]^{-1}\begin{bmatrix} \frac{\partial u}{\partial \xi} & \frac{\partial v}{\partial \xi} & \frac{\partial w}{\partial \xi} \\ \frac{\partial u}{\partial \eta} & \frac{\partial v}{\partial \eta} & \frac{\partial w}{\partial \eta} \\ \frac{\partial u}{\partial \zeta} & \frac{\partial v}{\partial \zeta} & \frac{\partial w}{\partial \zeta} \end{bmatrix} \quad (71)$$

The Jacobian matrix has its components defined from equation 62 and is represented as:

$$[\boldsymbol{J}] = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta} \end{bmatrix} \tag{72}$$

Using the previously defined equations, the global displacement derivatives can be determined numerically for every set of curvilinear coordinates. A further transformation to local displacement directions x', y' and z' will allow the strains and thus the matrix $[\boldsymbol{B}]$ to be evaluated.

In order to define a local coordinate system x', y' and z', it is necessary to uniquely define two vectors perpendicular to a known vector $\boldsymbol{V}_3$, normal to the reference surface at a defined point. Considering a vector î, as the unit vector along the x axis, a vector $\boldsymbol{V}_1$, perpendicular to the plane formed by the vectors $\boldsymbol{V}_3$ and $\boldsymbol{i}$ can be defined by the following equation:

$$\boldsymbol{V}_1 = \boldsymbol{i} \ x \, \boldsymbol{V}_3 \tag{73}$$

A vector $\boldsymbol{V}_2$ has to be orthogonal to both vectors $\boldsymbol{V}_1$ and $\boldsymbol{V}_3$, thus it is possible to define as:

$$\boldsymbol{V}_2 = \boldsymbol{V}_3 \, x \, \boldsymbol{V}_1 \tag{74}$$

The unit vectors in the three directions $\boldsymbol{v}_1$, $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$, are obtained by the division of $\boldsymbol{V}_1$, $\boldsymbol{V}_2$ and $\boldsymbol{V}_3$ by their scalar lengths.

If the direction of the vector $\boldsymbol{V}_3$ is exactly the same as the axis x, then the unit vector $\boldsymbol{i}$ should be replaced by a unity vector $\boldsymbol{j}$, along the axis y.

A cosine matrix can ben defined with the unit vectors in x', y' and z' directions, as:

$$[\boldsymbol{\theta}] = [\boldsymbol{v}_1 \quad \boldsymbol{v}_2 \quad \boldsymbol{v}_3] \tag{75}$$

The global derivatives of displacements u, v and w can then be transformed to the local derivatives of the local orthogonal displacements by a standard operation:

$$\begin{bmatrix} \frac{\partial u'}{\partial x'} & \frac{\partial v'}{\partial x'} & \frac{\partial w'}{\partial x'} \\ \frac{\partial u'}{\partial y'} & \frac{\partial v'}{\partial y'} & \frac{\partial w'}{\partial y'} \\ \frac{\partial u'}{\partial z'} & \frac{\partial v'}{\partial z'} & \frac{\partial w'}{\partial z'} \end{bmatrix} = [\boldsymbol{\theta}]^T \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial z} \end{bmatrix} [\boldsymbol{\theta}] \tag{76}$$

Defined the components of local strain, matrix $[\boldsymbol{B}']$ can be determined:

$$\{\boldsymbol{\varepsilon}'\} = [\boldsymbol{B}'] \begin{Bmatrix} \{\boldsymbol{\delta}_1\} \\ \{\boldsymbol{\delta}_1\} \\ \vdots \\ \{\boldsymbol{\delta}_j\} \end{Bmatrix} \tag{77}$$

where $\{\boldsymbol{\delta_i}\}$ is defined as:

$$\{\boldsymbol{\delta_i}\} = \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \alpha_i \\ \beta_i \end{Bmatrix} \tag{78}$$

where i represents each node of the element.

The infinitesimal volume is transformed from the global do the local coordinate system using the Jacobian, the determinant of Jacobian matrix, as follows:

$$dxdydz = determinan \ \ [\boldsymbol{J}]\ d\xi d\eta d\zeta \tag{79}$$

4 METHODOLOGY AND RESULTS

A set of different configurations of a shell is simulated in order to compare the influence of the curvature. The shell used in the simulation is a structure curved in one direction and flat in the other direction. The comparison provided in the following cases is of a curved shell with 180°, radius of 0,5m, and 0,1m of width, divided in 10 elements. The thickness of the structure was set as 0,1m, in order to have a thick shell with relation h/R=0,2 (thickness/radius). The dimensions of the shell are exposed in Figure 4.

