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Analysis of sensor placement in beams for crack identification

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ABSTRACT

The aim of the present work was to determine the most appropriate locations for installing sensors in beams in order to identify and characterize structural damages. The effect of different sizes and locations of cracks on the vibrational responses of beams was studied through a numerical (computational) model using the finite element model. The results show that, although most of the existing methodologies for identifying and characterizing damages in beams are based on an analysis of natural frequency variations, the vibration modes proved to be more sensitive for this purpose. Through the analysis of this vibrational response, it was possible to point out the most adequate sites for placing sensors aiming at identifying cracks in a beam in clamped-free boundary conditions.

Keywords: Structural Health Monitoring, modal analysis, harmonic analysis, crack identification

1 INTRODUCTION

A marked improvement in computational tools and an increased commitment of researchers worldwide have contributed significantly to produce great advances in the areas of Damage Prognosis (DP) and Structural Health Monitoring (SHM) in recent years. Such advances have allowed an increase in system security and a reduction in the operational costs of various industrial processes and activities. One of the most important steps for the successful implementation of these techniques is the identification and characterization of cracks found in structures. As the appearance of cracks causes changes in the vibrational parameters of a system, methods based on the analysis of those parameters have been proposed.

In general, such methods consist in comparing the vibrational responses or the modal parameters of damaged structures to responses of intact structures and – by observing variations, numerical models and optimization problems – in identifying the existence, location, and size of cracks. Beams, which are structures widely used in machinery, construction, mechanical engineering, shipbuilding, aviation industry, and so on so forth, are naturally among the most studied elements. Various researches have analyzed changes among natural frequencies, vibration modes, and frequency response functions (FRF) of intact beams and cracked beams aiming at identifying damages [1-21].

Analyzing those works, one notices that the monitoring of vibrational parameters is a feasible alternative for identifying cracks in beams. However, there still is no consensus about what technique would be the most recommended for each case, regarding the necessary precision, ease of measurement and other factors. There are different alternatives in almost all steps of the identifying cracks process, among which one can highlight: the vibrational parameters to be analyzed and controlled, the physical approach to be adopted, the type and location of the sensor to be used, the numerical modeling, the optimization technique used, the quantification of errors and uncertainties, etc. Depending on the approach, some of those steps may be omitted, interconnected or unfolded. Finally, the combination of all those alternatives generate a wide range of available possibilities for identifying cracks in beams, each presenting its own advantages, disadvantages, and peculiarities. This range of options renders the process of selecting the most adequate method for each case very difficult as well as the decision on which approach to use when developing a new SHM or DP technique for a specific application.

Most works found in the literature propose methods for identifying cracks in beams based on observation and analysis of change patterns in natural frequencies. However, the authors did not find papers stating that the use of this modal parameter is more recommended than the others for this purpose. On the contrary, some researchers report the difficulty one faces in identifying and monitoring cracks through natural frequencies, since they are almost insensitive to this type of structural damage [5, 12, 14].

It is known, therefore, that changes brought about by cracks on vibrational responses of structures can be subtle (especially in small surface cracks), and that the magnitude of those changes depends on the site being studied or monitored in the beam. From the above considerations, another relevant issue rises regarding the implementation of SHM technologies, which consists in determining the most adequate sites for installing sensors (such as accelerometers) in beams. For obvious reasons, the probability of correctly identifying the structural damage is greater when analyzing sites which are more sensitive to it.

A DP methodology for ordinary beams was proposed by the present research team in a previous paper and consists of two parts: identification of existing crack and prediction of remaining lifespan [22]. In the first part of the methodology, a finite element model was used combining elements of Timoshenko beam and Euler-Bernoulli beam. The crack was modeled using a torsion spring, and a modal analysis allowed obtaining the first vibration mode of the cracked beam; non-linear optimization techniques were implemented in order to adjust the mode obtained experimentally with its numerical equivalent in the wavelet domain. In the second part of the methodology, the Paris law was used to model the growth of cracks previously identified for different numbers of load cycles. The size of the critical crack was established, making it possible to estimate the remaining lifespan of the structure.

In this context, the present work evaluates the most appropriate sites for installing sensors in beams. It also analizes the influence of depth and location of cracks on natural frequencies, vibration modes, and FRF of a cantilever beam. To this end, a numerical model was used through the finite element method.

The present study does not intend to point at the best definitive alternative for identifying cracks in beams. Anyway, the results obtained allow researchers to make more conscious decisions regarding the development of techniques for identifying cracks, SHM, and damage prognosis.

2 REVIEW OF LITERATURE

The presence of cracks induces local flexibility in structural elements, changing the vibration responses [23]. It is possible to determine the location and size of a crack observing such changes, since the dynamic behavior of the structure is based on those parameters [1].

Reference [1] proposes a method for identifying location and size of cracks in cantilever beams by exciting harmonic vibrations into one of the natural vibration modes. The vibration amplitude was measured in two different sites using accelerometers, and the Newton-Raphson method was used to solve non-linear equations. The method was considered superior when compared to those using only natural frequency measurements, because of the comparatively higher occurrence of errors in measuring small changes using natural frequencies. However, the method needs periodic inspections, which hinders the monitoring of remote structures or structures with difficult access. Errors ranging from 5 to 8% were observed when the analysis was performed on small cracks (crack depth smaller than 10% of beam height).

Reference [5] has experimentally investigated the effects of cracks on structures aiming at determining their locations and depths. FRF measurements were taken using accelerometers in aluminum beams, with cracks produced by saws in seven different positions, with depths ranging from 10 to 70% of the beam height. A method to predict location and depth of cracks was proposed.

References [2, 3] presented simplified and analytical methods to calculate the fundamental frequency of Euler-Bernouilli beam with cracks submitted to bending. The crack was characterized as elastic springs, and the results found are in accordance with those numerically obtained using the finite element method. Reference [7] has proposed solutions for the natural frequencies of vibration of cracked Timoshenko beam. The beam was modeled as two segments connected by two springs of negligible mass.

