Displacement of Viscous Liquids by a Viscoplastic Material on Capillary Tubes

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Abstract: *The liquid-liquid displacement has been analysed since the pioneering work of Goldsmith & Mason (1963). This kind of problem has a number of practical applications as oil recovery in porous media and cementing of oil wells, in which the control of the liquid film attached to the wall is quite important. Considering two immiscible liquids, the physical mechanism is governed by the viscosity and density ratios, capillary number, Reynolds number, besides the rheological properties of each material. This work is a contribution on the understanding of the role played by viscoplastic materials in such phenomenon. In a simple experimental apparatus, a Carbopol solution is used to displace a Newtonian liquid in a capillary tube. The images of the interface are recorded during the test. From these images, we have the interface shape and the thickness of the displaced liquid attached to the wall as a function of a plastic number. By a simple volume balance, we also have a measure of the displacement efficiency. It is shown that the displacement efficiency and the liquid film attached to the wall are not directly related. In addition, instabilities were found along the length of the bubbles from the center through to the end. It was also observed that the displacement efficiency was greater than the equivalent Newtonian case, while the thickness of the displaced liquid attached to the wall was smaller. Finally it is shown that both parameters are lower than the equivalent Newtonian cases and they decrease with the increase of the plastic number.*
Keywords: *liquid-liquid displacement, capillary tubes, viscoplastic material, displacement efficiency*

1. INTRODUCTION

An important problem in the oil industry is the primary cementing of an oil well. After the drilling process, the cement slurry is deployed into the well in order to cement the narrow annular space formed by the outside wall of the steel casing and the inside wall of the drilled rock formation. The cement slurry displaces the drilling mud. Not only the drilling mud but also the cement slurry exhibits viscoplastic behavior, i.e., they are yield stress materials.

Another problem of interest in the oil industry is the displacement of oil by an injected fluid, in order to push the oil inside the reservoir. In the case of the so-called heavy oils, the oil exhibits viscoplasticity, i.e., it must be subjected to a minimum stress value in order to flow. Not taking into account this non-Newtonian feature of the material can lead to incorrect specifications of the hydraulic plant, such as the required power to pump the flow. Besides that the choice of the material used to displace the oil considerably affects the efficiency of the process.

These two problems are rather complex because they involve two different non-Newtonian materials, one displacing the other, and, sometimes, capillary effects due to the small length scale of the typical porous media and the interfacial forces between the fluids.

The gas–liquid (Newtonian) displacement has received considerable attention in the literature and was investigated by several authors such as Fairbrother and Stubbs (1935) (experimentally), Bretherton (1961) (theoretically), Taylor (1961) (experimentally) and Cox (1962) (experimentally).

The gas displacement of the non-Newtonian fluids has also received some attention. Hassager and Lauridsen (1998), Sousa *et al.* (2007) and Thompson *et al.* (2010) investigated the gas displacement of power-law fluids. Dimakopoulos and Tsamopoulos (2003),

The liquid–liquid displacement of two immiscible Newtonian fluids has been investigated since the work of Goldsmith and Mason (1963). Since than, important contributions were made by Hodges *et al.* (2004), Soares *et al.* (2005), Soares and Thompson (2009), Freitas *et al.* (2011b), and Soares *et al.* (2015), among others. In the liquid–liquid displacement problem, the conservation equations are solved in the displacing fluid domain also and the force balance at the interface needs an additional term that takes into account the extra normal stress at the displacing fluid side, besides the pressure.

A recent study of the Newtonian displacement by a non-Newtonian liquid when capillary stresses are important was conducted by Thompson and Soares (2010) and Freitas *et al.* (2011a) for a viscoplastic material and Thompson and Soares (2012) for the case of a power-law displacing fluid. Taghavi *et al.* (2012) studied the viscoplastic displacement by a Newtonian fluid. In connection to what was presented above, the objective of the present experimental study is to explore the displacement of a Newtonian fluid by a viscoplastic material in a capillary tube.

