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Resolvent analysis of coherent structures in the atmospheric boundary layer considering Coriolis effects

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Abstract. *The identification and characterization of large-scale coherent structures of turbulent flows is a problem that demands extensive field measurements or sophisticated numerical simulations, especially when it is referred to highly turbulent and variable flows like the atmospheric boundary layer (ABL). In this study, we explore the efficacy of Resolvent Analysis as a cost-effective means to model ABL coherent structures, leveraging only the mean flow field of the ABL. The Resolvent analysis uses the singular value decomposition of the linearized Navier-Stokes operator, treating nonlinear terms as external forcing. This decomposition yields response modes directly correlated with the most energetic coherent structures of the flow. To validate this approach, we compare its outcomes with modes obtained through Spectral Proper Orthogonal Decomposition (SPOD) applied to detailed numerical data from Large-Eddy Simulations (LES) of a theoretical channel, serving as an initial approximation of the ABL. The influence of the Coriolis force in the fluid flow is analyzed and results where resolvent analysis exhibits the closest agreement with SPOD demonstrate its potential as a cost-effective tool for characterizing ABL coherent structures.*

Keywords: *Turbulent flows, Atmospheric boundary-layer, Large-Eddy Simulations, Resolvent analysis, Proper orthogonal decomposition*

1. INTRODUCTION

The atmospheric boundary layer (ABL) is the lowest layer of the Earth's atmosphere, where the interaction between the Earth's surface and the atmosphere occurs. It is characterized by significant changes in wind properties, temperature, humidity, and other atmospheric variables. Within this layer, air movement is dominated by turbulent processes, resulting in an efficient mixing of heat, moisture, and momentum between the Earth's surface and the free atmosphere above it (Stull, 1988; Garratt, 1992).

Many phenomena occurring within the ABL are driven by turbulence, including vertical mixing of gases and particles, formation of low clouds and temperature variation with height. These phenomena have important implications in areas such as meteorology, air quality, pollutant dispersion, wind power generation, among others (Stull, 1988; Wyngaard, 2010), therefore the characterization of ABL turbulence is crucial for many applications. In addition to turbulence, an important effect in the study of ABL is the influence of the Coriolis force. The Coriolis force is a pseudo force caused

by the Earth's rotation and causes objects moving on its surface to be deflected from their trajectory. This phenomenon is fundamental in meteorology, influencing winds and ocean currents (Lutgens and Tarbuck, 1998). The intensity of the Coriolis force increases with the object's speed and latitude, being maximum at the poles and null at the equator (Wallace and Hobbs, 2006; Holton, 2004).

An important characteristic of turbulent flows is the presence of coherent structures (Brown and Roshko, 1971; Gupta *et al.*, 1971; Kline *et al.*, 1967), which are large, persistent vortices that play a crucial role in energy transfer, mass and momentum transport, as well as in the determination of flow patterns and statistical properties. The study of these structures is done through advanced analysis methods, such as the Spectral Proper Orthogonal Decomposition (SPOD), applied to detailed datasets of turbulent flows, allowing to decompose the velocity fluctuations into different modal components. Due to the high complexity and cost of them, however, techniques such as the Resolvent Analysis have been recently investigated, which are based only on the mean flow conditions and the governing equations of the flow.

The Resolvent method is a technique based on the theory of linear stability, which involves analyzing the evolution of perturbations in a linearized base flow over time. This approach decomposes the turbulent velocity field into coherent modes called resolvent modes (Jovanovic and Bamieh, 2005; Bagheri *et al.*, 2009; McKeon and Sharma, 2010). These resolvent modes are the optimal solutions of the linearized system and highlight the dominant coherent structures in the flow, along with the energy associated with each mode. The method involves analyzing the response of the linearized system to a specific forcing source. While the Resolvent method focuses on the response of the linearized system to a specific forcing, Spectral Proper Decomposition (SPOD) is a spectral analysis technique that decomposes velocity fluctuations into different frequencies and wavenumbers (Towne *et al.*, 2018). SPOD uses the Fourier transform to identify coherent modes that contribute significantly to energy at various spatial and temporal scales.