Reference [15] developed an analytic model for cracked beams, including various effects besides bending, such as axial stiffness, rotational inertia, shear deformation, and a combination of the last two. The developed model can be used to predict natural frequency variations for various crack locations and sizes along the beam not only for Euller-Bernouilli beams but also for Timoshenko beams. The analytical method developed may be a useful reference for comparing results obtained using numerical models.

Reference [12] has proposed an algorithm to identify single or double cracks in beams. As the researchers point out, the precise determination of natural frequencies (and its sequent relationship to the natural frequencies of intact beams) is crucial for a successful crack identification. The work uses a statistical approach aiming at minimizing deviations caused by sensitivity and lack of resolutions in measuring natural frequencies.

Reference [14] has presented a new method for identifying cracks in beams and for determining their locations and depths based on changes in natural frequencies. The researchers point out the difficulty in determining such frequencies due to the fact they have a low sensitivity to damage. Very often, inaccuracies in mathematical models cause greater changes in the compared frequencies of intact structures and damaged structures than the changes caused by the structural damage itself. The method consists in using an accelerometer and in obtaining the first 10 natural frequencies of a beam, making it possible to locate defects with 1% accuracy approximately.

References [5, 14, 15, 17] have reported similar effects: the presence of cracks lowers the natural frequency of beams, since they induce more flexibility to the system, reducing its stiffness. It has been found out that the pattern of changes of natural frequencies depend on the location of the crack, whereas the depth of the damage only amplifies that effect. Reference [16] has shown that the knowledge of the first two natural frequencies of cracked beams allows a one-time determination of severity and position of the damage in simply supported beams, regardless of the size of the crack.

Reviewing the literature, one can observe that the effect of the cracks on natural frequencies has been significantly more studied than the effect of cracks on vibrating modes and on damaged FRF beams. On the other hand, it is common knowledge that variations of natural frequencies of damaged beams are subtle, even when the cracks are comparatively large - it is common for cracks whose depth is 30% of beam height to cause changes less than 2% on the values of the natural frequencies. Very often, errors arising from the measurement system used have a higher magnitude than the very variation of the natural frequencies produced by cracks, which makes it difficult to identify them. Likewise, numerical and computational models used in analyses and prognoses can also introduce unacceptable errors due to the high precision required in those applications.

It is known that as cracks increase in depth there is also an increase in the variation of vibrational responses of the damaged structures when compared to the responses of intact structures. From a technological point of view, it is important to point out that, in various industrial applications, it is desirable to identify cracks when they are still at their initial stages since larger cracks may already have compromised the structure as a whole or they may be identified through a simple visual inspection.

Observing this scenario, it is possible to note that important advances in the techniques of identifying cracks in beams have been achieved in recent years. Other challenges still remain to be faced, such as the identification and characterization of small cracks

The present work aims at determining the most appropriate sites for installing sensors in beams, for a subsequent identification and characterization of structural damages. To this end, the sensitivity and variations of the natural frequencies, vibration modes, and FRF of beams were compared regarding different locations and depths of cracks. This knowledge makes it possible to have a more conscious development of techniques for SHM and DP, taking into account the applicability of the method and the necessary level of accuracy.

3 METHODOLOGY

In order to obtain data from vibrational responses, a computational model of the beam was created using the finite element method, following the approach of various works found in the literature [5, 12, 17, 24-29]. The following sections present details of the numerical model used and vibration responses obtained.

3.1 Numerical Model

First, a model of an intact Euler-Bernoulli beam was created, crack free, using the commercial ANSYS 14.5 software. This model contains information on the geometry and material of the structure.

The beam under study has a square cross-section and clamped-free boundary conditions, parameterized with length L, width w, and height h. The material considered was steel, with an elastic modulus E=206.8 GPa, density ρ=7830 kg/m³, and Poisson coefficient ν=0.33. The finite element mesh was created considering the longitudinal middle of the intact beam, using 32021 square solid elements of 8 nodes.

In order to make sure the numerical model was calibrated, the first five natural frequencies (fn) were obtained and compared to the natural frequencies obtained through the analytical equation, calculated according to [30]. The results obtained are presented in Table 1:

Table 1 Comparison between the analytical natural frequencies and those of the numerical model.

Natural frequency	Finite elements method (Hz)	Analytical solution (Hz)	Difference (%)
f_1	8.30230	8.301845519	0.00547
f_2	52.00500	52.02677765	0.04186
f_3	145.51000	145.6764842	0.11428
f_4	284.82000	285.4677671	0.22691
f_5	470.15000	471.8986737	0.37056

Analyzing the table, it is possible to observe that the greatest percentage difference between the model used and the analytical solution did not exceed 0.4% in any of the frequencies, thus indicating that the model is adequate.

Then, cracks were inserted to the numerical model. As the simulation of cracked structures requires further mesh refining and other special considerations, the mesh of the longitudinal sections of the structures with cracks contained 61 103 square elements of 8 nodes. The crack tip was modeled using 20 quarter-point type elements, the use of which is recommended by the literature because they presented the best results [31-33]. Figure 1 illustrates the mesh of finite elements used in the crack tip area.

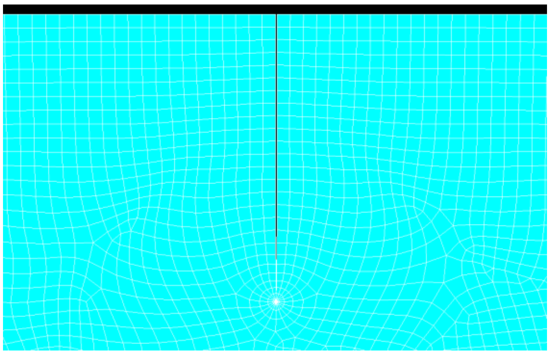


Figure 1: Finite elements mesh in the crack tip area.

Two crack parameters were varied in the model: depth a and location x_t. The shape of the crack and its width were defined based on the height of the beam, according to the schematic drawing in Figure 2.

