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LINEAR STABILITY ANALYSES OF GÖRTLER VORTICES IN NON-NEWTONIAN BOUNDARY LAYER FLOWS

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Abstract
Transition to turbulence in non-Newtonian fluids is important since a small fraction of polymers in a flow can significanly reduce drag in turbulent flows. Under the influence of centrifugal effects, Newtonian fluid flow transition may occur due to the development of Görtler vortices. In this paper, we conduct linear stability theory analyses to investigate the influence of curvature on the instability of non-Newtonian fluid flows using the Giesekus model. The results indicate that Görtler instability is also present in non-Newtonian fluid flows, with only one unstable and steady mode observed. In the cases examined, the non-Newtonian flows exhibit a greater destabilizing effect compared to Newtonian flows.

Keywords: *Görtler instability, linear stability theory, Giesekus model*

1. INTRODUCTION

In the present study, an investigation applying the Linear Stability Theory (LST) technique to non-Newtonian fluids incorporating a curvature term will be addressed. LST serves as a powerful tool in analyzing fluid flow stability, allowing for the exploration of how small disturbances develop over time and space.

To support this research, a comprehensive literature review was conducted to find previous studies dealing with the application of LST in non-Newtonian fluids of the Giesekus type with a curvature term. However, until now, no articles directly addressing this specific combination of techniques and conditions have been found in the literature. This gap in the literature suggests that the present study may bring novel contributions to the field of fluid mechanics, particularly in understanding stability behaviors in non-Newtonian fluids with curvature effects.

Several related research studies were surveyed. For example, in Furlan (2018), the laminar-turbulent transition due to Tollmien-Schlichting waves in two-dimensional incompressible Poiseuille flow of a viscoelastic fluid was studied using the Giesekus constitutive equation. Both Linear Stability Theory (LST) and Direct Numerical Simulation (DNS) were employed to assess stability to non-stationary disturbances. LST and DNS techniques proved effective in analyzing stability in Giesekus-type viscoelastic flows, providing a deep understanding of the influence of dimensionless parameters in these flows.

Similarly, Silva (2022) investigated the stability of non-Newtonian fluid flow modeled by the Linear Phan-Thien Tanner (LPTT) model. LST was used to assess the stability of viscoelastic fluid flows to non-stationary disturbances.

Linearization of equations for viscoelastic fluid was performed for LST analysis. Various numerical simulations were conducted to evaluate neutral stability curves, comparing them with Newtonian fluid.

Another study by Silva (2018) conducted a numerical simulation to investigate the stability of Giesekus fluid flows during laminar-turbulent transition. The focus was on analyzing Tollmien-Schlichting wave convection in incompressible flow between parallel plates using the Giesekus constitutive equation to describe the viscoelastic behavior of the fluid. Direct numerical simulation was adopted to evaluate flow stability to non-stationary disturbances. Additionally, computational experiments were conducted to verify the accuracy of the code used.

Furthermore, Fernandes (2019) focused on the instability of boundary layers over concave surfaces due to centrifugal instability, resulting in stationary counter-rotating vortices. The study aimed to complement previous research, particularly investigating the effect of pressure gradient on stability diagram and determining the transverse wavelength corresponding to the fastest vortex growth. Based on classical linear stability theory and parallel boundary layer approximation, the results are valid for Görtler numbers above 7. Using the Falkner-Skan solution, linear stability equations are solved for various acceleration parameters, revealing that favorable pressure gradients stabilize the system, while adverse gradients have the opposite effect.

In a study by Rogenski *et al.* (2013), recent numerical studies of centrifugal instability have attracted interest in the scientific community due to their engineering applications. The study investigated how curvature variations influence the development of primary instability using direct numerical simulations. The results are consistent with predictions based on the Parabolized Stability Equation. Convex curvature is identified as a means of controlling laminar flows, modifying longitudinal and transverse velocity profiles.

Therefore, this work aimed to investigate the application of Linear Stability Theory (LST) in non-Newtonian fluids. The absence of previous studies addressing this specific combination suggests a gap in the literature, making this research potentially contributive to understanding stability behaviors in non-Newtonian fluids. The literature review identified relevant studies, highlighting the effectiveness of LST and DNS techniques in analyzing stability in viscoelastic flows and the influence of curvature on primary instability.