When working with turbulent flow simulation, the use of the numerical solution of the Navier-Stokes equations, called Direct Numerical Simulation (DNS), tends to be very expensive in computational terms due to the increase in the variety of scales with the Reynolds number. (which indicates the intensity of the turbulence). An alternative is to use the Large-Eddy Simulation (LES) technique to solve the filtered Navier-Stokes equations, where only the large scales of the flow are solved directly, and the effects of the smaller scales are modeled by sub-grid scale models. This method allows the use of a coarser grid compared to DNS, significantly reducing the computational cost (Pope, 2000). Furthermore, LES can capture the main characteristics of coherent structures, including the impact of different forcing terms such as Coriolis and buoyancy forces in their shapes and magnitudes (Freire, 2022).

In this work, we perform a comparison between the methods Resolvent analysis and SPOD in capturing the coherent structures present in a theoretical half-channel, as a first approximation of the ABL, with the numerical database given by the LES. Furthermore, we show the influence of Coriolis force in the simulations.

2. MATHEMATICAL FORMULATION

In this work, the fluid flow is considered Newtonian, tri-dimensional, and incompressible. The governing equations are the continuity and Navier-Stokes equations with the presence of the Coriolis forcing term. The non-dimensional conservation equations governing the mean flow are

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[\nu_T \left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right) \right] - N_\tau \epsilon_{ijk} \frac{\Omega_j}{|\Omega|} u_k \quad (2)$$

where $-N_\tau \epsilon_{ijk} \frac{\Omega_j}{|\Omega|} u_k$ corresponds to the Coriolis force, with $N_\tau = 2|\Omega|/u_\tau$, where Ω is the angular speed of rotation frame, and u_τ is the friction velocity. ν_T is the total effective viscosity ($\nu_T = \nu_i + \nu$, with ν being the molecular viscosity). Here, we chose to use the model proposed by Cess (1958)

$$\frac{\nu_T}{\nu} = \frac{1}{2} \left(1 + \frac{\kappa^2 Re_\tau^2}{9} (1 - \eta^2)^2 (1 + 2\eta^2)^2 [1 - \exp(-|\eta| - 1) Re_\tau/A]^2 \right) + \frac{1}{2}, \quad (3)$$

where η is the non-dimensional wall distance in outer units, and the constants κ and A are given as 0.426 and 25.4, respectively (Pujals *et al.* (2009)).

2.1 Resolvent Analysis

The resolvent analysis has the objective of identifying the optimum modes that describe mechanisms of linear amplification in stable systems. When it is applied to a flow under some time-periodic forcing, this analysis can obtain information about the relevant structures and the nonlinear terms that excite them (Abreu *et al.*, 2020).

For a turbulent flow, Navier-Stokes equations can be rewritten and split into linear and non-linear terms that can be treated as forcing terms, the focus of resolvent analysis. Following the formulation presented in Cavalieri *et al.* (2019), the Navier-Stokes equations can be rearranged, leading to the linearized system in the input-output form:

$$\frac{\partial \mathbf{q}'}{\partial t} = \mathbf{L}_{\bar{\mathbf{q}}} \mathbf{q}' + \mathbf{f}, \quad (4)$$

where $\mathbf{q}' = [u', v', w']$ is the time-variant fluctuations ($\mathbf{q} = \bar{\mathbf{q}} + \mathbf{q}'$, with \mathbf{q} being the state vector of flow variables, and $\bar{\mathbf{q}}$ is the time-invariant base flow). $\mathbf{L}_{\bar{\mathbf{q}}}$ is the linearized Navier-Stokes operator about the base state $\bar{\mathbf{q}}$, and \mathbf{f} denotes the remaining non-linear terms, as well as any additional forcing terms added to the equations (Taira *et al.*, 2017; Towne *et al.*, 2018; McKeon and Sharma, 2010; House *et al.*, 2022).

The analysis of the unsteady flow is made in the frequency domain by analyzing the Fourier transform of Eq. (4) into

$$i\omega \hat{\mathbf{q}}_{\omega} = \mathbf{L}_{\bar{\mathbf{q}}} \hat{\mathbf{q}}_{\omega} + \hat{\mathbf{f}}_{\omega}, \quad (5)$$

where

$$\mathbf{q}(\mathbf{x}, t) = \hat{\mathbf{q}}_{\omega} e^{i\omega t} \quad \text{and} \quad \mathbf{f}(\mathbf{x}, t) = \hat{\mathbf{f}}_{\omega} e^{i\omega t}, \quad (6)$$

for temporal frequency ω . Thus, Eq. (5) can be written as

$$\hat{\mathbf{q}}_{\omega} = [i\omega \mathbf{I} - \mathbf{L}_{\bar{\mathbf{q}}}]^{-1} \hat{\mathbf{f}}_{\omega}, \quad (7)$$

where $\mathbf{A} = [i\omega \mathbf{I} - \mathbf{L}_{\bar{\mathbf{q}}}]^{-1}$ is the resolvent operator. \mathbf{A} relates a forcing input to a response of the state vector \mathbf{q} .