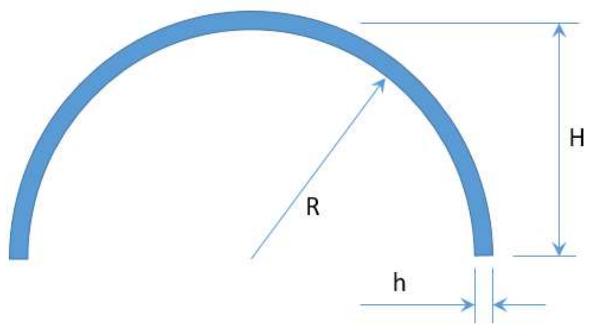

Figure 4 – Dimensions of the shell.

In all cases, the two nodes at position x=0 and z=0 were fixed (all five DOF fixed), as shown in Figure 5.

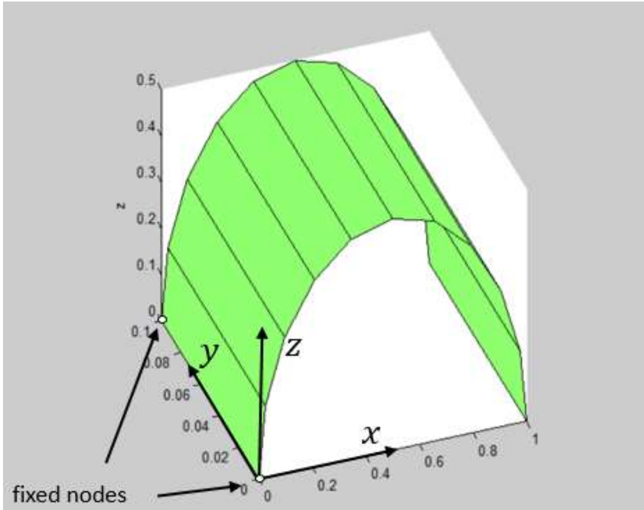

Figure 5 – Constraints applied to the shell.

Forces applied vary for each simulation and are shown in Figure 6.

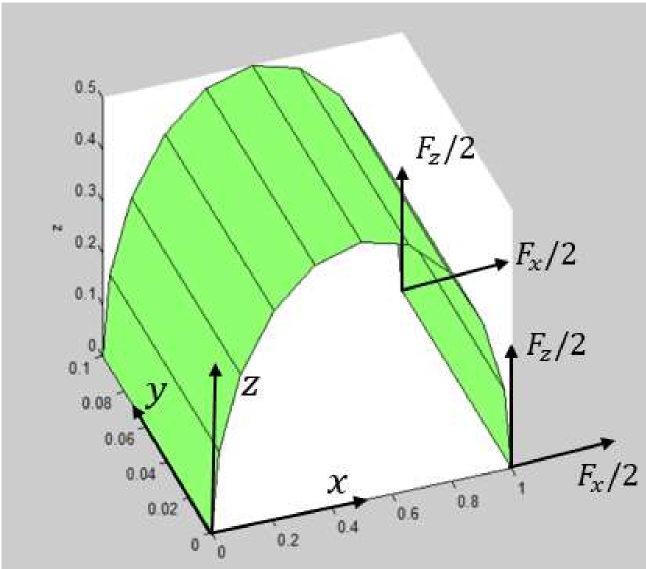

Figure 6 – Forces applied to the shell.

Two different modes of analysis were defined. In the first mode, the normal vectors of each node shared by different elements are averaged by the area of each element, and this is defined as "smooth normal". In a second mode, the normal vectors to each node are not averaged, thus the behaviour of the element is similar to a plate, and this was defined as "not smoothed normal'.

Elements in the shell are numbered starting from the side of the shell which lies in the position x=0, as shown in Figure 7. Each element has four nodes which follow the order presented also in Figure 7.