2. THEORETICAL ANALYSIS

The quantities m_e and m_g defined by Soares *et al.* (2015) were defined in a steady state regime for an observer attached to the tip of the interface as shown in Fig. 1. Therefore, we will present the analysis with this approximation, i.e. we assume that the front of he displacing liquid achieves a constant shape and that the thickness of the displaced fluid remains constant along certain length L much longer than the size of the nose of the drop. In addition, the flow is assumed to be isothermal, inertialess and incompressible with equal mass densities.

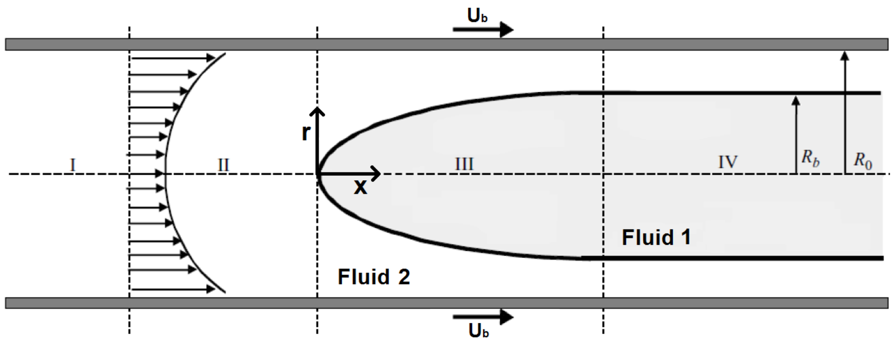

Figure 1: Schematic representation of the problem.

2.1 Dimensionless governing equation

Choosing the radius of the tube, R_0, as characteristic length and drop velocity U_b as characteristic velocity together with a characteristic stress, $\mu_2 U_b/R_0$, where μ_2 is the viscosity of the displaced Newtonian fluid. We can follow a traditional procedure to deduce the dimensionless forms of the governing equation, i.e. continuity, momentum balance, interface impermeability, and interface balance. The conservation of mass equation is given by

$$\nabla^* \cdot \mathbf{u}_k^* = 0 \tag{1}$$

while the equation of conservation of momentum is given by

$$\nabla^* \cdot \mathbf{T}_k^* = -\nabla^* p_k^* + \nabla^* \cdot \mathbf{T}_{\mathbf{k}}^{\mathbf{E}*} = 0 \tag{2}$$

where the subscript $k = 1, 2$ labels the two materials considered. In Eq. 1, $\mathbf{u}_k^*$ is the dimensionless velocity vector, $\mathbf{u}_k = U_b \mathbf{u}_k^*$, and in Eq. 2, $\mathbf{T}_k^* = -p_k^* \mathbf{1} + \mathbf{T}_{\mathbf{k}}^{\mathbf{E}*}$ is the total dimensionless stress vector, where p_k^* is the dimensionless pressure, $\mathbf{1}$ is the unit tensor, $\mathbf{T}_k^{\mathbf{E}*} = 2\eta_k^* \mathbf{D}_k^*$ is the viscous part of the stress tensor, and $\mathbf{D}_k$ is the symmetric part of the velocity gradient. Since phase 1 obeys the SMD equation proposed by Mendes and Dutra (2004) and phase 2 is Newtonian, Eq. 2 takes the form

$$-\nabla^* p_1^* + \frac{1}{N_\eta} \nabla^* \cdot \left\{ \left[\frac{Pl}{\dot{\gamma}_1^*} + (1 - Pl)\, \dot{\gamma}_1^{*(n-1)} \right] \left[1 - exp\left(-\eta_0 U_b \dot{\gamma}_1^* / \tau_0 R_0\right)\right] \right\} \mathbf{D}_1^* = 0 \tag{3}$$

in the displacing fluid domain, and

$$-\nabla^* p_2^* + \nabla^* \cdot \mathbf{D}_2^* = 0 \tag{4}$$

in the displaced fluid domain. The dimensionless quantities labeled with a superscript * are given by

