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Effective Elastic Properties for a Periodically Laminated Composite Considering Non-uniform Imperfect Adhesion

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ABSTRACT

The present work deals with the calculation of effective elastic properties for a periodically laminated composite considering non-uniform imperfect adhesion. The effective properties for two- and three-layers composites with imperfect and perfect interfaces respectively are calculated via Finite Element Method (FEM) and analytical technique based on the two-scale Asymptotic Homogenization Method (AHM). For this, it is used two-layers model with imperfect interface conditions (spring type), as well as three-layers model with perfect conditions, where an interface model is proposed for the computation of the interphase properties between the plies. Numerical simulations are carried out to illustrate graphically the effective properties. The effects of the non-uniform imperfection on the effective properties are observed. Therefore, when the thickness of the interphase is very small, the three-layers behave like two-phase laminated composites.

Keywords: Effective properties, Non-uniform imperfect adhesion, Asymptotic homogenization method, Finite element method

1 INTRODUCTION

Laminated composites have been successfully used in a wide range of applications, ranging from large structures, such as aerospace and racing cars, to small structures, such as golf clubs and artificial limbs [1]. The increasing use of laminated composites in different branches of engineering stimulates advances in the analysis of this kind of materials.

The bonding state at interfaces between two adjacent laminates clearly plays a critical role in determining the mechanical behavior of composite laminates. In most analytical and numerical work on composite materials, a perfect interface, which implies the continuity of both displacements and tractions across this idealized interlaminar interface, has been assumed. However, from a more rigorous physical point of view, the existence of a perfect interfacial bond in a real laminated is not an adequate model. One such situation is the presence of a thin layer or coating, enveloping the reinforcing constituent. Such an interfacial layer is generally referred to as interphase. It may be due to chemical interaction between the constituents or it may be introduced by design in order to improve the properties of the composite.

The macroscopic properties of composites depend upon the properties of the constituent phases and the interfacial bonding conditions, as well as the microstructures of the composites. Thus, the effect of the interfacial bonding conditions on the material properties of various composites has attracted a lot of attention of researchers in many fields, especially, in physics, materials science and technology, and mechanics. The prediction of the effective moduli taking into account interface effect is one of the fundamental problems in mechanics of composites [2-6]. This can be proved through new material interface models, which have been proposed by different researchers [7-16].

The imperfect bonding may be simulated by using a thin bond layer, which can have mechanical properties of the interface. Thus, the interfacial traction is assumed to be proportional to the displacement jump in terms of certain spring-type parameters. This kind of interface model has been proposed by Mal and Bose in [17], and it was investigated by different authors [18-28].

Based on the aspects pointed above, in the present work, three different approaches are developed in order to investigate laminated composites with interfacial imperfections. Thus, two-layer model with non-uniform imperfect adhesion (spring type) conditions and three-layer composite material with perfect mesophase conditions between the layers are proposed and evaluated. Analytical expressions for the elastic interphase are derived, so called interface model, which shown that the interphase properties can be simply related to constituent properties and geometry, constituting a model of the interaction behavior between layers. A three dimensional (3D) representative volume element (RVE) model is developed via FE package AbaqusTM for computing the effective properties of elastic laminate composite by using the proposed interface model. Thus, a computational procedure, based on Python language, is developed to calculate all effective coefficients from RVE. Moreover, two-scale Asymptotic Homogenization Method (AHM) provides an analytical expression of the effective properties for periodic layered composite. Finally, numerical analyses show that the three-layers model provides excellent approximation to the two-layers model. The effects of the non-uniform imperfection on the effective properties are illustrated graphically where a delamination process is simulated.

2 FORMULATION OF THE PROBLEM

Consider an anisotropic elastic body of a periodic structure occupying a bounded region Ω^ε in \mathbb{R}^3 space with Lipschitz boundary $\partial\Omega^\varepsilon = \overline{\partial_1\Omega^\varepsilon} \cup \overline{\partial_2\Omega^\varepsilon}$, such that $\partial_1\Omega^\varepsilon \cap \partial_2\Omega^\varepsilon = \emptyset$, where $\partial_1\Omega^\varepsilon$ and $\partial_2\Omega^\varepsilon$ are boundary portions. It was assumed that the region \mathcal{O} is made of periodic repetition of the unit cell Y in the parallelepiped form with dimensions $\varepsilon y_i (i=1,2,3)$ where ε the ratio of the unit cell size is (*i.e.* period of the structure) related to a typical length in the region. The Asymptotic Homogenization Method (AHM) is presented for a periodically layered composite with two-phase isotropic materials, considering non-uniform imperfect adhesion. Figure 1 shows the 3D layered composite, where Γ^ε is the interface separating of the composite layers, which depends on the slow variable \mathbf{x} . Γ is the interface in the unit cell, which is dependent of the fast variable \mathbf{y} . The medium is assumed to be layered in the x_1 direction, with all material parameters independent of x_2 and x_3 . Figure 2 shows the 2D unit cell with non-uniform interface where $\theta_r l_2$ is the length of partition “ $r=1,\dots,N$ ” for the interface, where N is the partitions number of the interface.

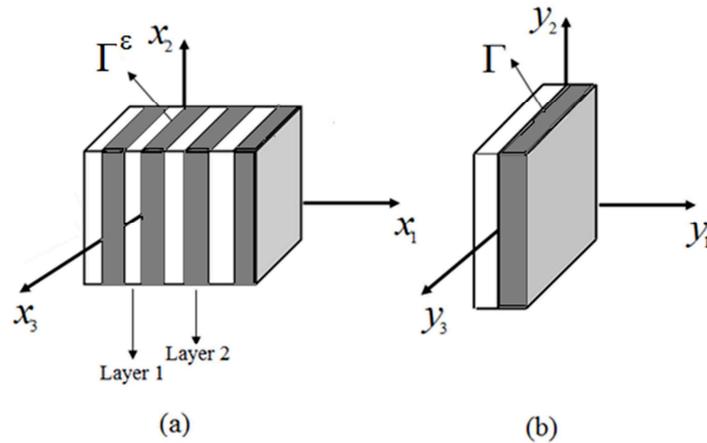


Figure 1: The 3D layered composite and unit cell: (a) layered composite, (b) unit cell.