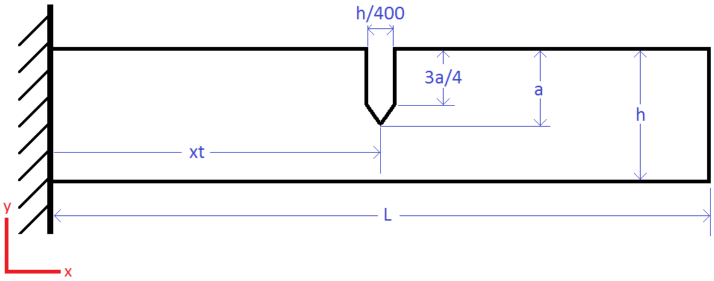


Figure 2: Sketch of beam format and width – not in scale.

During simulations, 4 depths and 7 locations were considered – according to Table 2. On the whole, 28 different scenarios (combinations of size and location) were analyzed.

Table 2 Positions and depths of cracks under study.

Crack position (m)		Crack Depth (m)		Proportion a/h
x_{t_1}	0.125	a_1	0.001	10%
x_{t_2}	0.250	a_2	0.003	30%
x_{t_3}	0.375	a_3	0.005	50%
x_{t_4}	0.500	a_4	0.007	70%
x_{t_5}	0.625			
x_{t_6}	0.750			
x_{t_7}	0.875			

3.2 Vibration Responses

For each of the 28 possible combinations of size and location of the crack, a modal analysis and a harmonic analysis were performed. The same analyses were performed for the crackless beam situation.

Through modal analysis, the first 5 natural frequencies and the beam's associated vibration modes were extracted. Studies were carried out on: vertical nodal displacements u_y of 10 different sites in the beam, located on $x = L/10$ (with increments of $L/10$ until total length L), height $y/h = 0.1$ and width $z/w = 0.5$. Such choice was made to make sure that the sites under study were always located in the remaining ligament of the beam, even regarding the largest crack, when $a/h = 0.7$. The site located in $x = 0$ was not studied because, by definition, its vertical displacement renders null by the boundary condition (free-clamped beam). Figure 3 presents a schematic view of the side and front views of the beam, showing all ten sites of study.

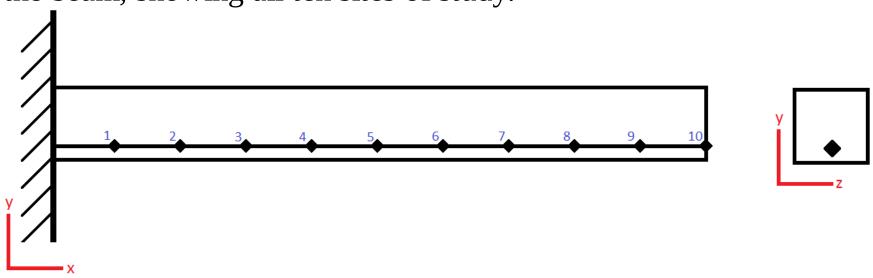


Figure 3: Side and front views of the beam showing the ten sites under study.

After simulations, the vertical displacement of the nodes concerning the different combinations of crack size and location were compared to the values obtained for the intact beam.

Harmonic analysis studied the FRF (receptance) of beam with 28 combinations of crack size and location for frequencies between 0 and 750 Hz, with increments of 1 Hz. In order to obtain those responses, incitations of unitary forces located at $x/L = 0.85$, height $y = h$ and width $z/w = 0.5$ were simulated. For each case, the responses were obtained in four different locations, according to Table 3. Those responses were obtained also for the intact beam situation.

Table 3 Location of obtained responses.

Response site	**Coordinate *x (m)***	**Coordinate *y (m)***	**Coordinate *z(m)***
1	0.2	0.005	0.005
2	0.4	0.005	0.005
3	0.6	0.005	0.005
4	0.8	0.005	0.005

The Figure 4 presents a schematic view of the excitation point and the 4 response sites of the studied situations.

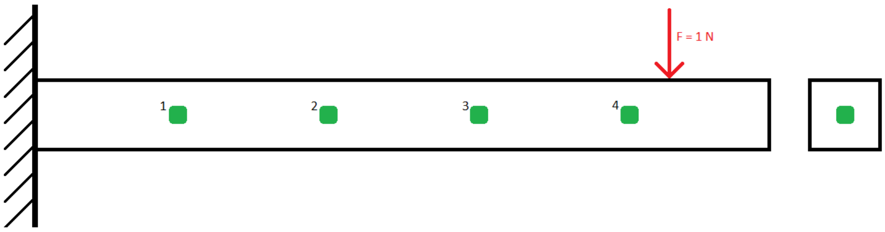


Figure 3: Excitation point and the 4 response sites.

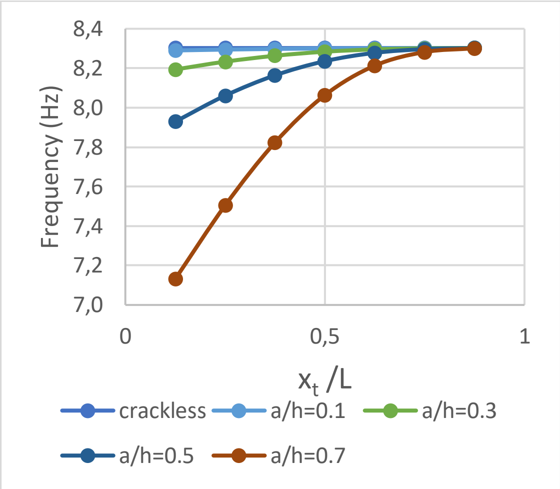
At the end of the simulations, the FRF curves obtained for beams containing cracks of different depths and locations were compared to the curves obtained for the intact beam.