$$\nabla^* \equiv R_0 \nabla; \quad p_k^* \equiv \frac{p_k R_0}{\mu_2 U_b}; \quad \eta_k^* \equiv \frac{\eta_k}{\mu_2}; \quad \dot{\gamma}_1^* \equiv \frac{\dot{\gamma}_1 R_0}{U_b}; \quad \mathbf{D}_k^* \equiv \frac{2R_0}{U_b} \mathbf{D}_k \tag{5}$$

in Eq. 3, we see the presence of the viscosity ratio, N_η, given by

$$N_\eta = \frac{\mu_2}{\tau_0 \left(\frac{R_0}{U_b}\right) + K \left(\frac{U_b}{R_0}\right)^{n-1}} \tag{6}$$

and the plastic number has the following expression:

$$Pl = \frac{\tau_0}{\tau_0 + K \left(\frac{U_b}{R_0}\right)^n} \tag{7}$$

The dimensionless parameters here presented were based on the criteria discussed by Thompson and Soares (2016) where the authors highlight the importance of taking the yield stress into consideration when defining viscoplastic dimensionless quantities. At the liquid-liquid interface there is no mass flow across the interface. This condition can be translated by

$$\mathbf{u}_1^* = \mathbf{u}_2^* = (\mathbf{u}_k^* \cdot \mathbf{t})\,\mathbf{t} \tag{8}$$

where $\mathbf{t}$ is the tangent vector. Furthermore, at the interface the traction balances the capillary pressure,

$$\mathbf{n}\,(p_1^* - p_2^*) + \mathbf{n}\left\{\mathbf{D}_2^* - \frac{1}{N_\eta}\left[\left[\frac{Pl}{\dot{\gamma}_1^*} + (1-Pl)\,\dot{\gamma}_1^{*(n-1)}\right][1 - exp\,(-\eta_0 U \dot{\gamma}_1^*/\tau_0 R_0)]\right]\mathbf{D}_1^*\right\} = \frac{1}{Ca}\frac{1}{R_m^*}\mathbf{n} \tag{9}$$

where p_1^* and p_2^* are the dimensionless pressures on fluids 1 and 2. The capillary number is given by

$$Ca = \frac{\mu_2 U_b}{\sigma} \tag{10}$$

where σ is the liquid-liquid interfacial tension and $R_m^* = R_m/R_0$ is the dimensionless radius of the curvature. Equation 9 carries the dimensionless number that govern the problem, namely Ca and N_η.

2.2 Geometrical residual mass fraction and displacement efficiency

The *geometrical residual mass fraction*, m_g, is the defined as the mass fraction of Fluid 2 that remains attached to the wall after the bubble of Fluid 1 has passed through the tube. Considering Fig. 1, it can be calculated by considering a cylinder of length Δx and radius R_0. Before the displacing fluid invades the cylinder, the mass of Fluid 2 is $\rho_2 \pi R_0^2 \Delta x$. After the Fluid 1 passes through, the mass of Fluid 2 inside the cylinder is $\rho_2 \pi (R_0^2 - R_b^2)\Delta x$. So, m_g is given by

$$m_g = \frac{\rho_2 \pi (R_0^2 - R_b^2)\Delta x}{\rho_2 \pi R_0^2 \Delta x} = 1 - \frac{R_b^2}{R_0^2} \tag{11}$$

The *mass displacement efficiency*, m_e, is defined as the complement of the mass fraction of displaced fluid that leaves the tube while the displacing fluid is injected. By definition m_e is given by

$$m_e = \frac{lost\,mass}{total\,mass} = 1 - \frac{recovered\,mass}{total\,mass} \tag{12}$$

3. EXPERIMENTS

3.1 Experimental setup

The liquid-liquid displacement experiments were conducted in the experimental setup depicted in Fig. 2. The elements that composed the setup were a syringe pump (2) controlled by a computer (1); a glass capillary tube (6) with a length $L = 1.5m$ and internal diameter of $D = 2mm$ supported by hooks attached to three metallic bars (3); an acrylic box (5) filled with glycerine; a photographic camera (4); a ruler (7); a precision scale (8); a glass beaker (9); a cylindrical tank (11) filled with the displaced fluid connected to an air compressor supply controlled by a valve (10).

