

Displacement of Viscous Liquids by a Newtonian Material on Capillary Tube

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Abstract: *The liquid-liquid displacement in capillary tubes has been constantly analyzed on the last decades. Currently, the comprehension of this kind of problem is of great relevance, as it involves several applications on oil recovery and on cementing of oil wells. The phenomenon has been explored at the present work in order to enlighten our understanding of the role played by the physical mechanisms that governs the problem. In a simple experimental apparatus, a PEG solution is used to displace a Newtonian liquid in a capillary tube. The images of the interface are recorded during the test. From these images, we have the interface shape and the thickness of the displaced liquid attached to the wall as a function of the capillary number and viscosity ratio. By a simple volume balance, we also have a measure of the displacement efficiency. It is shown that the physical mechanism is governed by the viscosity and density ratios, capillary number, Reynolds number. Finally, it is noticed that when viscosity ratio decreases, the thickness of the displaced liquid attached to the wall increases while the fraction of mass that is not recovered decreases.*

Keywords: *liquid-liquid displacement, capillary tube, displacement efficiency*

1. INTRODUCTION

Fluid-fluid displacement is an important subject of Fluid Mechanics that can be found in many application such as molding, injection of liquid medicine in blood vessels, oil recovery in porous media, and many others. In the present investigation we are focused on the analysis of immiscible fluids where interfacial forces are important, in laminar regime, where buoyancy and inertial effects are negligible. In addition, we are particularly interested in the case the displaced fluid wets the wall duct, and therefore, a layer of this fluids still remains attached to the wall.

This problem has its origins on the experimental investigation conducted by Taylor (1961) where the efficiency of the gas-liquid displacement was plotted against the capillary number. This work was generalized by considering the Newtonian-Newtonian liquid-liquid displacement (Goldsmith and Mason (1963); Hodges *et al.* (2004); Soares and Thompson (2009); Lac and Sherwood (2009); Freitas *et al.* (2011)).

As discussed by Soares *et al.* (2015) there are two quantities of interest that can be used in order to grasp the amount of liquid of displaced fluid that is "left behind". The so-called *mass displacement efficiency* and the so-called *geometrical residual mass fraction* which are coincident in the gas-liquid displacement problem, but are different measures in the liquid-liquid one.

The numerical scheme adopted by Soares *et al.* (2005); Soares and Thompson (2009) assumed that the flow is intrinsically transient, i.e. there is a reference frame that evaluates that this problem can be described in a steady state regime. That reference frame is the one attached to the tip of the interface. It is important to register that Soares and Thompson (2009) reported some difficulties on having converged results for a viscosity ratio in the range $N_\mu < 2$, i.e. when the viscosity of the displaced fluid is less than two times the viscosity of the displacing one. Lac and Sherwood (2009) and Soares and Thompson (2009) have independently shown that the Newtonian-Newtonian displacement problem has two branches of flow patterns where $N_\mu = 2$ is the border of the two cases. In particular, it was shown that it is impossible to have a *by-pass* regime when $N_\mu \leq 2$. In connection to what was presented above, the objectives of the present experimental study are to explore the $N_\mu < 2$ range in the Newtonian-Newtonian displacement problem.

2. THEORETICAL ANALYSIS

The quantities m_e and m_g defined by Soares *et al.* (2015) were defined in a steady state regime for an observer attached to the tip of the interface as shown in Fig. 1. Therefore, we will present the analysis with this approximation, i.e. we assume that the front of the displacing liquid achieves a constant shape and that the thickness of the displaced fluid remains constant along certain length L much longer than the size of the nose of the drop. In addition, the flow is assumed to be isothermal, inertialess and incompressible with equal mass densities.

2.1 Dimensionless governing equation

Choosing the radius of the tube, R_0 , as characteristic length and drop velocity U_b as characteristic velocity together with a characteristic stress, $\mu_2 U_b / R_0$, where μ_2 is the viscosity of the displaced Newtonian fluid. We can follow a