Given the resolvent operator \mathbf{A} , the singular value decomposition (SVD) technique is applied, given by

$$\mathbf{A} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^* \quad (8)$$

to obtain the corresponding eigenvalues and eigenvectors. Eigenvectors associated with the largest eigenvalues correspond to the main modes that represent the coherent structures of the flow, more sensitive to external forcing and perturbations.

Equation (8) gives a relationship between inputs and outputs. The superscript $*$ denotes the conjugate transpose of a matrix. \mathbf{V} represents the primary directions in which forcings are most effective, \mathbf{U} represents the responses these forcings will induce, and $\mathbf{\Sigma}$ represents the associated gains mapping between the forcing modes and the associated responses (House *et al.*, 2022).

2.2 Spectral Proper Orthogonal Decomposition

The Spectral Proper Orthogonal Decomposition (SPOD) is a statistical technique utilized for analyzing coherent structures within turbulent flows in the frequency domain. This method, after a data pre-processing step, involves applying a Fast Fourier Transform (FFT) to velocity fluctuations \mathbf{q}' in the homogeneous spatial directions (x and z) and time, which will enable the extraction of orthogonal modes that effectively represent turbulent kinetic energy at different frequencies. This transformation facilitates the assessment of energy distribution at distinct frequencies, denoted by ω . SPOD method is applied to this transformed field, which is equivalent to solving the integral equation

$$\int \mathbf{C}(\mathbf{x}, \mathbf{x}', \omega) \mathbf{\Psi}(\mathbf{x}', \omega) d\mathbf{x}' = \lambda \mathbf{\Psi}(\mathbf{x}, \omega), \quad (9)$$

where $\mathbf{\Psi}$ are the basis functions, also called SPOD modes, λ is the corresponding eigenvalue and \mathbf{C} is the two-point cross-spectral density. \mathbf{C} is a Hermitian matrix, and thus its eigenvalues are real and the eigenfunctions are orthogonal.

2.3 Large-Eddy Simulation

In this study, the LES code known as LESGO is used, which solves the half-channel flow forced by a constant mean pressure gradient in a fixed Cartesian grid (staggered in the vertical direction). The flow is periodic in both horizontal directions, where a spectral method is used to calculate the spatial derivatives. In the vertical direction, a second-order finite-difference method is used, and a wall model based on the logarithmic law of the wall is imposed as a bottom boundary condition. At the top of the domain, a stress-free condition is imposed, and the Adams-Bashforth method is used for time discretization. The Lagrangian-averaged scale-dependent subgrid-scale model is used (Bou-Zeid *et al.*, 2005), an improved version of the classical dynamic Smagorinsky model. This code has been used in many studies in the past years, in particular in the simulation of the ABL. Furthermore, the same code is used to simulate the ABL forced by a mean pressure gradient imposed in terms of a geostrophic wind, in the presence of the Coriolis and buoyancy forces that mimic the main drivers of the atmospheric turbulence (Kleissl *et al.*, 2006; Freire, 2022).

One simulation was made, corresponding to $Re_{\tau} = 5200$. Simulation parameters are detailed in Tab. 1. The results are presented in the next section.

Table 1. Simulation parameters for LES ($\delta = 1$ and $u^* = 1$).

domain size ($X \times Y \times Z$)	$2\pi\delta \times \delta \times 2\pi\delta$
number of grid points ($n_x \times n_y \times n_z$)	$128 \times 128 \times 128$
mean pressure gradient force ($F_i = \langle (1/\rho)(d\bar{p}/dx), 0, 0 \rangle$)	$\langle u^{*2}/\delta, 0, 0 \rangle$
simulation time step (Δt)	$0.0001\delta/u^*$
number of simulation time steps (N_t)	200 000