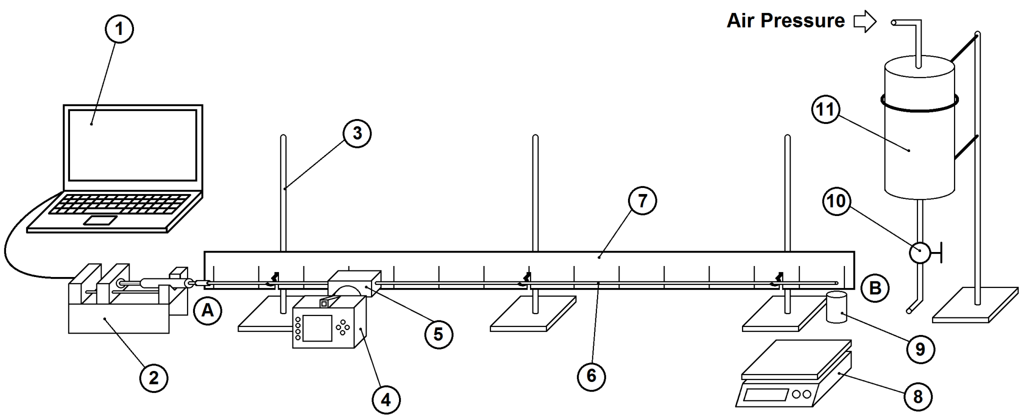

Figure 2: Scheme of the experimental setup.

3.2 Fluids employed

The displacing fluid used is a solution of water and hair gel 7.5% w/w. The properties of the displacing solution were obtained based on the SMD model curve adjusted to the solution's flow curve, as shown in Fig. 3 and presented in Table 1. The combination of displaced and displacing fluids along with the values of interfacial tension (σ) and density ratio (N_ρ) calculated with respect to the displaced fluid, are in Table 2.

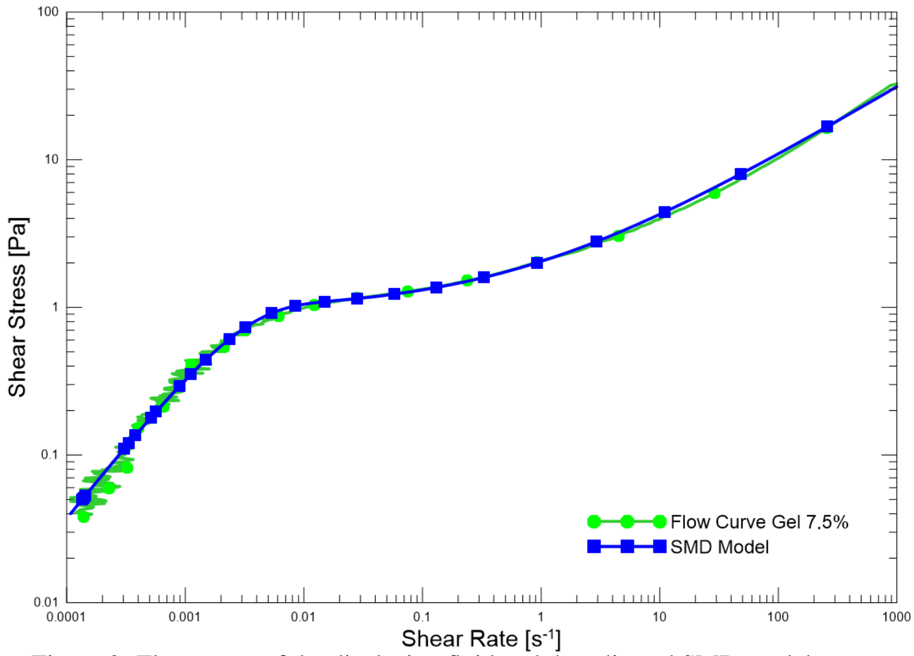

Figure 3: Flow curve of the displacing fluid and the adjusted SMD model curve.

Table 1: Properties of the viscoplastic solution.