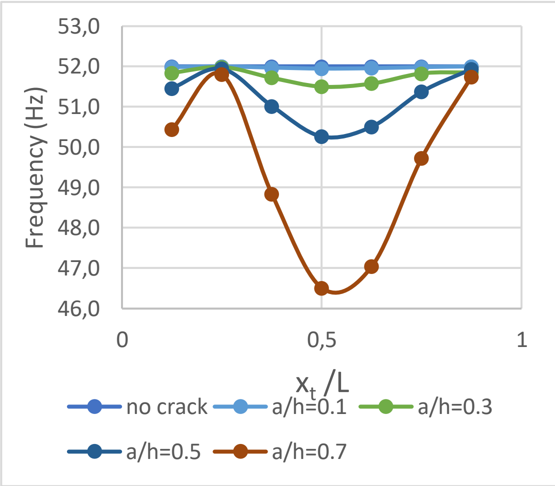
4 RESULTS AND DISCUSSION

The following sections present the results obtained from the simulations related to variations perceived in the natural frequencies, vibration modes, and FRFs of the beam.

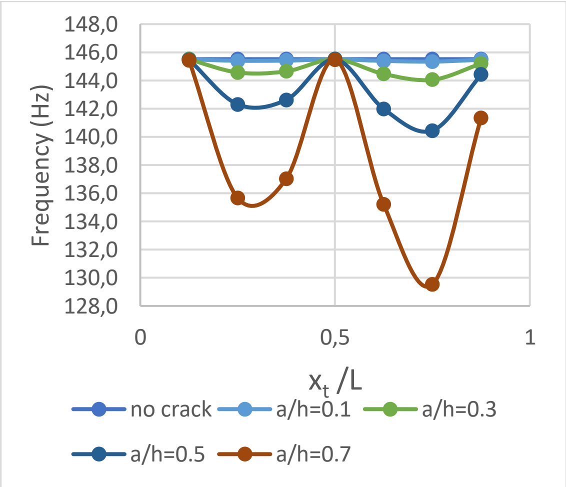
4.1 The Study of Natural Frequencies

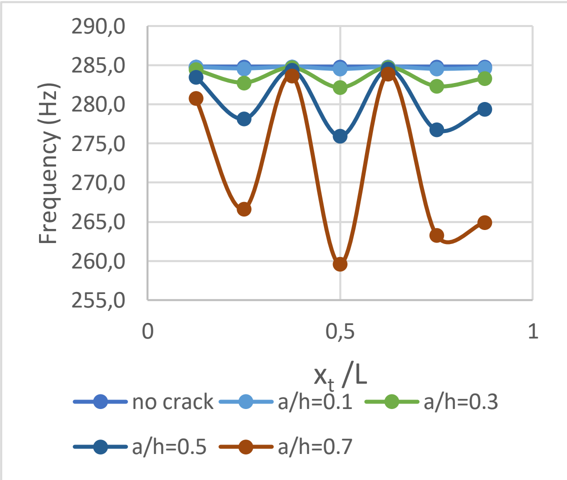
Figures 5 to 9 present the variation of the first five natural frequencies of the beam under study, according to crack location and size.





Figures 5 and 6: 1st and 2nd natural frequency based on the crack, respectively.





Figures 7 and 8: 3rd and 4th natural frequency based on the crack, respectively.

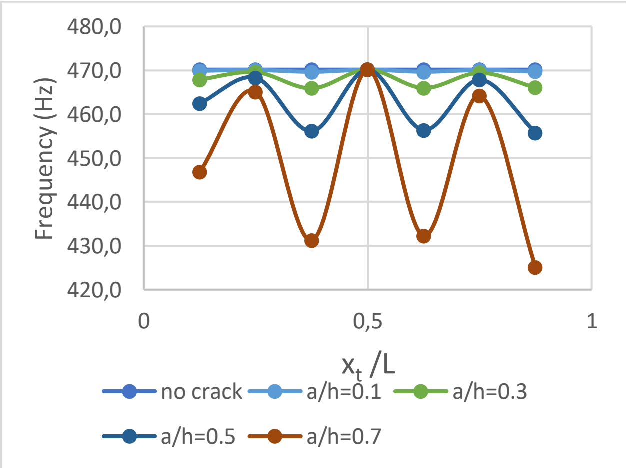


Figure 9: 5th natural frequency based on the crack.

Analyzing the charts, it is possible to observe that the presence of cracks produces decreases in the natural frequencies of the beams, when compared to intact beams. Such effect can be explained by the increase in local flexibility (and a consequent decrease in stiffness) produced by the crack.

One also notices that, due to the location of the crack, the curve of each natural frequency is related to the associated vibration mode. The increase in crack size just renders this effect more pronounced, that is, it increases the amplitude of the curve. Variations in the values of natural frequencies produced by less deep cracks ($a/h = 0.1$) are small, hardly reaching 0.5%. Those small percentage variations constitute a difficulty in identifying cracks through the monitoring of this parameter, for it is common that errors introduced by the measurement and data acquisition system and by computational numerical models are equal or bigger than that. Such results are in accordance with those presented by [5, 14, 15].

4.2 The Study of Vibration Modes

Figures 10 to 14 present three-dimensional charts of the first 5 vibration modes for each of the four crack depths establishing a relationship among: its location, the location of the analyzed node and the percentage variation between the nodal displacement of damaged beams and intact beams.