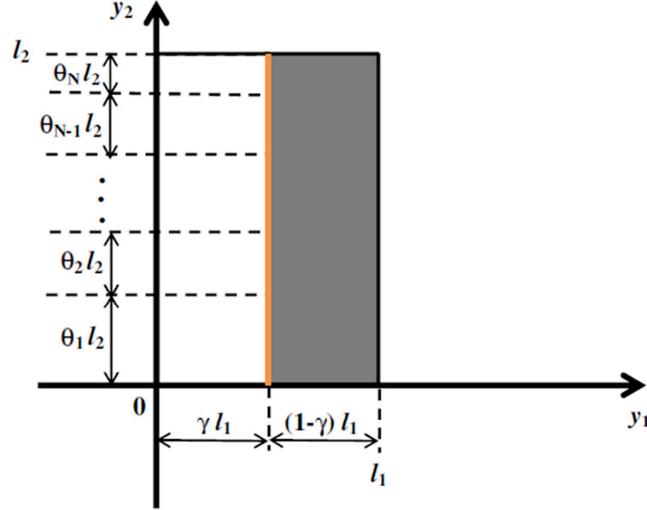


Figure 2: The 2D unit cell with nonuniform imperfect adhesion. $\sum_{r=1}^N \theta_r = 1$.

The problem is formulated in the bounded subset Ω^ε of \mathbb{R}^3 , and $Y = \{y = (y_1, y_2, y_3) \in \mathbb{R}^3 : 0 < y_i < l_i, i = 1, 2, 3\}$ denotes the reference cell, with l_i being given positive numbers. It is important to notice that $\Omega^\varepsilon = \varepsilon Y = \{x = (x_1, x_2, x_3) \in \mathbb{R}^3 : \varepsilon^{-1} x_i \in Y, i = 1, 2, 3\}$, where $y = x / \varepsilon$.

A general field variable f_i^ε is now dependent of both the macro and micro-scale, $f_i^\varepsilon(x) = f_i(x, y)$, and the partial derivatives take the form:

$$\frac{\partial f_i^\varepsilon}{\partial x_j} = \frac{\partial f_i}{\partial x_j} + \varepsilon^{-1} \frac{\partial f_i}{\partial y_j}. \quad (1)$$

Assuming the body forces equal to zero, then the elastic equilibrium equation is given by:

$$\frac{\partial \sigma_{ij}^\varepsilon}{\partial x_j} = 0 \text{ in } \Omega^\varepsilon, \quad (2)$$

$$\sigma_{ij}^\varepsilon n_j = K_{ij} \llbracket u_j^\varepsilon \rrbracket, \quad \llbracket \sigma_{ij}^\varepsilon \rrbracket n_j = 0 \text{ on } \Gamma^\varepsilon = \bigcup_{r=1}^N \Gamma_r^\varepsilon, \quad K_{ij} = \sum_{r=1}^N K_{ij}^r, \quad K_{ij}^r = 0, \quad i \neq j; \quad r = 1, 2, \dots, N \quad (3)$$

$$\sigma_{ij}^\varepsilon = C_{ijkl}(y) \frac{\partial u_k^\varepsilon}{\partial x_l}, \quad (4)$$

where $[[\bullet]]$ denotes the jump across the interface, *i.e.* $[[\bullet]] = (\bullet)^{(1)}(\mathbf{y}) - (\bullet)^{(2)}(\mathbf{y})$ for $\mathbf{y} \in \Gamma^\varepsilon$ and (henceforth, the Latin indices take values 1, 2 and 3); K_{ij} denotes the interface stiffness properties (with $K_{ij} \rightarrow \infty$ corresponding to perfect interface); $C_{ijkl} = C_{ijkl}^{(1)}$ for $0 < y_1 < \mathcal{V}_1$ and $C_{ijkl} = C_{ijkl}^{(2)}$ for $\mathcal{V}_1 < y_1 < l_1$ (\mathcal{V} is the volume fraction of layer 1 and l_1 is the length of the unit cell in the x_1 direction); and n_j is the unit vector in the outward normal direction.

2.1 Two Scales AHM: Two-Layer Elastic Composite with Imperfect Interface (Spring Type-2AHM)

The physical behavior of a heterogeneous medium (or composite), with a regular structure, is governed by differential equations with rapidly oscillating coefficients dependent on the material properties of the individual components. A mathematical framework from which to predict the mechanical behavior of regularly inhomogeneous media has been developed under the assumption that there is an ordered microstructure in such media, describable by a characteristic inhomogeneity dimension. Theoretical foundations of the method have been developed in scientific works [29-32].

The mechanical behavior of imperfect interface is modeled via a layer of mechanical springs of zero thickness. The spring constants $K_n = K_{11}$, $K_t = K_{22} = K_{33} = K_s$ are normal and tangential interface stiffness properties. It is seen that infinite values of the parameters imply vanishing of displacement jumps; and, therefore, perfect interface conditions. At the other extremity, zero values of the parameters imply vanishing of interface tractions and, therefore, debonding. Any finite positive values of the interface parameters define an imperfect interface.