2. FORMULATION AND NUMERICAL METHODS

Steady, incompressible and isothermal boundary layer flows of non-newtonian fluids are here represented by the mass equation

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

where $\mathbf{u}$ is the dimensional velocity field, and the Navier-Stokes system of equations

$$\rho\,(\mathbf{u}\cdot\nabla) + \kappa u_1^2\mathbf{j} = \nabla\cdot(-p\,\mathbf{I} + \eta_s\mathbf{D} + \mathbf{T}), \tag{2}$$

where ρ is the density, κ is the curvature, u_1 is the streamwise velocity component, p is the pressure and η_s is solvent viscosity. The tensors $\mathbf{I}$, $\mathbf{D}$ and $\mathbf{T}$ are, respectively, the identity, strain rate and extra-stress tensors. The extra-stress tensor is defined following the Giesekus model

$$\mathbf{T} + \nabla\cdot(\mathbf{u}\mathbf{T}) - \nabla\mathbf{u}\cdot\mathbf{T} - \mathbf{T}\cdot\nabla\mathbf{u}^T + \frac{\alpha_G}{\eta_p}(\mathbf{T}\cdot\mathbf{T}) = \frac{2\eta_p}{\lambda}\mathbf{D}, \tag{3}$$

where λ represents the relaxation time of the fluid and α_G is a mobility Giesekus parameter.

2.1 Linear Stability Theory

Solutions are consider to be a superposition of a two-dimensional boundary layer baseflow state and disturbances. By solving the system of equation assuming disturbances as normal modes, each relevant quantity g is expressed as

$$g_d(x,y,z) = g(y)e^{i(\alpha x + \beta_z z)}.$$

Assuming linearity, the system of equations for disturbances can be represented as a generalized eigenvalue problem in a general form

$$\mathbf{L}\,g = \alpha\,\mathbf{R}g,$$

where $g = [u\ v\ w\ p\ T_{xx}\ T_{xy}\ T_{xz}\ T_{yy}\ T_{yz}\ T_{zz}]^T$. Following spatial stability analyses, a negative value of the imaginary part of the eigenvalue α represents an unstable mode of vortices with wavenumber β_z.

2.2 The baseflow

The non-newtonian boundary layer baseflow solutions are numerically obtained using an in-house, high-order and compact finite difference code (Furlan, 2018).

3. DISCUSSION OF PRELIMINARY RESULTS

Results are presented in non-dimensional form. In the streamwise direction, velocity is normalized by the free stream velocity U_∞. A distance L from the leading edge is assumed to be the length scale in the streamwise direction. Both pressure and tensor components are normalized by ρU_∞^2. Normal and spanwise velocity are normalized by $Re^{-1}U_\infty$. Length scales in these two directions are normalized by $\delta = \sqrt{\eta L/(\rho U_\infty)}$. Adopted physical parameters are in agreement with classic experiments of Swearingen and Blackwelder (1987). We consider $L = 0.1\ m$, $R = 3.2\ m$, $U_\infty = 5\ m.s^{-1}$ and viscosity of $\eta/\rho = 1.5 \times 10^{-5}\ m^2.s^{-1}$. Here, we investigate vortical structures with spanwise wavelength of $18\ mm$.

Dimensionless relevent numbers are the Reynolds, Görtler and Wessember numbers, defined respectively as

$$Re = \frac{\rho U_\infty \delta}{\eta},\ Go_\delta = Re\sqrt{\delta\kappa},\ Wi = \frac{U_\infty \lambda}{L},$$

where η is the total viscosity of the fluid. The newtonian contribution to the flow is measured by the constant $\beta = \eta_s/\eta$.

Baseflow calculations are performed on a structured grid consisting of 633 points in the streamwise direction and 225 points in the normal direction. In the streamwise direction, a spatial step of $dx = 0.025$ is used. In the normal direction, the mesh distribution grows geometrically with a 1% ratio and an initial step size of 10^{-4}. Stability analyses are conducted using 121 Chebyshev polynomials. All results are independent of the discretization parameters.

In Fig. 1, amplification rates of the most unstable mode are presented against Go_δ for three different β parameters $0.7, 0.8$ and 0.9. Newtonian reference case is overlap in dashed style. One may observe the higher the non-newtonian contribution to the flow the higher the amplification rate. Observed differences increase as the boundary layer grows.