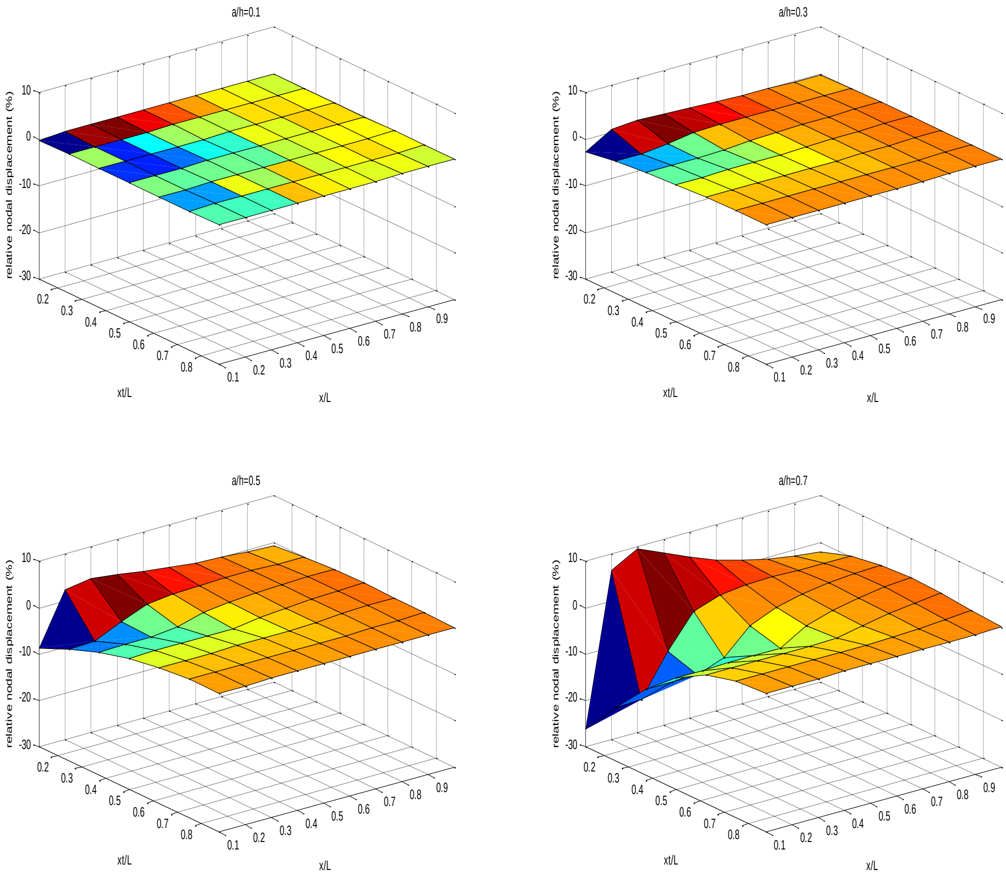


Figure 10: Variations in the 1st vibration mode based on the crack and on the site under study.

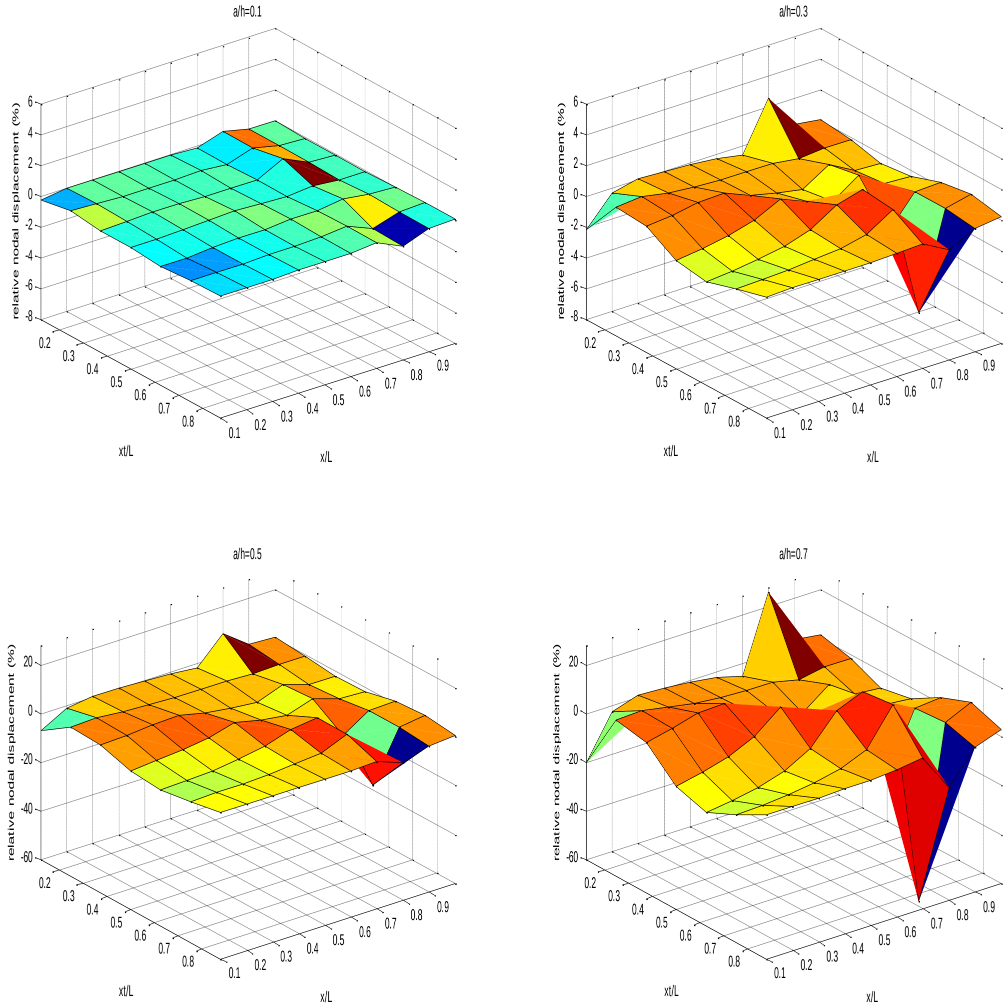


Figure 11: Variations in the 2nd vibration mode based on the crack and on the site under study.