Property	Value
$\%HairGel$	7.5
$\rho\ [kg/m]$	1008.9
$\tau_0[Pa]$	0.95
n	0.48
$K[Pa.s^n]$	1.1
$\eta_0[Pa.s]$	375

Table 2: Arrangements: dispacing and displaced fluids, interfacial tension (σ), and density ratio (N_ρ).

Displacing Fluid	Displaced Fluid	N_ρ	$\sigma[mN/m]$
Hair Gel Solution	Castor Oil	0.958	5.07

3.2.1 Experimental procedure

The end B of the capillary tube is connected with the displaced fluid tank and is filled with oil. The connection on end B is removed and the syringe filled with displacing fluid is connected to end A. The acrylic box is filled with glycerin and the camera is focused on the tube. The pump is set on the computer and the command to start the displacement is given. The moment the displacing drop reaches the 25 cm marking on the ruler, the beaker is placed on the end B of the tube, collecting the mass of the displaced fluid that leaves the tube from that moment on. When the drop reaches the 125 cm marking on the ruler, the beaker is removed from end B and the command to stop the pump is given. The recovered mass of displaced fluid is then weighed. Images of the displacement are captured during the test.

4. RESULTS AND DISCUSSIONS

As described in Subsection 2.2, m_g is measured directly from the captured images using the geometric relation given by Eq. 11, while m_e is measured directly from weighing the recovered mass. Figure 4 shows the experimental results obtained ind order to validate the experimental setup. The gas-liquid displacement problem explored by Taylor (1961) was reproduced in the present work for $N_\eta = 400$. It is shown that both curves are equivalents, which validates the experimental setup.

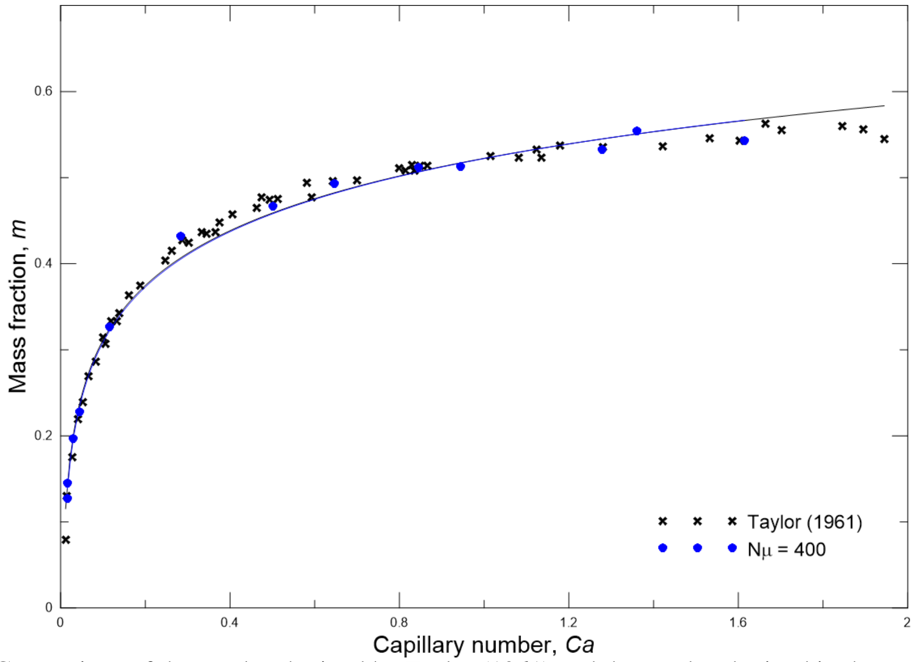

Figure 4: Comparison of the results obtained by Taylor (1961) and the results obtained in the present work.

Experimental results for the viscoplastic residual mass fraction and displacement efficiency compared to the equivalent Newtonian case (obtained in previous works) as a function of the plastic number, Pl, are shown in Fig. 5.