The displacements ($u_i^\varepsilon(\mathbf{x})$) is expressed in the form of the following two-scale asymptotic expansions [30-32]:

$$u_i^\varepsilon(\mathbf{x}) = u_i^{(0)}(\mathbf{x}) + \varepsilon \cdot u_i^{(1)}(\mathbf{x}, \mathbf{y}) + \varepsilon^2 \cdot u_i^{(2)}(\mathbf{x}, \mathbf{y}) + \dots \quad (5)$$

Substituting (5) into (4), it is possible obtain:

$$\sigma_{ij}^\varepsilon(\mathbf{x}) = \sigma_{ij}^{(0)}(\mathbf{x}, \mathbf{y}) + \varepsilon \cdot \sigma_{ij}^{(1)}(\mathbf{x}, \mathbf{y}) + \varepsilon^2 \cdot \sigma_{ij}^{(2)}(\mathbf{x}, \mathbf{y}) + \dots, \quad (6)$$

Where

$$\sigma_{ij}^{(m)}(\mathbf{x}, \mathbf{y}) = C_{ijkl}(\mathbf{y}) \frac{\partial u_k^{(m)}}{\partial x_l} + C_{ijkl}(\mathbf{y}) \frac{\partial u_k^{(m+1)}}{\partial y_l}, \quad m = 0, 1, 2, \dots \quad (7)$$

Substituting the expressions (5) and (6) into (2) and rearranging the terms of equal exponent ε , it is obtained for ε^{-1} and ε^0 the following equations:

$$\frac{\partial \sigma_{ij}^{(0)}}{\partial y_j} = 0, \quad (8)$$

$$\frac{\partial \sigma_{ij}^{(0)}}{\partial x_j} + \frac{\partial \sigma_{ij}^{(1)}}{\partial y_j} = 0. \quad (9)$$

The recurrent system of partial differential equations (8)-(9) yields local problems with l_1 -periodic unknown functions $N_{ijk}^{2AHM}(\mathbf{y})$, where the notation 2AHM is associated to the two-layer spring problem.

The effective coefficients are calculated using the formula:

$$C_{ijkl}^* = \left\langle C_{ijkl} + C_{ijhs} \frac{\partial N_{hkl}^{2AHM}}{\partial y_s} \right\rangle = \sum_{r=1}^N \theta_r C_{ijkl}^{*,r} = \sum_{r=1}^N \theta_r \left\langle C_{ijkl} + C_{ijh1} \frac{\partial N_{hkl}^{2AHM,r}}{\partial y_1} \right\rangle, \quad (10)$$

The local functions $N_{ijk}^{2AHM,r}$ ($r=1, \dots, N$) are determined from the following local problems:

$$\frac{\partial}{\partial y_j} \left(C_{ijkl} \frac{\partial N_{kpq}^{2AHM,r}}{\partial y_l} + C_{ijpq} \right) = 0 \quad \text{in } Y_r, \quad (11)$$

with interface conditions

$$\left(C_{ijkl} \frac{\partial N_{kpq}^{2AHM,r}}{\partial y_l} + C_{ijpq} \right) n_j = K_{ij}^r \left[N_{ipq}^{2AHM,r} \right], \quad \left[C_{ijkl} \frac{\partial N_{kpq}^{2AHM,r}}{\partial y_l} + C_{ijpq} \right] n_j = 0 \quad \text{on } \Gamma_r. \quad (12)$$

where

$$Y_1 = \{ \mathbf{y} = (y_1, y_2, y_3) \in \mathbb{R}^3 : 0 < y_i < l_i, i=1,3 \text{ and } 0 < y_2 < \theta_1 l_2 \} \text{ for } r=1,$$

$$Y_r = \left\{ \mathbf{y} = (y_1, y_2, y_3) \in \mathbb{R}^3 : 0 < y_i < l_i, i=1,3 \text{ and } l_2 \sum_{i=1}^{r-1} \theta_i < y_2 < l_2 \sum_{i=1}^r \theta_i \right\} \text{ for } r \geq 2,$$

and Γ_r is the interface of the unit cell portion Y_r .

Using in (10), non-zero local functions N_{ijk}^{2AHM} are calculated for isotropic constituents. The effective moduli can be written as

$$\begin{aligned}
 C_{1111}^{*2AHM} &= \langle C_{1111}^{-1} \rangle^{-1}, \quad C_{1122}^{*2AHM} = C_{1133}^{*2AHM} = C_{1111}^{*2AHM} \langle C_{1111}^{-1} C_{1122} \rangle, \\
 C_{2222}^{*2AHM} &= C_{3333}^{*2AHM} = \langle C_{1111} \rangle + \frac{(C_{1122}^{*2AHM})^2}{C_{1111}^{*2AHM}} - \langle C_{1111}^{-1} C_{1122}^2 \rangle, \\
 C_{2233}^{*2AHM} &= \langle C_{1122} \rangle + \frac{(C_{1122}^{*2AHM})^2}{C_{1111}^{*2AHM}} - \langle C_{1111}^{-1} C_{1122}^2 \rangle, \\
 C_{1212}^{*2AHM} &= C_{1313}^{*2AHM} = \langle C_{1212}^{-1} \rangle^{-1}, \quad C_{2323}^{*2AHM} = \frac{1}{2} [C_{2222}^{*2AHM} - C_{2233}^{*2AHM}],
 \end{aligned} \tag{13}$$

