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# STUDY OF NON-NEWTONIAN FLUID FLOW STABILITY MODELED BY LPTT

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***Abstract.*** *The influence of elasticity on the inertial flows of viscoelastic fluids is not yet completely understood, with research underway to better understand the mechanisms involved. This knowledge is crucial for practical applications in industrial processes and materials development. This study investigates the laminar-turbulent transition, focusing on Tollmien-Schlichting wave convection in a two-dimensional incompressible Poiseuille flow for a viscoelastic fluid, using the linear Phan-Thien Tanner (PTT) constitutive equations. The Navier-Stokes equations and the LPTT constitutive equations are solved with high-order compact finite difference methods. Numerical simulations will be carried out varying the dimensionless parameters to compare the distribution of extra stress tensors in flows between parallel fluid plates modeled by LPTT.*

***Keywords:*** *Laminar-Turbulent Transition, Viscoelastic Fluids, PTT Model, Flow Stability, Linear Stability Theory.*

## 1. INTRODUCTION

Fluids are substances that flow and deform continuously under shear stress. The study of non-Newtonian fluid flows is essential in several industries, including chemical, food, petroleum and biomedical engineering. Understanding the behavior of these fluids is crucial for optimizing industrial processes and developing new materials.

Advances in computational technology have made numerical simulations a valuable tool for studying viscoelastic fluid flows in industrial applications. They are an economical alternative to laboratory experiments, offering detailed and accurate results that reflect the real behavior of these fluids.

Modeling viscoelastic flows is complex due to the nonlinear nature of the rheological properties of fluids. Advanced experimental approaches and computer simulation techniques are needed to predict the behavior of these fluids under different flow conditions and specific geometries. Continuous research in this area is vital for industrial progress and the development of more efficient and durable products. Investigations into new polymer formulations, improved processing techniques, and flow control strategies are essential to improving the treatment of viscoelastic fluid flows.

Despite the challenges in numerical simulation of viscoelastic fluid flows due to the complexity of the constitutive equations, this approach provides accurate results and is a cost-efficient alternative compared to laboratory experiments. The development of efficient numerical methods and the appropriate choice of constitutive models are active areas of research, with the aim of improving the understanding and management of viscoelastic fluid flows in industrial applications.

Several studies on constitutive models of viscoelastic fluids address both linear and nonlinear models. The most popular linear viscoelastic model is the Maxwell model Beris *et al.* (1987), Mompean and Deville (1997), which combines characteristics of an elastic solid and a viscous Newtonian fluid. Among the main nonlinear viscoelastic models found in the literature are the differential models: Oldroyd-B Brasseur *et al.* (1998), Alves *et al.* (2003), White-Metzner White and Metzner (1963), Giesekus Giesekus (1982), Leonov Leonov (1976), FENE type models Bird *et al.* (1980), Bird and DeAguiar (1983), Christainsen and Bird (1977), Stevenson and Bird (1971), Warner Jr (1972), Oliveira (2002), PTT Thien and Tanner (1977), Alves *et al.* (2003) and derivatives, PomPom Luo and Tanner (1988) and derivatives; and the integral models: Maxwell Kaye (1962) and K-BKZ Luo and Tanner (1986), Luo and Tanner (1988).

Elastic instabilities have been widely studied in recent years through experimental and theoretical work using linear stability analysis(Larson *et al.* (1990), Shaqfeh *et al.* (1992), Larson (1992)).Experimental studies generally involve physical tests on prototype structures, applying different loads and observing behavior and instabilities. Theoretical studies use mathematical modeling and analytical or numerical methods to predict behaviors and identify conditions of instability. Furthermore, several studies on flows in viscoelastic fluid channels are found in the literature. Poiseuille and Couette flows are generally used as benchmarks for new constitutive equations, with great interest in instabilities in these types of flows. (Avgousti and Beris (1993), Sureshkumar and Beris (1995), Mak (2009), Gervazoni (2016), Zhang *et al.* (2013), Souza *et al.* (2012), Silva (2018)). Poiseuille flow in viscoelastic fluids, including the LPTT model, remains an interesting and challenging research problem, with many stability questions still open, requiring further investigation. Experimental and theoretical studies specific to these fluids are essential to advance our understanding.