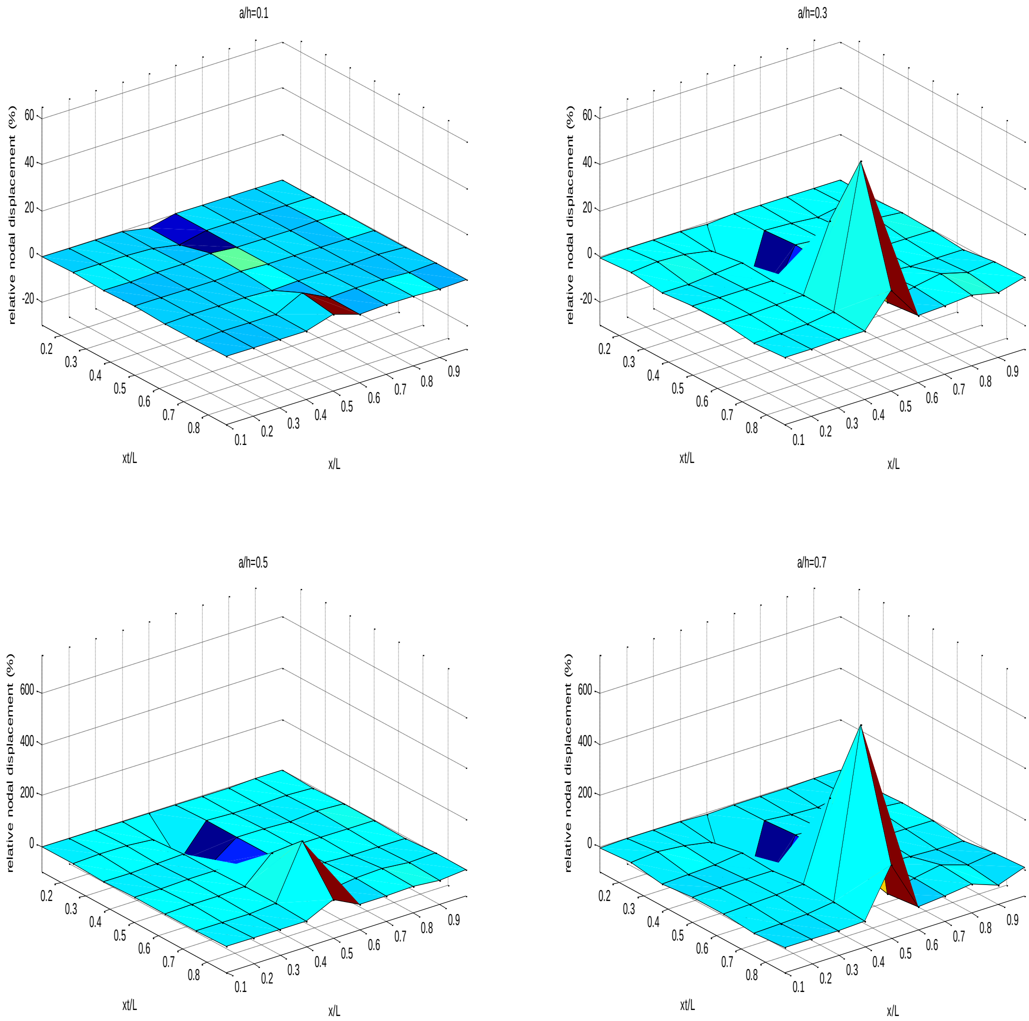


Figure 12: Variations in the 3rd vibration mode based on the crack and on the site under study.

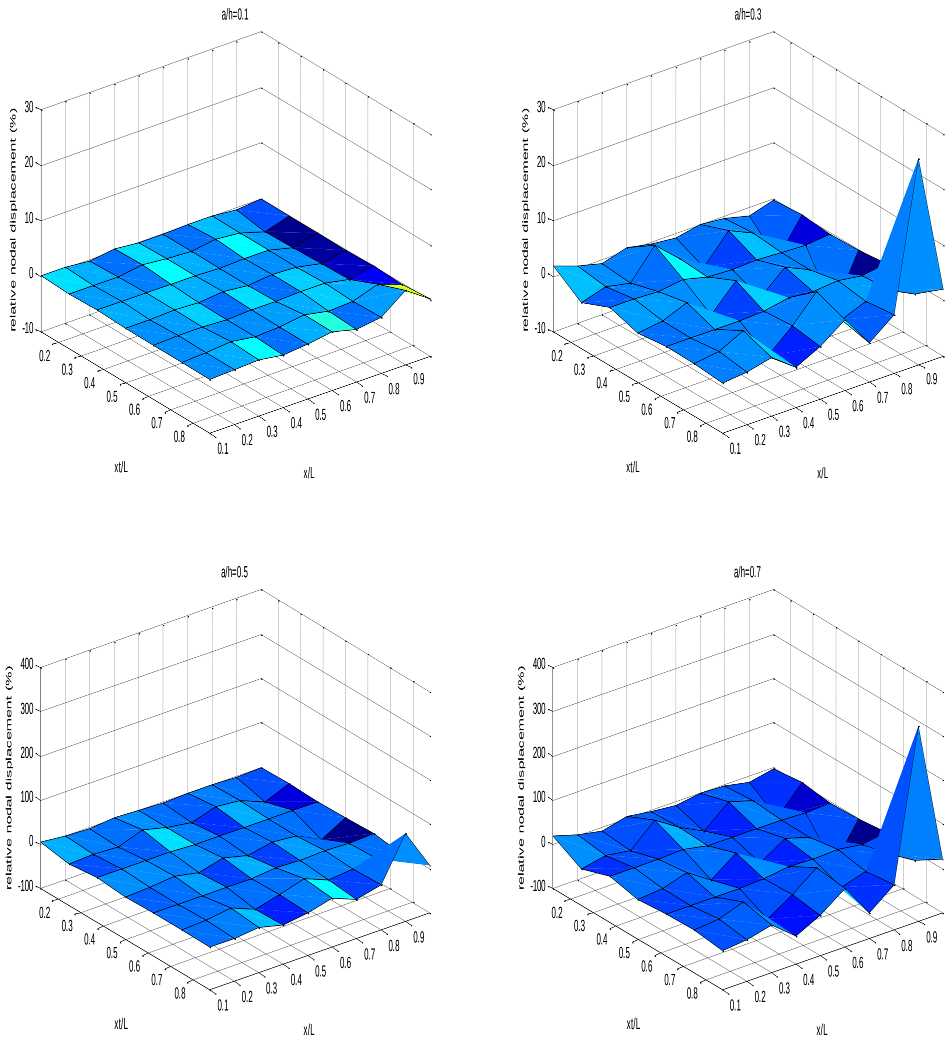


Figure 13: Variations in the 4th vibration mode based on the crack and on the site under study.