Where

$$\begin{aligned}
 \langle C_{1111}^{-1} \rangle^{-1} &= \sum_{r=1}^N \frac{\theta_r C_{1111}^{(1)} C_{1111}^{(2)} l_1 K_n^r}{[(1-\gamma)C_{1111}^{(1)} + \gamma C_{1111}^{(2)}] l_1 K_n^r + C_{1111}^{(1)} C_{1111}^{(2)}}, \quad \langle C_{ijkl} \rangle = \sum_{r=1}^N \theta_r [\gamma C_{ijkl}^{(1)} + (1-\gamma) C_{ijkl}^{(2)}], \\
 \langle C_{1111}^{-1} C_{1122} \rangle &= \sum_{r=1}^N \frac{\theta_r [(1-\gamma)C_{1111}^{(1)} C_{1122}^{(2)} + \gamma C_{1111}^{(2)} C_{1122}^{(1)}]}{C_{1111}^{(1)} C_{1111}^{(2)}}, \quad \langle C_{1111}^{-1} C_{1122}^2 \rangle = \sum_{r=1}^N \frac{\theta_r [(1-\gamma)C_{1111}^{(1)} (C_{1122}^{(2)})^2 + \gamma C_{1111}^{(2)} (C_{1122}^{(1)})^2]}{C_{1111}^{(1)} C_{1111}^{(2)}}, \\
 \langle C_{1212}^{-1} \rangle^{-1} &= \sum_{r=1}^N \frac{\theta_r C_{1212}^{(1)} C_{1212}^{(2)} l_1 K_t^r}{[(1-\gamma)C_{1212}^{(1)} + \gamma C_{1212}^{(2)}] l_1 K_t^r + C_{1212}^{(1)} C_{1212}^{(2)}}.
 \end{aligned}$$

It is worth to highlight that the effective coefficient C_{1212}^* is function of the tangential interface parameter K_t^r , whereas others are dependent of the normal interface parameter K_n^r .

2.2 Two Scales AHM: Three-Layer Elastic Composite (3AHM)

Three-layer composites consist on showing an interphase between layers. The superscript assumes to be 1, I and 2 for the layer 1, interphase (mesophase) and layer 2, respectively. The effective coefficients are calculated using the formula $C_{ijkl}^* = \langle C_{ijkl} + C_{ijhs} \partial N_{hkl}^{3AHM} / \partial y_s \rangle$, with perfect conditions ($K_n = K_t = \infty$) at the interface Γ , and the notation 3AHM is associated to the three-layer problem. Non-zero local functions N_{ijk}^{3AHM} are determined and the effective moduli are listed as follows,

$$\begin{aligned}
C_{1111}^{*3AHM} &= \langle C_{1111}^{-1} \rangle^{-1}, \quad C_{1122}^{*3AHM} = C_{1133}^{*3AHM} = C_{1111}^{*3AHM} \langle C_{1111}^{-1} C_{1122} \rangle, \\
C_{2222}^{*3AHM} &= C_{3333}^{*3AHM} = \langle C_{1111} \rangle + \frac{(C_{1122}^{*3AHM})^2}{C_{1111}^{*3AHM}} - \langle C_{1111}^{-1} C_{1122}^2 \rangle, \\
C_{2233}^{*3AHM} &= \langle C_{1122} \rangle + \frac{(C_{1122}^{*3AHM})^2}{C_{1111}^{*3AHM}} - \langle C_{1111}^{-1} C_{1122}^2 \rangle, \\
C_{1212}^{*3AHM} &= C_{1313}^{*3AHM} = \langle C_{1212}^{-1} \rangle^{-1}, \quad C_{2323}^{*3AHM} = \frac{1}{2} [C_{2222}^{*3AHM} - C_{2233}^{*3AHM}],
\end{aligned} \tag{14}$$

where

$$\begin{aligned}
\langle C_{1111}^{-1} \rangle^{-1} &= \sum_{r=1}^N \frac{\theta_r C_{1111}^{(1)} C_{1111}^{(I)} C_{1111}^{(2)}}{v^{(2)} C_{1111}^{(1)} C_{1111}^{(I)} + v^{(I)} C_{1111}^{(1)} C_{1111}^{(2)} + v^{(1)} C_{1111}^{(I)} C_{1111}^{(2)}}, \quad \langle C_{ijkl} \rangle = \sum_{r=1}^N \theta_r (v^{(1)} C_{ijkl}^{(1)} + v^{(I)} C_{ijkl}^{(I)} + v^{(2)} C_{ijkl}^{(2)}), \\
\langle C_{1111}^{-1} C_{1122} \rangle &= \sum_{r=1}^N \frac{\theta_r (v^{(2)} C_{1111}^{(1)} C_{1111}^{(I)} C_{1122}^{(2)} + v^{(I)} C_{1111}^{(1)} C_{1111}^{(2)} C_{1122}^{(I)} + v^{(1)} C_{1111}^{(I)} C_{1111}^{(2)} C_{1122}^{(1)})}{C_{1111}^{(1)} C_{1111}^{(I)} C_{1111}^{(2)}}, \\
\langle C_{1111}^{-1} C_{1122}^2 \rangle &= \sum_{r=1}^N \frac{\theta_r [v^{(2)} C_{1111}^{(1)} C_{1111}^{(I)} (C_{1122}^{(2)})^2 + v^{(I)} C_{1111}^{(1)} C_{1111}^{(2)} (C_{1122}^{(I)})^2 + v^{(1)} C_{1111}^{(I)} C_{1111}^{(2)} (C_{1122}^{(1)})^2]}{C_{1111}^{(1)} C_{1111}^{(I)} C_{1111}^{(2)}}, \\
\langle C_{1212}^{-1} \rangle^{-1} &= \sum_{r=1}^N \frac{\theta_r C_{1212}^{(1)} C_{1212}^{(I)} C_{1212}^{(2)}}{v^{(2)} C_{1212}^{(1)} C_{1212}^{(I)} + v^{(I)} C_{1212}^{(1)} C_{1212}^{(2)} + v^{(1)} C_{1212}^{(I)} C_{1212}^{(2)}},
\end{aligned}$$

and $v^{(1)}$, $v^{(I)}$, $v^{(2)}$ are related to layer 1, interphase and layer 2 volume fraction, respectively.