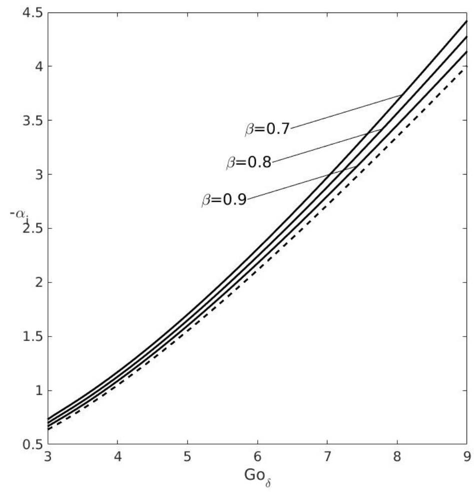


Figure 1: Amplification rate curves against Go_δ for $\beta = 0.7, 0.8$ and 0.9. The most unstable mode is followed for cases where $\alpha_G = 0.30$ and $Wi = 1$. Other parameters remain the same as those used in the Newtonian case. Dashed line represents the newtonian reference case.

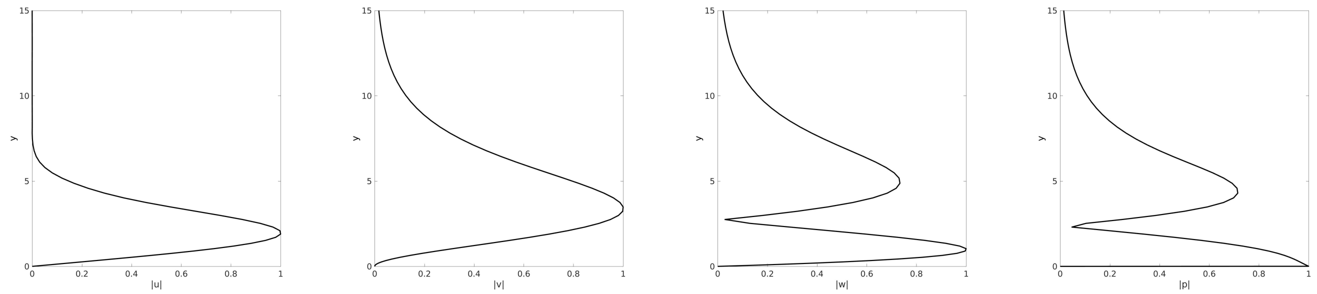
These observed unstable modes can be related to newtonian unstable Görtler mode. At $Go_\delta = 4.5$, eigenfuctions for the non-newtonian case $\beta = 0.7$, $\alpha_G = 0.30$ and $Wi = 1$ are presented in Fig. 3. Each function is normalized by their maximum values. Disturbance profiles are in agreement with the ones observed in stabilities studies for newtonian fluid flows. In addition, Fig. 3 presents eigenfuctions for each component of the non-Newtonian tensor.

The eigenvalue spectra to the case is presented in Fig. 4. One may notice that there is only one observed purely imaginary unstable mode ($\alpha_i < 0$).

In the adopted range of parameters, the elasticity parameter, given by the Weissenberg number, shows a weakly stabilization effect. In Tab. 1, amplification rates of the most unstable mode decreases with the increment of Wi at the streamwise position $Go_\delta = 4.5$. For these cases $\alpha_G = 0.30$ and $\beta = 0.7$.

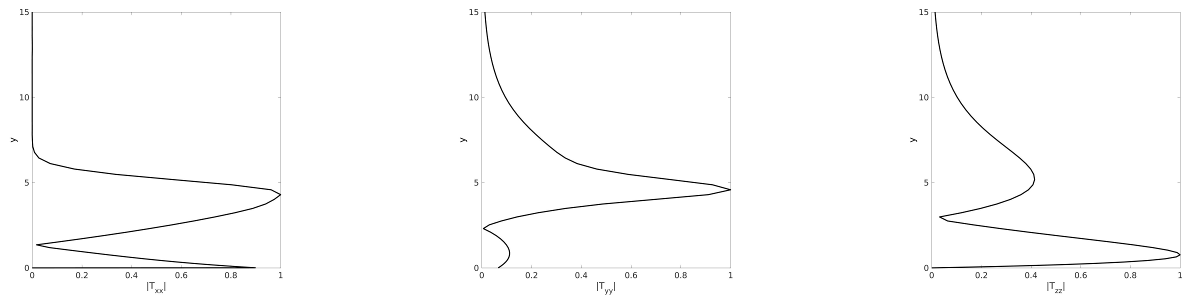
Table 1: Amplification rate of most unstable mode at $Go_\delta = 4.5$ for $Wi = 1, 5, 10$ and 15. Non-newtonian parameters are $\beta = 0.7$ and $\alpha_G = 0.30$.