Direct Numerical Simulation (DNS) is a computational technique that solves the Navier-Stokes equations at all spatial and temporal scales of a flow. The main advantage of DNS is the ability to study in detail the processes that lead to turbulence, directly modeling the nonlinear interactions responsible for it, without simplifications or statistical modeling used in other approaches. Thus, DNS is especially useful for gaining an in-depth understanding of the specifications and characteristics of a turbulent flow.

Linear Stability Theory (LST) provides a framework for understanding the growth rate of instabilities in relation to the frequency of a base flow. Based on the continuity and Navier-Stokes equations, LST considers some hypotheses about the flow and how disturbances propagate Lacerda *et al.* (2018). It predicts the conditions under which instabilities form and provides information on spatial and temporal scales, identifying the type of instability that occurs. Stability analysis is directly linked to the transition to turbulence, providing information about which conditions make a flow unstable and prone to becoming turbulent.

The importance of the proposed project is the development of a diagram of neutral stability curves for viscoelastic flows using the Linear Stability Theory. The contribution of scientific research is innovative, as it aims to add knowledge and promote the scientific and technological improvement of laminar-turbulent transition, the development of codes and the construction of efficient techniques for simulating incompressible flows of viscoelastic fluids. Thus, this work contributes to current results in the sense of providing an important tool for verifying the stability of two-dimensional flows using the LPTT fluid.

The main objective of this work is to study the transition characteristics to turbulence of the incompressible, two-dimensional Poiseuille flow of a viscoelastic fluid, of the LPTT type. The investigation of this discovery is carried out through the analysis of Tollmien-Schlichting wave convection for the considered flow, using Linear Stability Theory techniques, in order to analyze the flow stability of the viscoelastic fluid of the LPTT type.

## 2. METHODOLOGY

TThe mathematical modeling of incompressible and isothermal flows can be carried out using the conservation of mass (continuity) and conservation of momentum equations, presented below:

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\rho \left[ \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{uu}) \right] = \nabla \cdot \sigma, \tag{2}$$

where $\mathbf{u}$ is the velocity vector, $t$ is the time, $\rho$ is the fluid density and $\sigma$ is the total stress tensor, defined by:

$$\sigma = \tau - p\mathbf{I}, \tag{3}$$

where $p$ is the pressure, $\mathbf{I}$ is the identity tensor and $\tau$ is the symmetric stress tensor, which can be determined from the constitutive equation of the fluid considered. In the two-dimensional case, $\mathbf{u} = [u \;\; v]^T$ depicts the velocity components in the $x$ and $y$ directions, respectively, and the symmetric stress tensor is defined by: $\tau = \begin{bmatrix} \tau^{xx} & \tau^{xy} \\ \tau^{xy} & \tau^{yy} \end{bmatrix}$

For modeling Newtonian fluids, the symmetric stress tensor is linearly proportional to the strain rate tensor, which is given by:

$$\tau = 2\eta_s \mathbf{D}, \tag{4}$$

where $\eta_s$ is the dynamic viscosity of the fluid and $\mathbf{D}$ is the deformation tensor, given by:

$$\mathbf{D} = \frac{1}{2}(\nabla u + (\nabla u)^T). \tag{5}$$

For viscoelastic fluid modeling, the tensor can be defined as:

$$\tau = 2\eta_s \mathbf{D} + \mathbf{T}, \tag{6}$$

where $\mathbf{T}$ is the extra-stress tensor (symmetric) representing the non-Newtonian (polymeric) contribution, which is given by:

$$\mathbf{T} = \begin{bmatrix} T^{xx} & T^{xy} \\ T^{xy} & T^{yy} \end{bmatrix}. \tag{7}$$

### 2.1 PTT model

The constitutive equation for this model is written as follows:

$$f(tr(\mathbf{T}))\mathbf{T} + \lambda \overset{\nabla}{\mathbf{T}} = 2\eta_p \mathbf{D}, \tag{8}$$

where $D$ is the strain rate tensor, $\lambda$ is the fluid relaxation time, $\eta_p$ is the viscosity contributed by the polymer, and $f(tr(\mathbf{T}))$ is considered to be the stress coefficient function. The PTT model is considered in linear form, i.e. the function $f(tr(\mathbf{T}))$ can be given by:

$$f(tr(\mathbf{T})) = 1 + \frac{\lambda\epsilon}{\eta_p} tr(\mathbf{T}), \tag{9}$$

The notation $f(tr(\mathbf{T}))\mathbf{T}$ is used to represent the trace of the extra stress tensor $\mathbf{T}$ and the symbol $\overset{\nabla}{\mathbf{T}}$ represents the convected derivative given by:

$$\overset{\nabla}{\mathbf{T}} = \frac{D\mathbf{T}}{Dt} - \mathbf{T}\cdot\mathbf{L} - \mathbf{L}^T\cdot\mathbf{T}, \tag{10}$$

where the difference $\mathbf{L} = \nabla\mathbf{u} - \xi\mathbf{D}$ is called the effective velocity gradient, $\epsilon$ and $\xi$ are positive parameters of the model.