2.3 Interface Model

Analytical expressions for the elastic interphase, which represent interphase properties, can be written based on the constituent properties and geometry. Thus, two phase model with imperfect interface conditions (spring type) is modeled as a three-phase material with perfect conditions. Hence, equaling the effective coefficients C_{1111}^* , C_{1122}^* and C_{1212}^* from the formulas (13) and (14), interface moduli for the r -partition can be written as:

$$\begin{aligned}
C_{1111}^{(I)r} &= \frac{v^{(I)} t^{(I)} K_n^r C_{1111}^{(1)} C_{1111}^{(2)}}{[(1-\gamma-v^{(2)})C_{1111}^{(1)} + (\gamma-v^{(1)})C_{1111}^{(2)}] t^{(I)} K_n^r + v^{(I)} C_{1111}^{(1)} C_{1111}^{(2)}}, \\
C_{1122}^{(I)r} &= \frac{(v^{(2)} C_{1111}^{(1)} C_{1122}^{(I)r} + v^{(I)} C_{1111}^{(1)} C_{1122}^{(2)} + v^{(1)} C_{1111}^{(I)r} C_{1122}^{(1)}) [(1-\gamma)C_{1111}^{(1)} C_{1122}^{(2)} + \gamma C_{1111}^{(2)} C_{1122}^{(1)}] t^{(I)} K_n^r}{v^{(I)} C_{1111}^{(1)} C_{1122}^{(2)} \{ [(1-\gamma)C_{1111}^{(1)} + \gamma C_{1111}^{(2)}] t^{(I)} K_n^r + v^{(I)} C_{1111}^{(1)} C_{1122}^{(2)} \}} \\
&\quad - \frac{v^{(2)} C_{1111}^{(I)r} C_{1122}^{(2)}}{v^{(I)} C_{1111}^{(2)}} - \frac{v^{(1)} C_{1111}^{(I)r} C_{1122}^{(1)}}{v^{(I)} C_{1111}^{(1)}}, \\
C_{1212}^{(I)r} &= \frac{v^{(I)} t^{(I)} K_t^r C_{1212}^{(1)} C_{1212}^{(2)}}{[(1-\gamma-v^{(2)})C_{1212}^{(1)} + (\gamma-v^{(1)})C_{1212}^{(2)}] t^{(I)} K_t^r + v^{(I)} C_{1212}^{(1)} C_{1212}^{(2)}},
\end{aligned} \tag{15}$$

2.4 FEM: Three-Layer Elastic Composite

Based on Representative Volume Element (RVE) concept combined to Finite Element Method (FEM), effective properties of elastic composites are calculated considering interface effects. Thus, a three-layer composite is studied in which the layers have homogeneous and isotropic properties. In addition, the interface has homogeneous and isotropic properties, which are obtained by using the formula (15).

As it is known, RVE is the smallest portion of the actual composite, which has same elastic constants and volume fraction of the investigated material. Thus, a macro-structural model is defined in order to represent the periodic layered composite as a homogenized macroscopic continuum. Therefore, the proper choice of the RVE determines largely the accuracy for modelling a heterogeneous material. The Representative Volume Element is meshed by using solid elements. This numerical model is used to determine a homogeneous equivalent medium for the original composite, and it comprises the smallest portion of the composite, which keeps the most representative combination of its main materials. In addition, it is assumed that the average mechanical properties of the RVE are equal to the average properties of the composite material as follow:

$$\bar{\sigma}_{ij}^r = \langle \sigma_{ij}^r \rangle = \frac{1}{|V|} \int_V \sigma_{ij}^r dV, \quad \bar{\varepsilon}_{ij}^r = \langle \varepsilon_{ij}^r \rangle = \frac{1}{|V|} \int_V \varepsilon_{ij}^r dV, \tag{16}$$

where $|V|$ is the unit cell volume.

Discretizing Eq. (16) using the Finite Element Method (FEM), the average values can be calculated by:

$$\bar{\sigma}_{ij}^r = \frac{1}{|V|} \sum_{m=1}^{nel} \sigma_{ij}^{(m)r} V^{(m)}, \quad \bar{\varepsilon}_{ij}^r = \frac{1}{|V|} \sum_{m=1}^{nel} \varepsilon_{ij}^{(m)r} V^{(m)}, \tag{17}$$

where, nel is the number of finite elements of the complete unit cell; $V^{(m)}$ is the volume of the m^{th} element, $\sigma_{ij}^{(m)r}$ and $\varepsilon_{ij}^{(m)r}$ are the respective stress and strain tensors evaluated in the m^{th} element.

For a complete description of a differential problem in order to determine effective material properties, it is necessary to formulate appropriate boundary conditions. For any parallelepiped RVE models, $\Delta x_k^j = x_i^{+j} - x_i^{-j}$ is constant, therefore, the following unified periodic boundary conditions is obtained:

$$u_i^{+j} - u_i^{-j} = c_i^j \quad (i, j = 1, 2, 3) \quad (18)$$

where, u_i^j denotes the displacement along the i -direction of one node located at the boundary face whose normal vector is along the j -direction. The index “ $+j$ ” means along the positive X_j direction, and “ $-j$ ” means along the negative X_j direction.