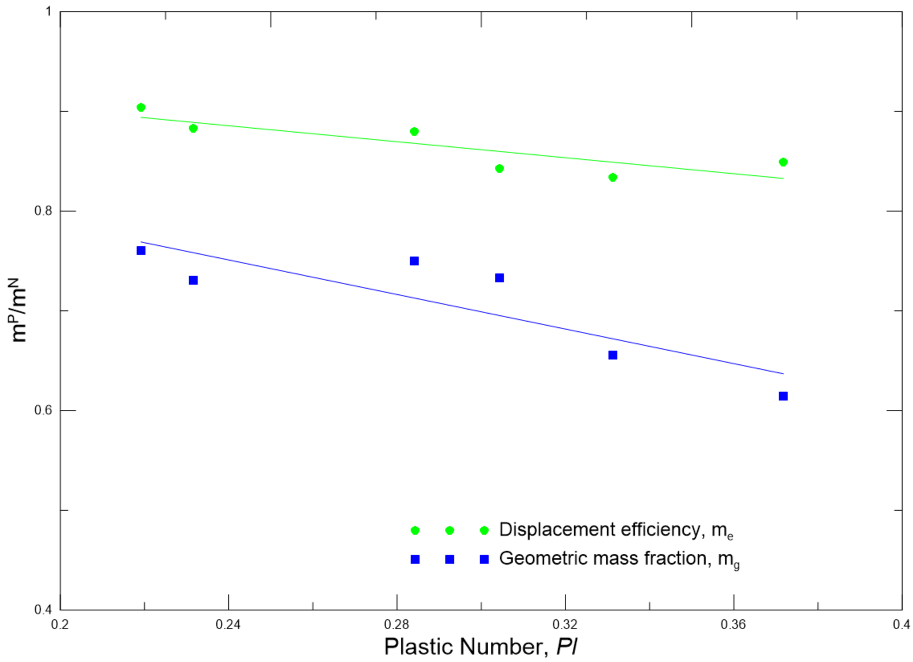

Figure 5: Comparison between the Newtonian and viscoplastic cases as a function of the plastic number.

It is shown that the geometric mass fraction and the displacement efficiency are smaller for the viscoplastic case. In fact, as the plastic number increases, the viscoplastic effects are more significant and the results departs from the Newtonian case. Furthermore, images of the interfaces were captured during the tests and it is noticed that the viscoplastic nature of the material causes disturbs the shape of the interface, as can be seen in Fig. 6, which compares the viscoplastic case with a equivalent Newtonian one (obtained in previous works).

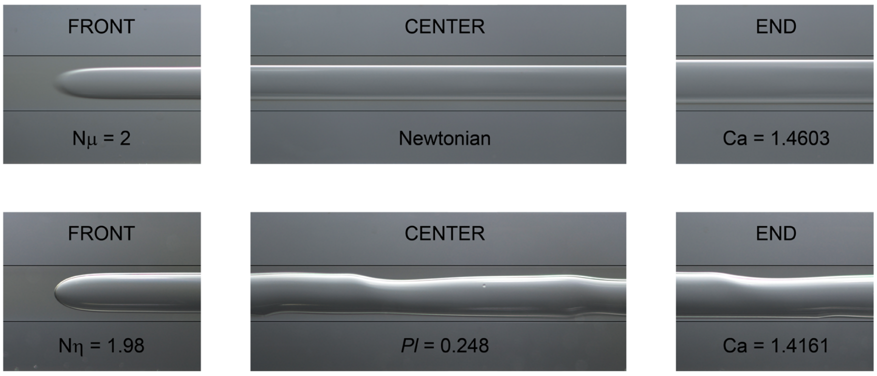

Figure 6: Comparison between the shape of the interface in Newtonian and viscoplastic cases for N_η = 2.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

Bretherton, F.P., 1961. "The motion of long bubbles in tubes". *Journal of Fluid Mechanics*, Vol. 10, pp. 166–188.

Cox, B.G., 1962. "On driving a viscous fluid out of a tube". *Journal of Fluid Mechanics*, Vol. 14, pp. 81–96.