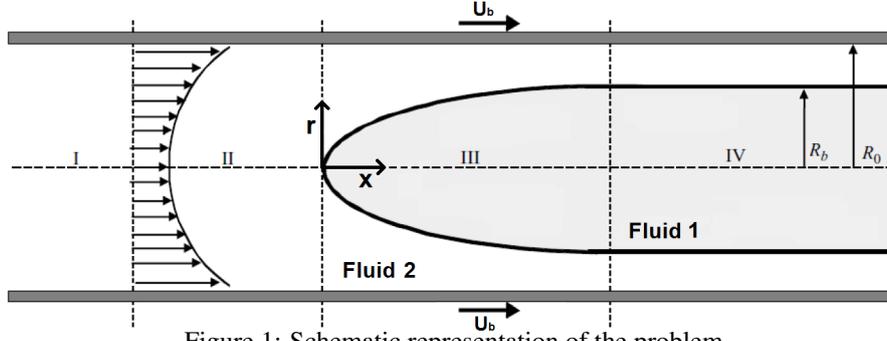


Figure 1: Schematic representation of the problem.

traditional procedure to deduce the dimensionless forms of the governing equation, i.e. continuity, momentum balance, interface impermeability, and interface balance. The conservation of mass equation is given by

$$\nabla^* \cdot \mathbf{u}_k^* = 0 \quad (1)$$

while the equation of conservation of momentum is given by

$$\nabla^* \cdot \mathbf{T}_k^* = -\nabla^* p_k^* + \nabla^* \cdot \mathbf{T}_k^{\mathbf{E}*} = 0 \quad (2)$$

where the subscript $k = 1, 2$ labels the two materials considered. In Eq. 1, \mathbf{u}_k^* is the dimensionless velocity vector, $\mathbf{u}_k = U_b \mathbf{u}_k^*$, and in Eq. 2, $\mathbf{T}_k^* = -p_k^* \mathbf{1} + \mathbf{T}_k^{\mathbf{E}*}$ is the total dimensionless stress vector, where p_k^* is the dimensionless pressure, $\mathbf{1}$ is the unit tensor, $\mathbf{T}_k^{\mathbf{E}*} = 2\eta_k^* \mathbf{D}_k^*$ is the viscous part of the stress tensor, and \mathbf{D}_k^* is the symmetric part of the velocity gradient. Since both fluids are Newtonians, $\eta_1^* = \mu_1/\mu_2$ and $\eta_1^* = 1$. Hence, Eq. 2 takes the form

$$-\nabla^* p_1^* + \frac{1}{N_\mu} \nabla^* \cdot \mathbf{D}_1^* = 0 \quad (3)$$

in the displacing fluid domain, and

$$-\nabla^* p_2^* + \nabla^* \cdot \mathbf{D}_2^* = 0 \quad (4)$$

in the displaced fluid domain. The dimensionless quantities labeled with a superscript $*$ are given by

$$\nabla^* \equiv R_0 \nabla; \quad p_k^* \equiv \frac{p_k R_0}{\mu_2 U_b}; \quad \eta_k^* \equiv \frac{\mu_k}{\mu_2}; \quad \mathbf{D}_k^* \equiv \frac{2R_0}{U_b} \mathbf{D}_k \quad (5)$$

in Eq. 3, we see the presence of the viscosity ratio, N_μ , given by

$$N_\mu = \frac{\mu_2}{\mu_1} \quad (6)$$

At the liquid-liquid interface there is no mass flow across the interface. This condition can be translated by

$$\mathbf{u}_1^* = \mathbf{u}_2^* = (\mathbf{u}_k^* \cdot \mathbf{t}) \mathbf{t} \quad (7)$$

where \mathbf{t} is the tangent vector. Furthermore, at the interface the traction balances the capillary pressure,

$$\mathbf{n} (p_1^* - p_2^*) + \mathbf{n} \left(\mathbf{D}_2^* - \frac{1}{N_\mu} \mathbf{D}_1^* \right) = \frac{1}{Ca} \frac{1}{R_m^*} \mathbf{n} \quad (8)$$

where p_1^* and p_2^* are the dimensionless pressures on fluids 1 and 2. The capillary number is given by

$$Ca = \frac{\mu_2 U_b}{\sigma} \quad (9)$$

where σ is the liquid-liquid interfacial tension and $R_m^* = R_m/R_0$ is the dimensionless radius of the curvature. Equation 8 carries the dimensionless number that govern the problem, namely Ca and N_μ .