Thus, the RVE is analyzed by FEM for different loadings with suitable boundary conditions [6] applied in a unique way using AbaqusTM combined to Python language. This procedure has been developed to systematically calculate all RVE effective coefficients, thereby reducing exhaust manual work, saving time, and diminishing the chance of numerical errors. In addition, the FEM–RVE model shows the three phases, which are modeled by solid elements with linear interpolation, *i.e.* eight-node brick element with three degrees of freedom per node and eight integration points (C3D8E – Abaqus nomenclature). It is important to highlight that AbaqusTM calculates the quantities for the tensors in the Gauss points and it uses numerical techniques to integrate various quantities over the volume of each element to feed the Eq. (17). In addition, for numerical analyses, it was investigated the influence of the mesh density. Thus, three different meshes were used. Firstly, to evaluate the effective coefficients, RVE was meshed by using approximately 6000 elements. After that, it was used 8000, 12000 and 16000 elements. However, the results showed that the differences for the effective coefficients are of 10⁻⁴ order, considering the used mesh densities. Therefore, all the results are shown only for the meshed RVE by using approximately 6000 elements.

3 RESULTS ANALYSIS

In order to illustrate the effect of the nonuniform imperfection on the effective properties calculated using numerical (FEM) and analytical (AHM) method, some numerical examples are presented. The material properties used in the calculations are listed in Table 1, which are given in the works [33, 34]. For almost all calculations, the interface volume fraction is assumed to be equal 10⁻³. For example, in Table 2 it is shown the effective coefficients for volume fraction of layer 2 equal 0.5, considering the two limit cases, *i.e.* perfect bonding ($K_n^2 = K_t^2 = 10^{12}$, $\theta_1 = 0$) and complete separation ($K_n^1 = K_t^1 = 10^{-3}$, $\theta_1 = 1$) of the interface (complete imperfection). And, it is observed that the results of the three-layer problem provide excellent approximation with the results of the two-layer spring problem.

Table 1. Material properties

Materials	E [GPa]	ν
Titanium Ti6Al4V (layer 1)	110.3	0.30
Ceramic AD-96 (layer 2)	303.0	0.21

Table 2. Effective coefficients for limit cases with volume fraction of layer 2 equal 0.5 and interphase thickness $t^{(I)} = 10^{-3}$

θ_1	C_{11}^* (GPa)		
	FEM	3AHM	2AHM
0	211.137193	211.134069	211.134069
1	0.001000	0.001000	0.001000
	C_{12}^* (GPa)		
	FEM	3AHM	2AHM
0	75.493084	75.490791	75.490791
1	0.000358	0.000358	0.000358
	C_{23}^* (GPa)		
	FEM	3AHM	2AHM
0	79.190981	79.188582	79.18858231
1	52.160375	52.159220	52.197048
	C_{33}^* (GPa)		
	FEM	3AHM	2AHM
0	246.495981	246.494431	246.494431
1	219.381185	219.380869	219.502896
	C_{66}^* (GPa)		
	FEM	3AHM	2AHM
0	63.012861	63.0115782	63.01157820
1	0.00100	0.00100	0.00100

A study for different values for the interphase thickness $t^{(I)} = 10^{-3}$ with volume fraction of layer 2 equal 0.5 (Table 3) is considered. In this case, the effective coefficient C_{23}^* is calculated for different values of the parameter θ_1 . It is verified the assumptions that the interface region of the two-layer problem occupies a zero volume fraction in the composite. This can be explained due to the increasing of the interphase thickness and the imperfection. Thus, the relative difference between three-layer and two-layer models as increases as the interface thickness and imperfection increase.

Table 3 Elastic effective coefficient C_{23}^* for different values of the parameters θ_1 (degree of imperfection), and interphase thickness $t^{(l)}$ with volume fraction of layer 2 equal 0.5 and spring constants $K_n^1 = K_t^1 = 10^{-3}$ and $K_n^2 = K_t^2 = 10^{12}$.

$t^{(l)}$	θ_1	(A) FEM	(B) 3AHM	(C) 2 AHM	$\frac{ (A)-(C) }{(A)} \times 100\%$	$\frac{ (B)-(C) }{(B)} \times 100\%$
10^{-3}	0.2	73.784860	73.782710	73.790275	0.007339	0.010254
	0.4	68.378739	68.376837	68.391969	0.019348	0.022129
	0.6	62.972618	62.970965	62.993662	0.033418	0.036044
	0.8	57.566497	57.565092	57.595355	0.050130	0.052571
10^{-2}	0.2	73.716752	73.714619	73.790275	0.099738	0.102634
	0.4	68.242542	68.240655	68.391969	0.218963	0.221735
	0.6	62.768333	62.766692	62.993662	0.358985	0.361609
	0.8	57.294123	57.292728	57.595355	0.525764	0.528211
10^{-1}	0.2	73.035689	73.033710	73.790275	1.033175	1.035913
	0.4	66.880590	66.878838	68.391969	2.259817	2.262496
	0.6	60.725491	60.723965	62.993662	3.735122	3.737728
	0.8	54.570391	54.569093	57.595355	5.543232	5.545744

Figs. 3–7 show the effective coefficients for different cases, such as complete separation of the interface and perfect bonding (limit cases) and different values of the parameter θ_1 , simulating a delamination process. In each plot, three-layer results are compared to the imperfect interphase results. It is seen that the plots are numerically indistinguishable. For all elastic effective coefficients, except the effective coefficients C_{23}^* and C_{33}^* , in the case of perfect interface, the coefficients increase as the volume fraction of layer 2 increases, and remain close to zero for complete separation of the interface. Different behavior is shown for the effective coefficients C_{23}^* and C_{33}^* .