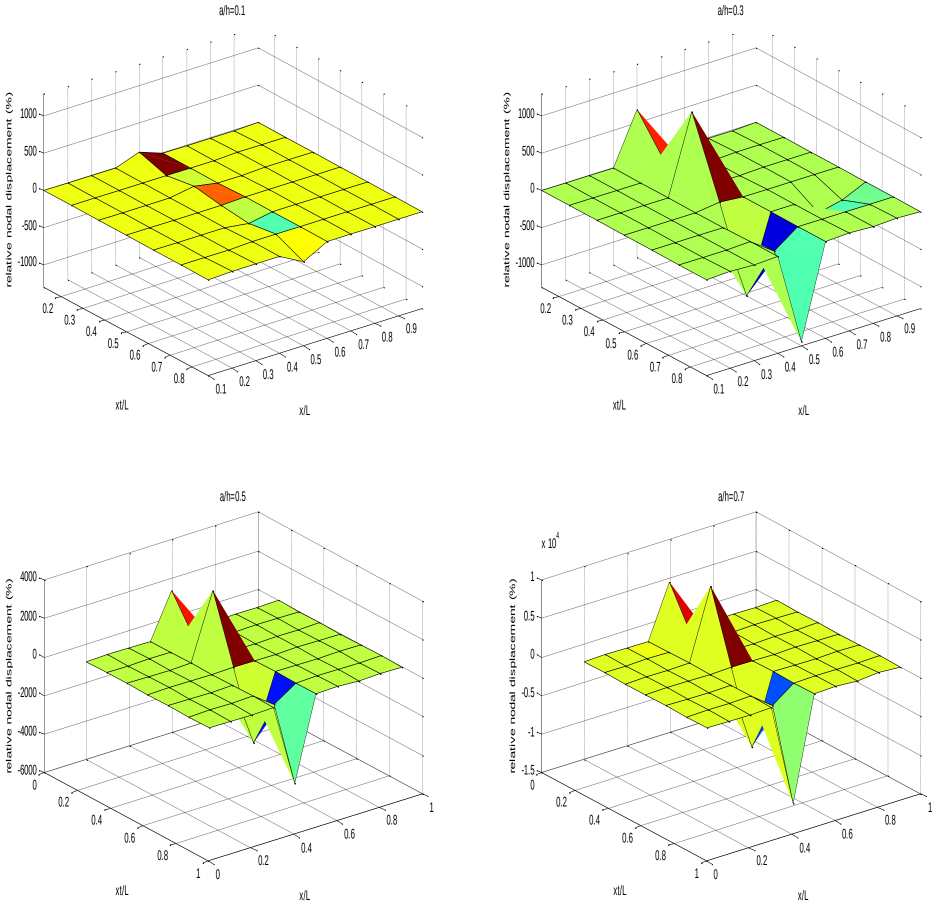


Figure 14: Variations in the 5th vibration mode based on the crack and on the site under study.

Analyzing those charts, it is possible to observe that the variation of the vertical displacement of nodes produced by cracks strongly depends on the location of x/L of the observed node, vibration mode, location and size of the crack. For the third and fifth vibration modes (Figures 12 and 14), for example, cracks produced significant variations in the vertical displacement of nodes located in the middle section of the beam, that is, $x/L = 0.5$. For the first vibration mode (Figure 10), the most significant variations are observed in the nodes located at $x/L = 0.1$. As for the second vibration mode (Figure 11), the effect is more pronounced at $x/L = 0.8$.

Such information provides data about the location of beams which are more sensitive to changes in vibration modes produced by the appearance of cracks. Those sites could be used preferably for installing sensors aiming at monitoring actual structures and identifying cracks, since, at those sites, the relative differences are amplified and the noise effect and measurement errors are smaller.

The charts also show that variation in the displacement of nodes between intact and damaged structures present a similar behavior for the same node locations and crack locations. As larger cracks appear, this effect becomes more pronounced. Such behavior is analogous to that observed in the variation of natural frequencies according to different crack sizes: the shape of the curves is the same, but, the larger the cracks, the larger the amplitude of the curves.

It is interesting to point out that even small cracks ($a/h = 0.1$) produce significant changes on the vibration mode of the beam. In the third mode, for example, this crack size produces 5% changes over the values of the vertical displacements of nodes, when compared to intact beams; in the fourth mode, the changes were above 3%. It was found out that the vibration modes are more sensitive than both natural frequencies and the FRF of beams for a damage of the same magnitude.

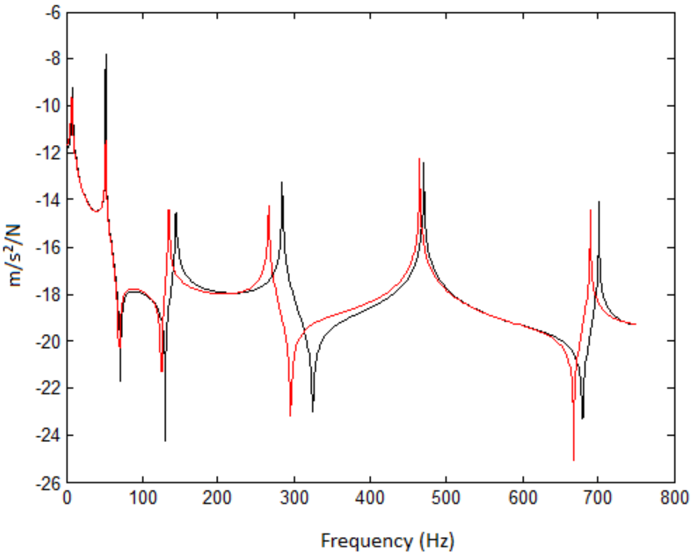
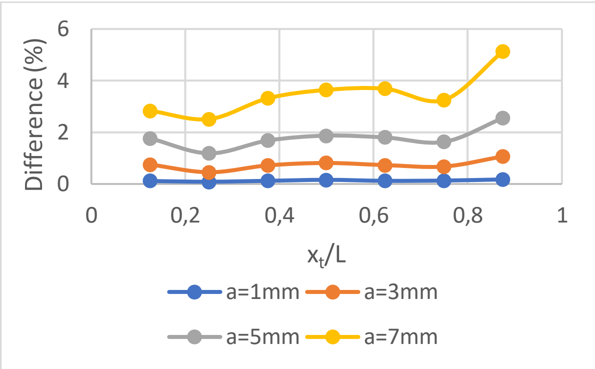