3. NUMERICAL RESULTS AND DISCUSSION

For a forced linear system with white noise, SPOD and Resolvent modes must be identical, which makes the comparison between these modes pertinent. Fig. 1 shows the mean velocity profiles of the flow (used in the Resolvent Analysis) in addition to the energy spectrum considering the z -direction, which relates the energy of the coherent structures to their wavelength. The snapshots of the instantaneous flow field in x - and z -direction are shown in Fig. 2, where it can be seen the presence of typical coherent structures in the z -direction caused by the effect of Coriolis force. Here, we adopted a Coriolis force of magnitude 0.091. Based on the Fig. 1b) and Fig. 2, their representative wavelengths were chosen as $(\lambda_x, \lambda_z) \approx (0.7, 3.16)$. The temporal wavelength $\lambda_t = 0.2$ was chosen with the objective to obtain a phase velocity $c = \omega/\alpha$ compatible with the mean velocity profile of the flow, in order to capture its coherent structures. α represents the x wavenumber direction. For $\lambda_t = 0.2$, we obtain a phase velocity $c = 3.296$, and frequency $\omega = 29.45$.

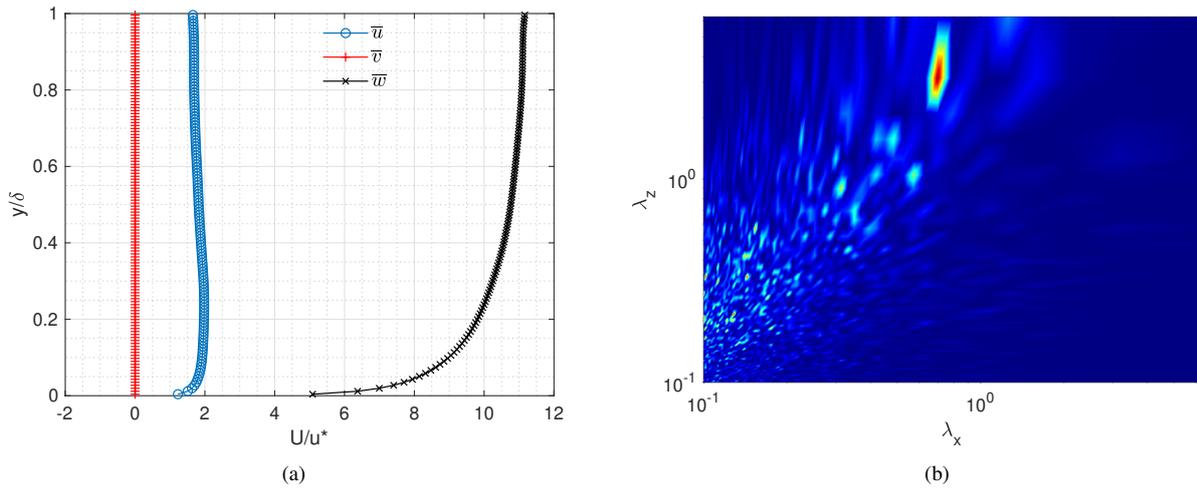


Figure 1. a) The mean streamwise velocity profile, and b) premultiplied streamwise energy spectra $\alpha\beta E_{ww}$ in $y/\delta = 0.0352$ plane.