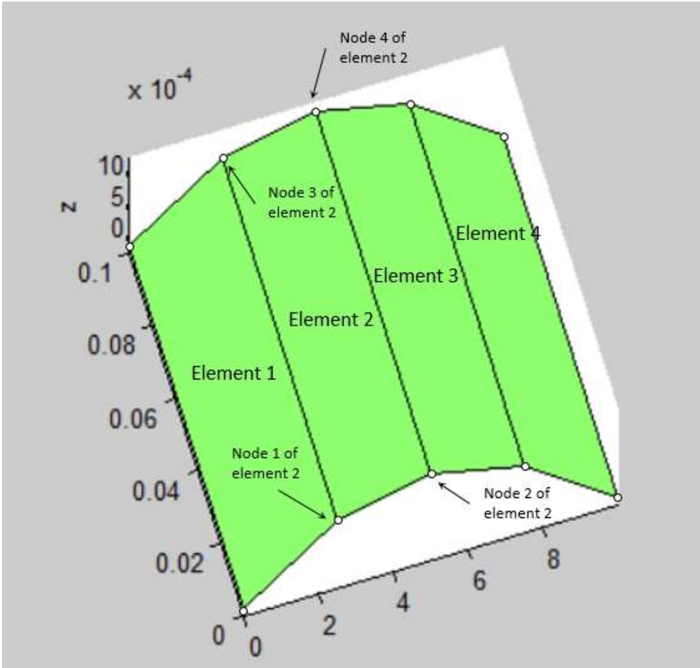

Figure 7 – Example of the distribution of elements in a structure, and nodes inside each element.

The couple stress resultants obtained for each case are presented in tables, and a relation between the resultants obtained with and without the application of the curvature in the calculation is also presented. This relation is obtained for each node of each element with the following equation:

$$relation = \frac{M_{curved} - M_{flat}}{M_{flat}} \quad (80)$$

where M_{curved} are the resultants for each node of each element obtained considering the effect of the curvature, and M_{flat} are the resultants for each node of each element obtained without the curvature.

The relation between the resultants is evaluated, searching for the maximum value, as well as the resultants in the fixed nodes.

Since the structures are usually developed in order to withstand the maximum reaction values, the variations with and without the consideration of the effect of the curvature, are also compared to the maximum reaction obtained in the whole structure, defining the impact the variation has on the whole structure.

4.1 Case 1

The first case simulated of a thick shell, with $\frac{h}{R} = 0{,}2$, and application of load in the x direction of a structure whose normal of each element are smooth. The general data related to this simulation is presented in Table 1.

Table 1 - General data for simulation 1.

Dimensional properties				Loads		
Thickness [m]	h =	0,2		Fx [N]	Fx =	1
Radius [m]	R =	1,0		Fz [N]	Fz =	0
Height [m]	H =	1,0				
Angle [degree]	θ =	180°				
Relation h/R	h/R =	0,2				

The analysis with the proposed geometry and loads for this case has the results presented in the following tables. Table 2 contains the couple stresses M_{yy} calculated without consideration of the curvature of each element, Table 3 contains the couple stresses M_{yy} calculated considering the curvature of each element, and Table 4 is the relation between couple stresses calculated with and without the consideration of the curvature for each node of each element.

Table 2 – Couple stress resultants M_{yy} without the curvature of the elements.

Myy - flat		Node 1	Node 2	Node 3	Node 4
Element	1	-0,0518	-0,0434	-0,0518	-0,0434
	2	-0,1224	-0,1220	-0,1224	-0,1220
	3	-0,1703	-0,1538	-0,1703	-0,1538
	4	-0,2270	-0,1104	-0,2270	-0,1104
	5	-0,1819	-0,0776	-0,1819	-0,0776
	6	-0,0776	-0,1819	-0,0776	-0,1819
	7	-0,1104	-0,2270	-0,1104	-0,2270
	8	-0,1538	-0,1702	-0,1538	-0,1702
	9	-0,1218	-0,1231	-0,1218	-0,1231
	10	-0,0442	-0,0493	-0,0442	-0,0493

Table 3 – Couple stresses results M_{yy} considering the curvature of the elements.

Myy - curved		Node			
		1	2	3	4
Element	1	-0,0513	-0,0442	-0,0513	-0,0442
	2	-0,1211	-0,1207	-0,1211	-0,1207
	3	-0,1695	-0,1551	-0,1695	-0,1551
	4	-0,2270	-0,1084	-0,2270	-0,1084
	5	-0,1799	-0,0725	-0,1799	-0,0725
	6	-0,0725	-0,1799	-0,0725	-0,1799
	7	-0,1085	-0,2270	-0,1085	-0,2270
	8	-0,1551	-0,1694	-0,1551	-0,1694
	9	-0,1206	-0,1218	-0,1206	-0,1218
	10	-0,0450	-0,0488	-0,0450	-0,0488

Table 4 – Relation between couple stresses resultants M_{yy} with and without the curvature of the elements.