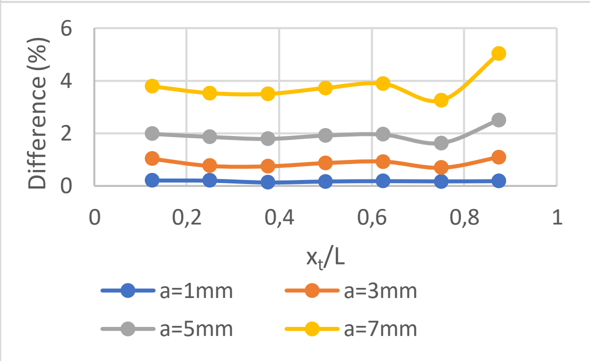
Figure 15: Comparison between the FRF receptance of an intact and a damaged beam.

4.3 The Study of FRFs

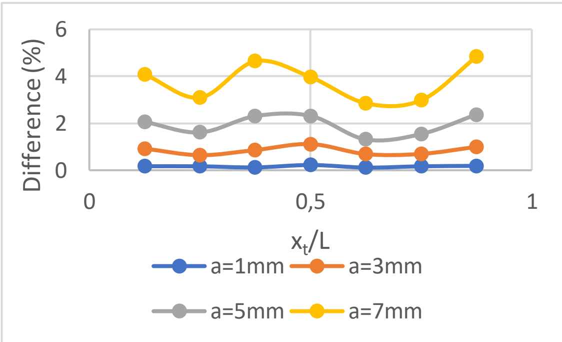
As an example, Figure 15 presents the receptance FRF obtained in site $x/L = 0.6$ for an intact beam (black curve) and for a beam with one crack located at $x_t/L = 0.7$ and size $a/h = 0.7$ (red curve). The estimated difference between those two curves is approximately 3%.

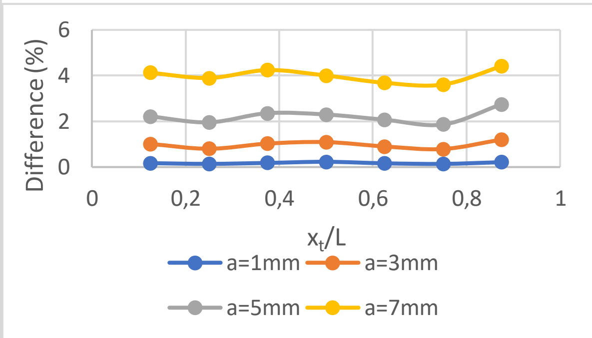
On the whole, the differences between curves for 112 situations like this were calculated, representing the 28 possible combinations of location and size of cracks for 4 different sites for obtaining FRF. The results are presented in Figures 16 to 19, which correlate the percentage variation of FRF curves of intact and damaged beams with location and size of cracks, for each response site.





Figures 16 and 17: Percentage variations of FRF in the response sites x/L=0.2 and x/L=0.4, respectively





Figures 18 and 19: Percentage variations of FRF in the response sites x/L=0.6 and x/L=0.8, respectively

Analyzing the charts, one can observe that, as the cracks get larger, so does the percentage difference between the FRF curves of intact and damaged beams. This effect is in accordance with what was observed for natural frequencies and vibration mode, that is, the greater the crack depth, the greater the change produced by it on the vibrational responses of the structure. For small cracks ($a/h = 0.1$), the percentage difference between the curves did not reach 3% in any case. Analogously to what was discussed in the section about natural frequencies, these low variations constitute a problem when using FRF in identifying cracks, since errors introduced by the measurement and data acquisition system and by the computational models used are usually above that.

The charts also indicate that, as the site for obtaining FRF gets closer to the crack location, there is a reduction in the percentage difference between the curves of intact and damaged beams (see Figure 16, for example). Although more studies need to be conducted in order to confirm this behavior, such effect suggests that sensors (such as accelerometers) should be placed at sites sufficiently distant from the location where the cracks appeared, aiming at making it easier to identify them by capturing differences introduced in the FRF curves.

The comparison of superposed FRF curves for intact and damaged beams corroborates the phenomenon already observed in the analysis of the natural frequencies of beams: the introduction of cracks produces a reduction in the natural frequencies of beams. This effect can be visually observed in the chart in Figure 15.

5 CONCLUSIONS

The effect of different crack locations and sizes on the vibration responses of beams was studied using a computational numerical model. The results show that both location and depth of a crack produce changes in natural frequencies, vibration modes, and FRFs of beams. It was detected that the amplitude of the changes regarding those parameters is proportional to the size of the observed crack.

Cracks whose depths are 10% the height of the beam produced changes lower than 0.4% on the values of the natural frequencies, and lower than 0.3% on the FRF curves, when comparing intact beams to damaged beams. On the other hand, the percentage differences produced by cracks of the same depth $(a/h = 0.1)$ on the vibration modes reached more significant values - above 5% depending on the location of the site under study.

Using three-dimensional charts correlating crack location, analyzed beam position, and amplitude of variations on the vibration modes, the most adequate sites for installing sensors on beams were determined keeping in mind the subsequent aim of identifying the damage. Those sites are given by those locations which are most sensitive to the variation of vibration modes, where the effects become more evident and the identification is made easier.

Using three accelerometers, for example, it would be convenient to place them at $x/L = 0.1$ (first vibration mode monitoring), $x/L = 0.5$ (third and fifth vibration modes monitoring), and $x/L = 0.8$ (second vibration mode monitoring). Depending on the vibration mode analyzed and the number of available sensors, one could think of other advantageous combinations. The authors have already been working on a methodology to identify cracks through the recognition of a pattern change produced by them on the vibration modes of beams.

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