2.2 Geometrical residual mass fraction and displacement efficiency

The *geometrical residual mass fraction*, m_g , is defined as the mass fraction of Fluid 2 that remains attached to the wall after the bubble of Fluid 1 has passed through the tube. Considering Fig. 1, it can be calculated by considering a cylinder of length Δx and radius R_0 . Before the displacing fluid invades the cylinder, the mass of Fluid 2 is $\rho_2 \pi R_0^2 \Delta x$. After the Fluid 1 passes through, the mass of Fluid 2 inside the cylinder is $\rho_2 \pi (R_0^2 - R_b^2) \Delta x$. So, m_g is given by

$$m_g = \frac{\rho_2 \pi (R_0^2 - R_b^2) \Delta x}{\rho_2 \pi R_0^2 \Delta x} = 1 - \frac{R_b^2}{R_0^2} \quad (10)$$

The *mass displacement efficiency*, m_e , is defined as the complement of the mass fraction of displaced fluid that leaves the tube while the displacing fluid is injected. By definition m_e is given by

$$m_e = \frac{\text{lost mass}}{\text{total mass}} = 1 - \frac{\text{recovered mass}}{\text{total mass}} \quad (11)$$

3. EXPERIMENTS

3.1 Experimental setup

The liquid-liquid displacement experiments were conducted in the experimental setup depicted in Fig. 2. The elements that composed the setup were a syringe pump (2) controlled by a computer (1); a glass capillary tube (6) with a length $L = 1.5m$ and internal diameter of $D = 2mm$ supported by hooks attached to three metallic bars (3); an acrylic box (5) filled with glycerine; a photographic camera (4); a ruler (7); a precision scale (8); a glass beaker (9); a cylindrical tank (11) filled with the displaced fluid connected to an air compressor supply controlled by a valve (10).

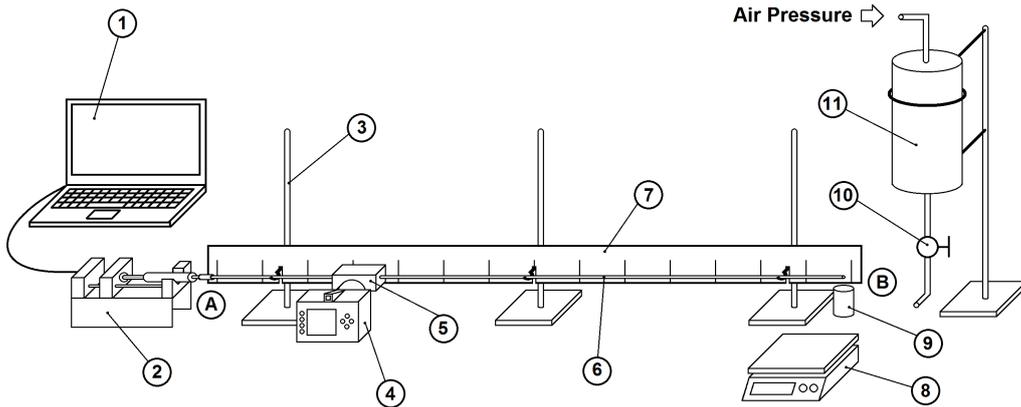


Figure 2: Scheme of the experimental setup.

3.2 Fluids employed

The different displacing fluids were obtained from solutions of water, polyethylene-glycol and ethanol. The polyethylene-glycol and the ethanol were used to control, respectively, the level of viscosity and the mass density of the displacing fluid. As displaced fluids it was used solutions from castor oil and soybean oil to control the level of viscosity. The properties of the solution are presented in Table 1 and the combination of displaced and displacing fluids along with the values of interfacial tension (σ), N_μ and density ratio ($N_\rho = \rho_2/\rho_1$) calculated with respect to the displaced fluid, are in Table 2.

Table 1: Solutions used: composition, viscosity and mass density.

Sample	Composition	ρ [kg/m]	μ [mPa.s]
S _{1A}	Water 41.7% w/w + Ethanol 16.6% w/w + PEG 41.7% w/w	1035.8	103.3
S _{1B}	Water 69% w/w + Ethanol 31% w/w	971.5	2.4
S _{1C}	Water 21% w/w + Ethanol 51% w/w + PEG 28% w/w	944.8	30
S _{2A}	Castor Oil 100% w/w	966.5	961.5
S _{2B}	Soybean Oil 100% w/w	924.3	61
S _{2C}	Castor Oil 73.7% w/w + Soybean Oil 26.3% w/w	955.7	410.4

Table 2: Arrangements: displacing and displaced fluids, interfacial tension (σ), viscosity ratio (N_μ) and density ratio (N_ρ).