Wi	1	5	10	15
$-\alpha_i$	1.725	1.715	1.713	1.712

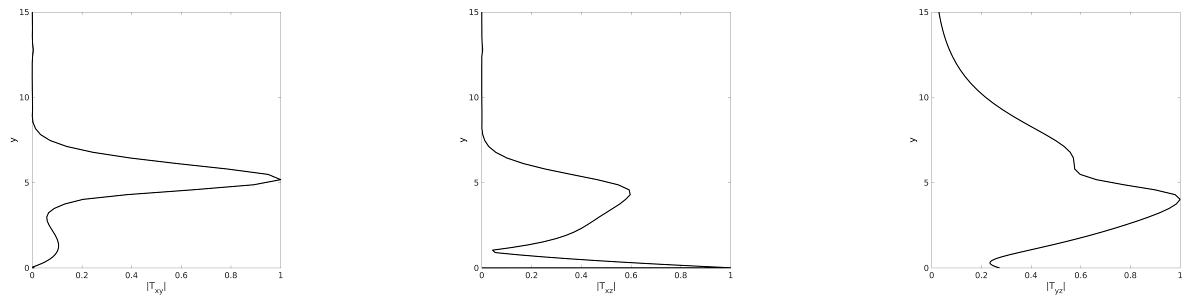


(a) u (b) v (c) w (d) p

Figure 2: Distributions of the absolute values of the disturbance velocity and pressure eigenfunctions for the most unstable mode at $Go_\delta = 4.5$. Here $\beta = 0.7$, $\alpha_G = 0.30$ and $Wi = 1$.



(a) T_{xx} (b) T_{yy} (c) T_{zz}



(d) T_{xy} (e) T_{xz} (f) T_{yz}

Figure 3: Distributions of absolute values of the eigenfunctions of each disturbance tensor component for the most unstable mode at $Go_\delta = 4.5$. Here $\beta = 0.7$, $\alpha_G = 0.30$ and $Wi = 1$.

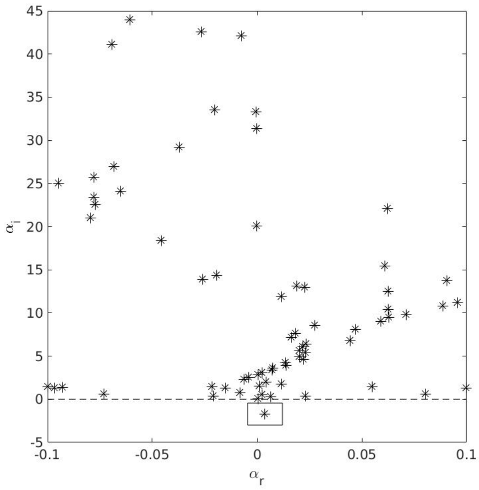


Figure 4: Eigenvalue spectra for case $\beta = 0.7$, $\alpha_G = 0.30$ and $Wi = 1$. Displayed results are obtained using 121 Chebyshev polynomials. Other parameters remain the same as those used in the newtonian case.

4. CONCLUSION

This study investigated the application of Linear Stability Theory (LST) in non-Newtonian fluids, with a specific focus on the Giesekus model, considering the effect of curvature. The researchers used a combination of numerical methods, including Direct Numerical Simulations (DNS) and the solution of a generalized eigenvalue problem, to analyze the amplification rates and eigenfunctions of unstable modes. The main results and conclusions of this study are: the influence of non-Newtonian behavior, which revealed that the presence of non-Newtonian properties in the fluid leads to an increase in the amplification rate of unstable modes compared to Newtonian fluids, with this difference becoming even more pronounced as the boundary layer develops; the effect of elasticity, where the research examined the impact of the elasticity parameter, represented by the Weissenberg number (Wi), within a specific range of parameters, indicating that an increase in Wi has a weak stabilizing effect, resulting in a slight decrease in the amplification rates of the most unstable mode; and the comparison with the Newtonian Görtler mode, where the perturbation profiles observed for the non-Newtonian case, specifically with $\beta = 0.70$, $\alpha_G = 0.30$ and $Wi = 1$ at $Go_\delta = 4.5$, showed similarities to those observed in stability studies for Newtonian fluid flows, suggesting a possible link between the unstable modes observed in this study and the Görtler mode, a well-known instability mode found in Newtonian fluids.

5. ACKNOWLEDGEMENTS

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