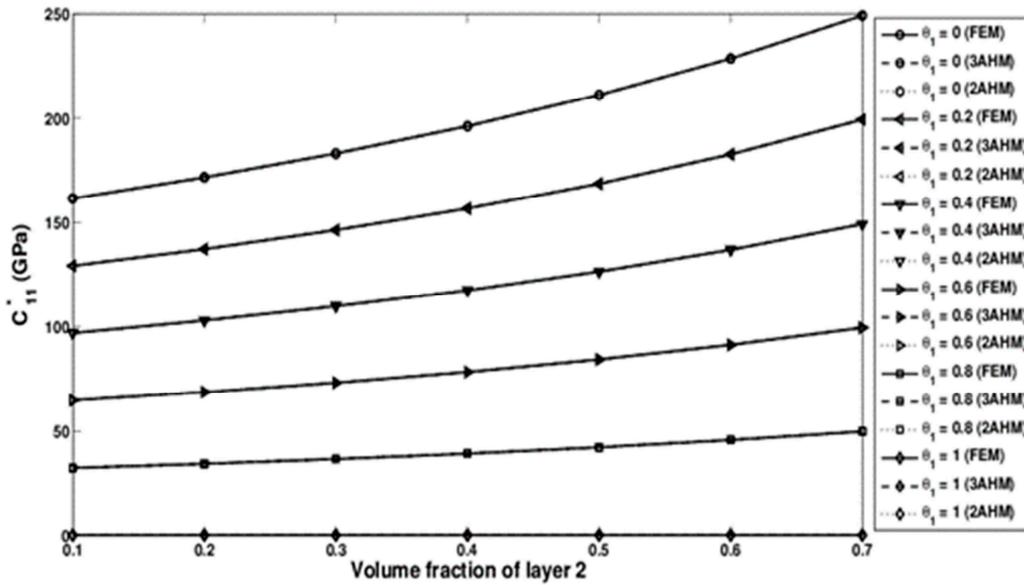


Figure 3: Evolution of elastic property C_{11}^* as a function of the volume fraction of layer 2 for different values of the parameter θ_1 .

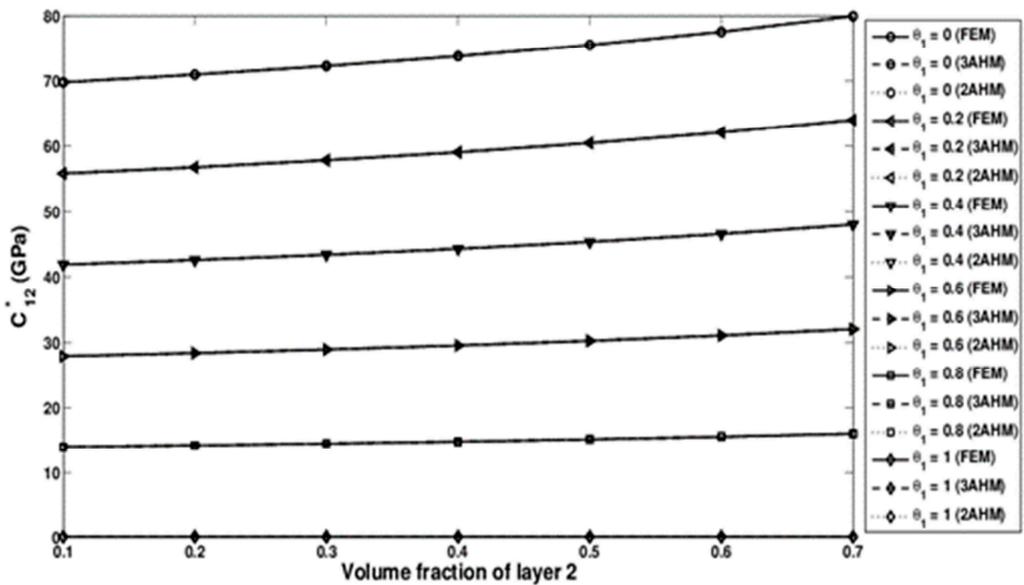


Figure 4: Evolution of elastic property C_{12}^* as a function of the volume fraction of layer 2 for different values of the parameter θ_1 .

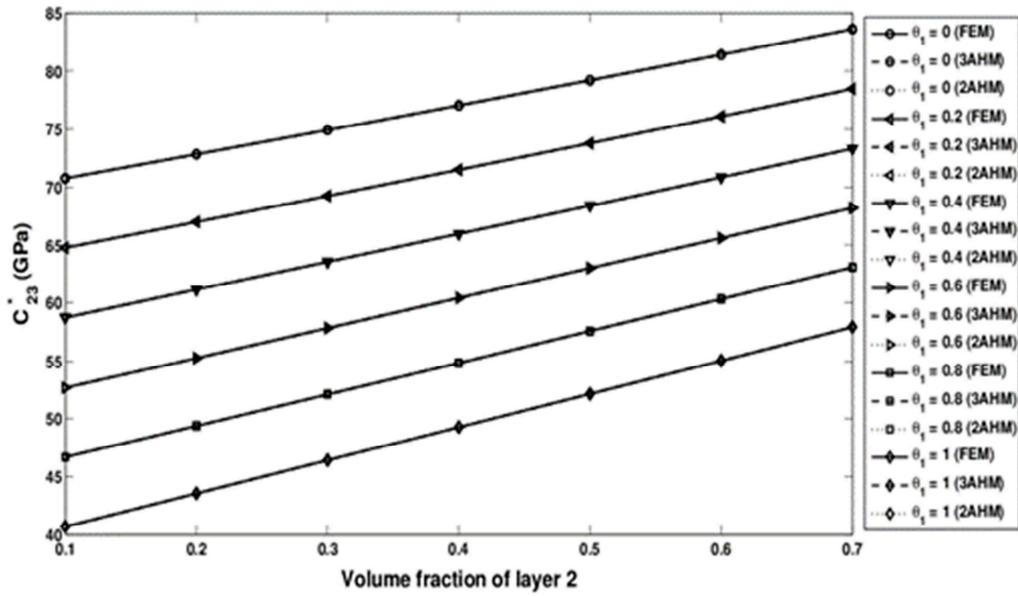


Figure 5: Evolution of elastic property C_{23}^* as a function of the volume fraction of layer 2 for different values of the parameter θ_1 .