Displacing Fluid	Displaced Fluid	$\sigma [mN/m]$	N_μ	N_ρ
S _{1B}	S _{2A}	16.41	400	0.995
S _{1A}	S _{2C}	6.75	4	0.923
S _{1C}	S _{2B}	5.85	2	0.978

3.2.1 Experimental procedure

The end B of the capillary tube is connected with the displaced fluid tank and is filled with oil. The connection on end B is removed and the syringe filled with displacing fluid is connected to end A. The acrylic box is filled with glycerin and the camera is focused on the tube. The pump is set on the computer and the command to start the displacement is given. The moment the displacing drop reaches the 25 cm marking on the ruler, the beaker is placed on the end B of the tube, collecting the mass of the displaced fluid that leaves the tube from that moment on. When the drop reaches the 125 cm marking on the ruler, the beaker is removed from end B and the command to stop the pump is given. The recovered mass of displaced fluid is then weighed. Images of the displacement are captured during the test.

4. RESULTS AND DISCUSSIONS

As described in Subsection 2.2, m_g is measured directly from the captured images using the geometric relation given by Eq. 10, while m_e is measured directly from weighing the recovered mass. Figure 3 shows the experimental results obtained in order to validate the experimental setup. The gas-liquid displacement problem explored by Taylor (1961) was reproduced in the present work for $N_\mu = 400$. It is shown that both curves are equivalent, which validates the experimental setup.

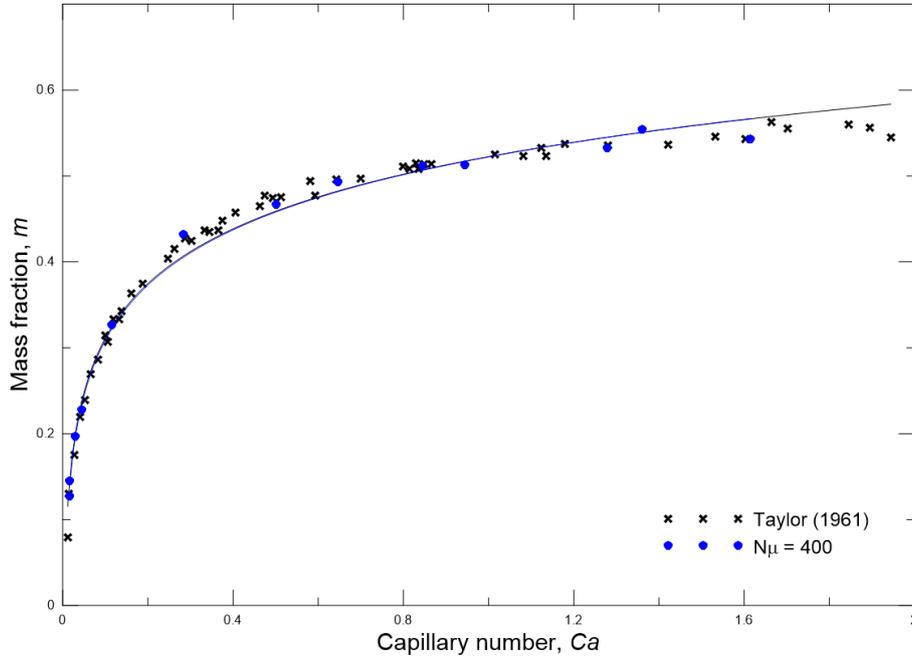


Figure 3: Comparison of the results obtained by Taylor (1961) and the results obtained in the present work.

Experimental results for the residual mass fraction and displacement efficiency are shown in Fig. 4. The experimental test were carried out for a wide range of capillary numbers at fixed viscosity ratios, $N_\mu = 0.5; 1; 2; 4$. The gas-liquid displacement is represented by the results of Taylor (1961). Each point obtained corresponds to a different experimental test.

Based on Fig. 4, it was noticed that decreasing the viscosity ratio induces m_g to increase, while m_e decreases. In other words, fixing the displaced fluid, the optimization of a displacement process leads to opposite decisions if one wants to minimize the layer of residual displaced liquid or if one wants to maximize the amount of recovered original liquid. For all values of N_μ , when the importance of the interfacial forces falls (large Ca values), m_g and m_e tend to an asymptotic value. The two forms of mass fraction depart from the gas-liquid data for $N_\mu = 4$ (circles). A more pronounced difference occurs when the viscosity ratio is decreased to $N_\mu = 2$.