Myy - relation		Node			
		1	2	3	4
Element	1	0,0094	-0,0184	0,0094	-0,0184
	2	0,0100	0,0101	0,0100	0,0101
	3	0,0047	-0,0086	0,0047	-0,0086
	4	0,0000	0,0180	0,0000	0,0180
	5	0,0109	0,0660	0,0109	0,0660
	6	0,0660	0,0109	0,0660	0,0109
	7	0,0180	0,0000	0,0180	0,0000
	8	-0,0085	0,0046	-0,0085	0,0046
	9	0,0099	0,0104	0,0099	0,0104
	10	-0,0169	0,0088	-0,0169	0,0088

The maximum variation between the couple stress resultants with and without the consideration of the curvature of the element is 6,60%, in nodes 1 and 3 of element 8. The variation in the constraint region (fixed nodes) is of 0,94%. Nodes 2 and 4 of element 5 are the nodes where the variation is most representative in relation to maximum resultants in the whole structure. The variation in these nodes represents 2,26% of maximum values of resultants calculated in the structure.

4.2 Case 2

The second case simulated is geometrically identical to the first case. It is a thick shell, with $\frac{h}{R} = 0{,}2$, and application of load in the x direction, but in this case the normal vectors of each node of each element are not smoothed. The general data related to this simulation is presented in Table 5.

Table 5 – General data for simulation 2.

Dimensional properties				Loads		
Thickness [m]	h =	0,2		Fx [N]	Fx =	1
Radius [m]	R =	1,0		Fz [N]	Fz =	0
Height [m]	H =	1,0				
Angle [degree]	θ =	180°				
Relation h/R	h/R =	0,2				

The analysis with the proposed geometry and loads for this case has the results presented in the following tables. Table 6 contains the couple stresses M_{yy} calculated without consideration of the curvature of each element, Table 7 contains the couple stresses M_{yy} calculated considering the curvature of each element, and Table 8 is the relation between couple stresses calculated with and without the consideration of the curvature for each node of each element.

Table 6 - Couple stress resultants M_{yy} without the curvature of the elements.

Myy - flat		**Node**			
		1	**2**	**3**	**4**
Element	**1**	-0,0462	-0,0367	-0,0462	-0,0367
	2	-0,1195	-0,1170	-0,1195	-0,1170
	3	-0,1758	-0,1597	-0,1758	-0,1597
	4	-0,2280	-0,1181	-0,2280	-0,1181
	5	-0,1674	-0,0692	-0,1674	-0,0692
	6	-0,0692	-0,1674	-0,0692	-0,1674
	7	-0,1181	-0,2280	-0,1181	-0,2280
	8	-0,1597	-0,1757	-0,1597	-0,1757
	9	-0,1169	-0,1200	-0,1169	-0,1200
	10	-0,0603	-0,0676	-0,0603	-0,0676

Table 7 - Couple stresses results M_{yy} considering the curvature of the elements.

Myy - curved		**Node**			
		1	**2**	**3**	**4**
Element	**1**	-0,0458	-0,0376	-0,0458	-0,0376
	2	-0,1195	-0,1156	-0,1195	-0,1156
	3	-0,1743	-0,1597	-0,1743	-0,1597
	4	-0,2283	-0,1177	-0,2283	-0,1177
	5	-0,1671	-0,0667	-0,1671	-0,0667
	6	-0,0667	-0,1671	-0,0667	-0,1671
	7	-0,1177	-0,2283	-0,1177	-0,2283
	8	-0,1597	-0,1742	-0,1597	-0,1742
	9	-0,1155	-0,1199	-0,1155	-0,1199
	10	-0,0612	-0,0672	-0,0612	-0,0672

Table 8 - Relation between couple stresses resultants M_{yy} with and without the curvature of the elements.

Myy - relation		**Node**			
		1	**2**	**3**	**4**
Element	**1**	0,0081	-0,0256	0,0081	-0,0256
	2	0,0002	0,0117	0,0002	0,0117
	3	0,0086	0,0002	0,0086	0,0002
	4	-0,0014	0,0033	-0,0014	0,0033
	5	0,0013	0,0356	0,0013	0,0356
	6	0,0356	0,0013	0,0356	0,0013
	7	0,0033	-0,0014	0,0033	-0,0014
	8	0,0002	0,0086	0,0002	0,0086
	9	0,0116	0,0004	0,0116	0,0004
	10	-0,0151	0,0051	-0,0151	0,0051

The maximum variation between the couple stress resultants with and without the consideration of the curvature of the element is 3,56%, in nodes 1 and 3 of element 6. The variation in the constraint region (fixed nodes) is of 0,81%. Nodes 2 and 4 of element 5 are the nodes where the variation is most representative in relation to maximum resultants in the whole structure. The variation in these nodes represents 1,08% of maximum values of resultants calculated in the structure.