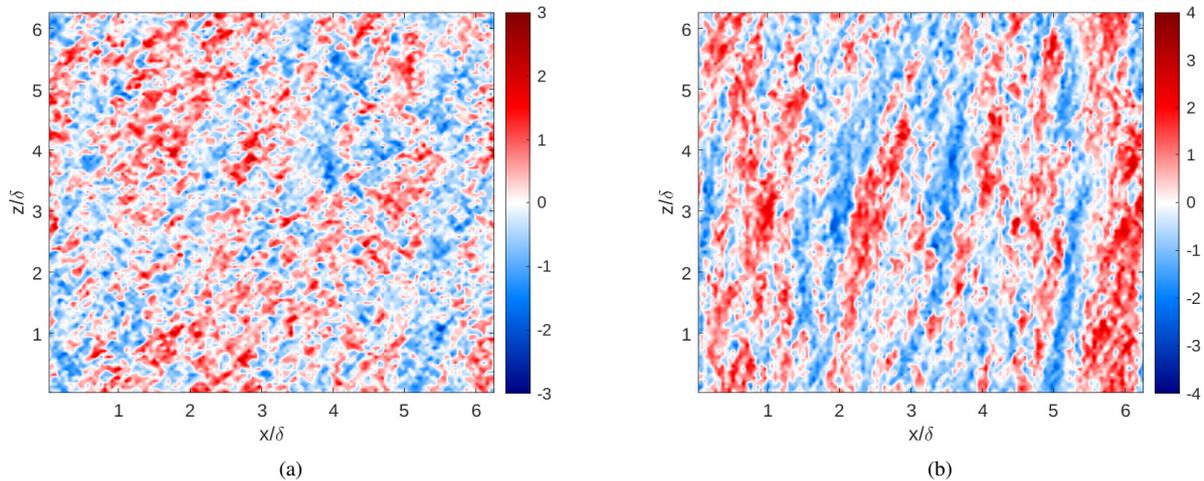


Figure 2. Instantaneous velocity fluctuations a) u' and b) w' field in the wall-parallel plane.

Figure 3 shows the comparisons between the first two modes obtained from simulations of SPOD and Resolvent methods. The results obtained from the Resolvent and SPOD methods for the first mode demonstrate great agreement. Additionally, they exhibit peaks concentrated in the first points of y/δ . However, for the first mode, there is a noticeable difference in the velocity component v , particularly for $0.05 < y/\delta < 0.4$, where the magnitude of the results between the Resolvent and SPOD cases differs. Nevertheless, it is notable that both exhibit similar behavior, with a peak concentrated at the initial y/δ points followed by a decay. Regarding the second mode, it is apparent that despite the initial similarity in the domain of the w component, the results generally display differing behaviors. We suspect that this effect is due to the resolution or parameters chosen for the resolvent analysis, which requires further investigation.

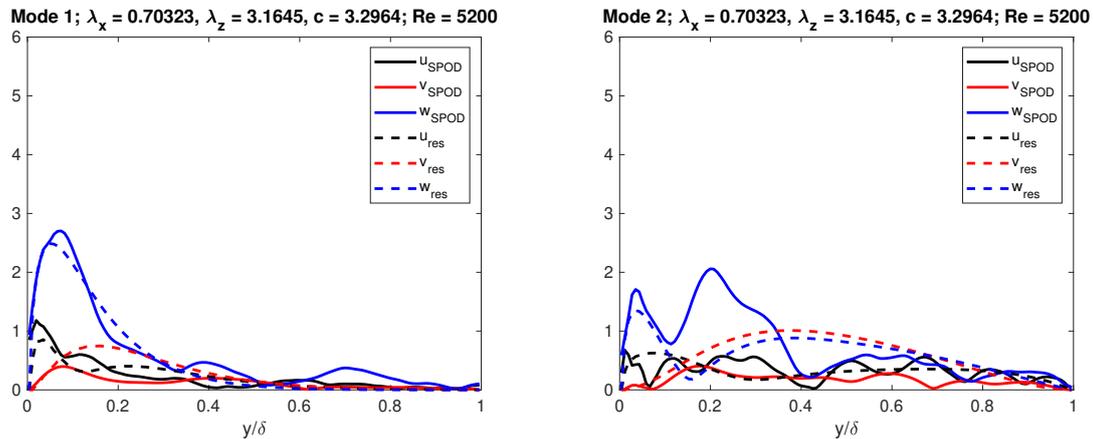


Figure 3. Comparison between the first two modes of SPOD and Resolvent methods of u' , v' , and w' , for $(\lambda_x, \lambda_z, \lambda_t) = (0.7, 3.16, 0.2)$, and $Re_\tau = 5200$.

4. CONCLUSION

The present work shows a comparison between the Resolvent analysis and SPOD methods in capturing the coherent structures present in a turbulent half-channel with $Re_\tau = 5200$ under the influence of Coriolis force with the numerical database given by the LES. The adopted methods are responsible for identifying the coherent modes of the turbulent velocity field, obtaining the optimal solutions of the linearized system, and thus valuable information about the dominant coherent structures.

The influence of Coriolis force in the fluid flow is analyzed, by comparing the similarity of results obtained using Resolvent and SPOD methods, for fixed wavelengths $(\lambda_x, \lambda_z, \lambda_t)$. We observed that, for the first mode, the Resolvent and SPOD methods presented great agreement, indicating that Resolvent analysis can be used to observe energetic structures in this kind of flow.

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