Images of the interfaces were captured during the tests. The results presented in Fig. 4 regarding the geometric mass fraction can be visualized in Fig. 5.

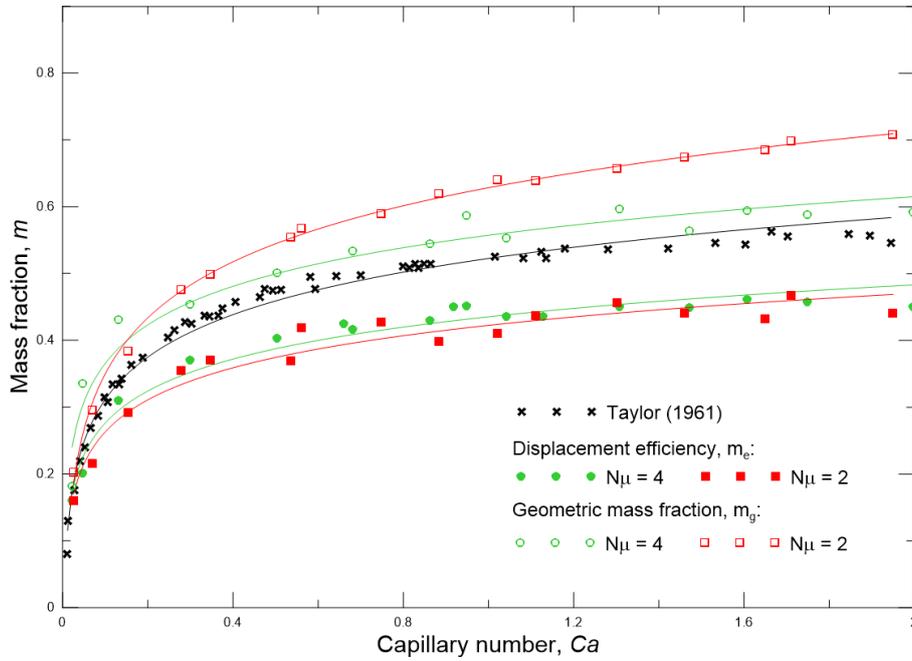


Figure 4: Mass fraction as a function of capillary number and viscosity ratios.

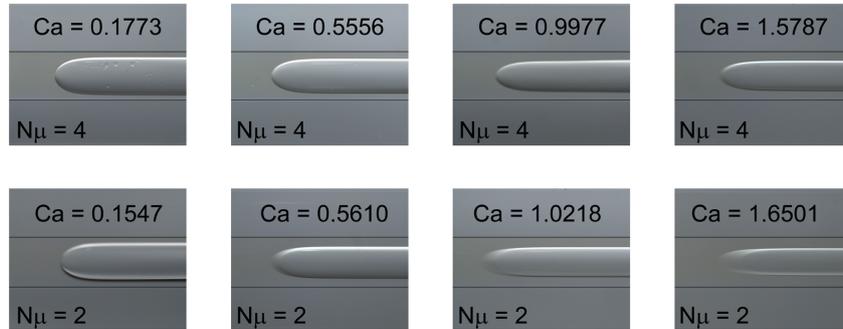


Figure 5: Images of the front of the interface as a function of capillary number and viscosity ratios.

5. FINAL REMARKS

We experimentally analyzed the liquid-liquid displacement of two immiscible fluids in a capillary tube. The problem was studied for a wide range of Ca and the quantities of interest were computed in term of the mass fraction attached to the wall (m_g) and the fraction of mass not recovered when the displacing fluid crosses the tube section analyzed (m_e). Taylor's data were recovered using a pair of fluid with $N_\mu = 400$.

The main result obtained in the present paper is the fact that the when the viscosity ratio decreases, while the fraction of mass not recovered decreases, the residual mass fraction increases. In a practical sense, if one wants to remove a large amount of liquid per unit of time, one has to choose a more viscous liquid as the displacing one. However, the thickness of the displaced fluid that remains attached to the wall is larger using this strategy. Images of the interfaces were used to visually express the results regarding the geometric mass fraction.

6. ACKNOWLEDGEMENTS

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