4.3 Case 3

The third case simulated is geometrically identical to the first case. It is a thick shell, with $\frac{h}{R} = 0{,}2$, but the load is applied in the z direction of a structure whose normal of each element are smooth. The general data related to this simulation is presented in Table 9.

Table 9 – General data for simulation 3.

Dimensional properties				Loads		
Thickness [m]	h =	0,2		Fx [N]	Fx =	0
Radius [m]	R =	1,0		Fz [N]	Fz =	1
Height [m]	H =	1,0				
Angle [degree]	θ =	180°				
Relation h/R	h/R =	0,2				

The analysis with the proposed geometry and loads for this case has the results presented in the following tables. Table 10 contains the couple stresses M_{yy} calculated without consideration of the curvature of each element, Table 11 contains the couple stresses M_{yy} calculated considering the curvature of each element, and Table 12 is the relation between couple stresses calculated with and without the consideration of the curvature for each node of each element.

Table 10 - Couple stress resultants M_{yy} without the curvature of the elements.

Myy - flat		Node			
		1	2	3	4
Element	1	-0,6388	-0,5675	-0,6388	-0,5675
	2	-0,4624	-0,5008	-0,4624	-0,5008
	3	-0,4014	-0,3822	-0,4014	-0,3822
	4	-0,3662	-0,2318	-0,3662	-0,2318
	5	-0,1987	-0,1314	-0,1987	-0,1314
	6	-0,0254	-0,1647	-0,0254	-0,1647
	7	-0,0408	-0,1447	-0,0408	-0,1447
	8	-0,0664	-0,0763	-0,0664	-0,0763
	9	-0,0335	-0,0325	-0,0335	-0,0325
	10	-0,0063	-0,0081	-0,0063	-0,0081

Table 11 - Couple stresses results M_{yy} considering the curvature of the elements.

Myy - curved		Node 1	Node 2	Node 3	Node 4
Element	1	-0,6353	-0,5695	-0,6353	-0,5695
	2	-0,4622	-0,4956	-0,4622	-0,4956
	3	-0,3994	-0,3832	-0,3994	-0,3832
	4	-0,3648	-0,2273	-0,3648	-0,2273
	5	-0,1962	-0,1254	-0,1962	-0,1254
	6	-0,0208	-0,1635	-0,0208	-0,1635
	7	-0,0410	-0,1463	-0,0410	-0,1463
	8	-0,0693	-0,0771	-0,0693	-0,0771
	9	-0,0340	-0,0325	-0,0340	-0,0325
	10	-0,0066	-0,0081	-0,0066	-0,0081

Table 12 - Relation between couple stresses resultants M_{yy} with and without the curvature of the elements.

Myy - relation		Node 1	Node 2	Node 3	Node 4
Element	1	0,0056	-0,0035	0,0056	-0,0035
	2	0,0006	0,0104	0,0006	0,0104
	3	0,0050	-0,0025	0,0050	-0,0025
	4	0,0038	0,0195	0,0038	0,0195
	5	0,0129	0,0457	0,0129	0,0457
	6	0,1800	0,0076	0,1800	0,0076
	7	-0,0069	-0,0113	-0,0069	-0,0113
	8	-0,0442	-0,0108	-0,0442	-0,0108
	9	-0,0131	-0,0022	-0,0131	-0,0022
	10	-0,0514	0,0051	-0,0514	0,0051

The maximum variation between the couple stress resultants with and without the consideration of the curvature of the element is 18,0%, in nodes 1 and 3 of element 6. The variation in the constraint region (fixed nodes) is of 0,56%. Nodes 2 and 4 of element 5 are the nodes where the variation is most representative in relation to maximum resultants in the whole structure. The variation in these nodes represents 0,94% of maximum values of resultants calculated in the structure.

4.4 Case 4

The fourth case is similar to case 3. It is a thick shell, with $\frac{h}{R} = 0{,}2$, and application of load in the z direction, but in this case the normals of each node of each element are not smoothed. The general data related to this simulation is presented in Table 13.