Dimakopoulos, Y. and Tsamopoulos, J., 2003. "Transient displacement of a viscoplastic material by air in straight and suddenly constricted tubes". *Journal of Non-Newtonian Fluid Mechanics*, Vol. 112, pp. 43–75.

Fairbrother, F. and Stubbs, A.E., 1935. "Studies in electro-endosmosis. part vi. the bubble-tube method of measurement". *Journal of Chemical Society*, pp. 527–529.

Freitas, J.F., Soares, E.J. and Thompson, R.L., 2011a. "Immiscible newtonian displacement by a viscoplastic material in a capillary plane channel". *Rheologica Acta*, Vol. 50, pp. 403–422.

Freitas, J.F., Soares, E.J. and Thompson, R.L., 2011b. "Residual mass and flow regimes for the immiscible liquid-liquid displacement in a plane channel". *International Journal of Multiphase Flow*, Vol. 37, pp. 640–646.

Goldsmith, H.L. and Mason, S.G., 1963. "The flow of suspensions through tubes. ii. single large bubbles". *Journal of Colloid Science*, Vol. 18, pp. 237–261.

Hassager, O. and Lauridsen, T.L., 1998. "Singular behavior of power-law fluids in hele shaw flow". *Journal of Non-Newtonian Fluid Mechanics*, Vol. 29, pp. 337–346.

Hodges, S.R., Jensen, E.O. and Rallison, M.J., 2004. "The motion of a viscous drop through a cylindrical tube". *Journal of Fluid Mechanics*, Vol. 501, pp. 279–301.

Mendes, P.R.S. and Dutra, E.S.S., 2004. "A viscosity function for viscoplastic liquids". *Annual Transactions of the Nordic Rheology Society*, Vol. 12.

Soares, E.J., Carvalho, M.S. and Mendes, P., 2005. "Immiscible liquid-liquid displacement in capillary tubes". *Journal of Fluid Engineering*, Vol. 127, pp. 81–96.

Soares, E.J. and Thompson, R.L., 2009. "Flow regimes for the immiscible liquid–liquid displacement in capillary tubes with complete wetting of the displaced liquid". *Journal of Fluid Mechanics*, Vol. 641, pp. 63–84.

Soares, E.J., Thompson, R.L. and Niero, D.C., 2015. "Immiscible liquid–liquid pressure-driven flow in capillary tubes: Experimental results and numerical comparison". *Physics of Fluids*, Vol. 27.

Sousa, D.A., Soares, E.J., Queiroz, R.S. and Thompson, R.L., 2007. "Numerical investigation on gas-displacement of a shear-thinning liquid and a visco-plastic material in capillary tubes". *Journal of Non-Newtonian Fluid Mechanics*, Vol. 144, pp. 149–159.

Taghavi, S.M., Alba, K., Moyers-Gonzalez, M. and Frigaard, I.A., 2012. "Incomplete fluid–fluid displacement of yield stress fluids in near-horizontal pipes: Experiments and theory". *Journal of Non-Newtonian Fluid Mechanics*, Vol. 167–168, pp. 59–74.

Taylor, G.I., 1961. "Deposition of a viscous fluid on the wall of a tube". *Journal of Fluid Mechanics*, Vol. 10, pp. 161–165.

Thompson, R.L. and Soares, E.J., 2010. "Fraction of mass attached to the wall in the liquid-liquid displacement in capillary tubes when the displacing liquid is a power-law non-newtonian fluid". *Brazilian Conference on Rheology*.

Thompson, R.L. and Soares, E.J., 2012. "Motion of a power-law long drop in a capillary tube filled by a newtonian fluid". *Chemical Engineering Science*, Vol. 72, pp. 126–141.

Thompson, R.L. and Soares, E.J., 2016. "Viscoplastic dimensionless numbers". *Journal of Non-Newtonian Fluid Mechanics*.

Thompson, R.L., Soares, E.J. and Bacchi, R.D.A., 2010. "Further remarks on numerical investigation on gas-displacement of a shear-thinning liquid and visco-plastic material in capillary tubes". *Journal of Non-Newtonian Fluid Mechanics*, Vol. 165, pp. 448–452.