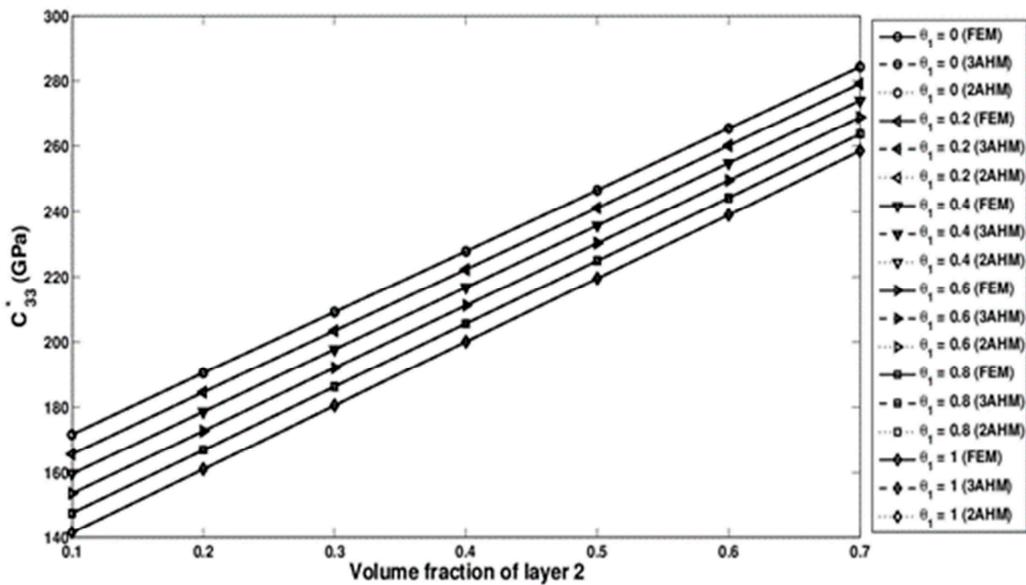


Figure 6: Evolution of elastic property C_{33}^* as a function of the volume fraction of layer 2 for different values of the parameter θ_1 .

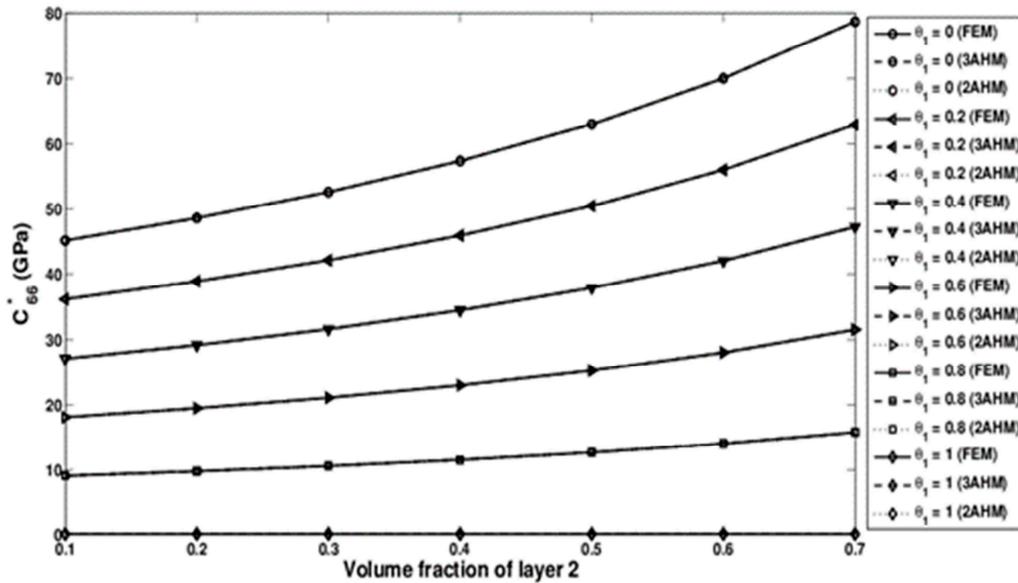


Figure 6: Evolution of elastic property C_{66}^* as a function of the volume fraction of layer 2 for different values of the parameter θ_1 .

4 CONCLUSION

This work focus on the analysis of two approaches in order to model nonuniform imperfect interfaces in a periodic layered composite. Using the results of the two layer model with nonuniform imperfect interface conditions (spring type) and the three layer material with perfect conditions, an interface model was derived for the computation of the interphase properties in the case of three-layer composites. It is shown that the results of the three-layer model provide excellent approximation with the results of the two-layer spring problem.

The numerical results can confirm some remarks and conclusions pointed above. For different values for the interphase thickness $t^{(I)}$ with volume fraction of layer 2 equal 0.5, the effective coefficient C_{23}^* is calculated using different values of the parameter θ_1 (degree of imperfection). The relative difference between three-layer and two-layer models as increases as the interface thickness and imperfection increase. In addition, the imperfection increases when the values of the parameter θ_1 increases simulating a delamination process. Therefore, the developed approaches provide alternatives to study the separation along the interfaces of the layers, *i.e.* delamination. In other words, they can be used as tools to aid the concept design of smart composite structures. Finally, for future work, the authors propose to perform a study of the effective coefficients increasing the number of partitions of the interface, as well as considering intermediate values of K_n and K_t to model real problems.

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