Table 13 – General data for simulation 4.

Dimensional properties				Loads		
Thickness [m]	h =	0,2		Fx [N]	Fx =	0
Radius [m]	R =	1,0		Fz [N]	Fz =	1
Height [m]	H =	1,0				
Angle [degree]	θ =	180°				
Relation h/R	h/R =	0,2				

The analysis with the proposed geometry and loads for this case has the results presented in the following tables. Table 14 contains the couple stresses M_{yy} calculated without consideration of the curvature of each element, Table 15 contains the couple stresses M_{yy} calculated considering the curvature of each element, and Table 16 is the relation between couple stresses calculated with and without the consideration of the curvature for each node of each element.

Table 14 - Couple stress resultants M_{yy} without the curvature of the elements.

Myy - flat		Node 1	Node 2	Node 3	Node 4
Element	1	-0,5894	-0,5117	-0,5894	-0,5117
	2	-0,4644	-0,5009	-0,4644	-0,5009
	3	-0,4154	-0,4041	-0,4154	-0,4041
	4	-0,3679	-0,2420	-0,3679	-0,2420
	5	-0,1820	-0,1174	-0,1820	-0,1174
	6	-0,0228	-0,1528	-0,0228	-0,1528
	7	-0,0480	-0,1478	-0,0480	-0,1478
	8	-0,0677	-0,0789	-0,0677	-0,0789
	9	-0,0323	-0,0323	-0,0323	-0,0323
	10	-0,0415	-0,0440	-0,0415	-0,0440

Table 15 - Couple stresses results M_{yy} considering the curvature of the elements.

Myy - curved		Node 1	2	3	4
Element	1	-0,5882	-0,5163	-0,5882	-0,5163
	2	-0,4692	-0,4960	-0,4692	-0,4960
	3	-0,4105	-0,4028	-0,4105	-0,4028
	4	-0,3665	-0,2402	-0,3665	-0,2402
	5	-0,1814	-0,1145	-0,1814	-0,1145
	6	-0,0205	-0,1531	-0,0205	-0,1531
	7	-0,0491	-0,1496	-0,0491	-0,1496
	8	-0,0697	-0,0795	-0,0697	-0,0795
	9	-0,0325	-0,0327	-0,0325	-0,0327
	10	-0,0419	-0,0439	-0,0419	-0,0439

Table 16 - Relation between couple stresses resultants M_{yy} with and without the curvature of the elements.

Myy - relation		Node 1	2	3	4
Element	1	0,0021	-0,0090	0,0021	-0,0090
	2	-0,0102	0,0097	-0,0102	0,0097
	3	0,0119	0,0033	0,0119	0,0033
	4	0,0037	0,0071	0,0037	0,0071
	5	0,0035	0,0246	0,0035	0,0246
	6	0,0995	-0,0022	0,0995	-0,0022
	7	-0,0240	-0,0127	-0,0240	-0,0127
	8	-0,0302	-0,0075	-0,0302	-0,0075
	9	-0,0087	-0,0126	-0,0087	-0,0126
	10	-0,0092	0,0012	-0,0092	0,0012

The maximum variation between the couple stress resultants with and without the consideration of the curvature of the element is 9,95%, in nodes 1 and 3 of element 6. The variation in the constraint region (fixed nodes) is of 0,21%. Nodes 1 and 3 of element 3 are the nodes where the variation is most representative in relation to maximum resultants in the whole structure. The variation in these nodes represents 0,84% of maximum values of resultants calculated in the structure.

5 CONCLUSION

The simulation results presented indicate that the curvature of the element cannot simply be excluded from the simulation without loss of precision.

The analysis with a thick shell shows large variations along the shell which could have a big impact locally, since this variation was around 20% in some regions. It is important to notice that the variation in the value of the couple stress resultants was not large, though.

The impact of the variation for each node of each element was then analyzed by a comparison with the maximum resultants in the complete structure, and the result of this comparison, within the four cases analyzed shown a maximum of 2,26%, which is not a large value. In a case where the resultants in a specific point of the surface is important, the maximum local variation can be important.

Based on the results, the consideration of the curvature of the element can result in considerable differences in the resultants and should be included in the calculations. This is the main contribution of this work, since the theory and finite element implementation of shell structures is well stablished, but simulation programs usually do not consider the curvature of the elements in the analysis of the results.

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