The above equations are written in dimensionless form by introducing the following dimensionless variables.

$$\mathbf{x}^* = \frac{\mathbf{x}}{L}, \;\; \mathbf{u}^* = \frac{\mathbf{u}}{U}, \;\; \mathbf{t}^* = \frac{tU}{L}, \;\; p^* = \frac{p}{\rho U^2}, \;\; \mathbf{T}^* = \frac{\mathbf{T}}{\rho U^2}, \tag{11}$$

By applying the changes of variables (11), the dimensionless numbers appear: Reynolds number $(Re)$, Weissenberg number $(Wi)$ and the constant $\beta$.
Reynolds number $(Re)$:

$$Re = \frac{\rho U L}{\eta_0}, \tag{12}$$

where $\eta_0$ is the total dynamic viscosity of the fluid, given by $\eta_0 = \eta_s + \eta_p$.
Weissenberg number$(Wi)$:

$$Wi = \frac{\lambda U}{L}, \tag{13}$$

Constant $\beta$: $\beta \in (0, 1)$

$$\beta = \frac{\eta_s}{\eta_0}. \tag{14}$$

### 2.2 Linear stability theory

The stability analysis of the flows in this work is carried out using Linear Stability Theory for viscoelastic fluid flows, together with the LPTT constitutive equation for the viscoelastic model. the instantaneous flow is decomposed into two parts, a base flow and a disturbance flow. The base flow is invariant in the direction of flow and the normal velocity component of the base flow v is zero. The disturbances must be infinitesimal, so that the non-linear terms can be neglected in comparison with the linear terms. The disturbance components are represented by $\tilde{u}, \tilde{v}, \tilde{p}, \tilde{T}$.

The decomposed flow variables are substituted into the non-Newtonian continuity, momentum conservation and extra-stress tensor equations. Then, by subtracting the equations describing the basic flow, the following disturbance equations are obtained

$$\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0, \tag{15}$$

$$\frac{\partial \tilde{u}}{\partial t} + U\frac{\partial \tilde{u}}{\partial x} + \tilde{v}\frac{\partial U}{\partial y} = -\frac{\partial \tilde{p}}{\partial x} + \frac{\beta}{Re}\left(\frac{\partial^2 \tilde{u}}{\partial x^2} + \frac{\partial^2 \tilde{u}}{\partial y^2}\right) + \frac{\partial \tilde{T}^{xx}}{\partial x} + \frac{\partial \tilde{T}^{xy}}{\partial y}, \tag{16}$$

$$\frac{\partial \tilde{v}}{\partial t} + U\frac{\partial \tilde{v}}{\partial x} = -\frac{\partial \tilde{p}}{\partial y} + \frac{\beta}{Re}\left(\frac{\partial^2 \tilde{v}}{\partial x^2} + \frac{\partial^2 \tilde{v}}{\partial y^2}\right) + \frac{\partial \tilde{T}^{xy}}{\partial x} + \frac{\partial \tilde{T}^{yy}}{\partial y}. \tag{17}$$

For the equation of the non-Newtonian tensor $T^{xx}$, it follows that:

$$f(tr(T)) = \left[1 + \underbrace{\epsilon\frac{ReWi}{(1-\beta)}}_{(C)\ constant}(T^{xx}+T^{yy})\right]T^{xx} = \left[1 + C(T^{xx}+\tilde{T}^{xx}) + (T^{yy}+\tilde{T}^{yy})\right](T^{xx}+\tilde{T}^{xx}) = \tilde{T}^{xx} + C\left(2T^{xx}\tilde{T}^{xx} + T^{xx}\tilde{T}^{yy} + T^{yy}\tilde{T}^{xx}\right). \tag{18}$$

$$f(tr(T))\tilde{T}^{xx} + Wi\left(\frac{\partial\tilde{T}^{xx}}{\partial t} + U\frac{\partial\tilde{T}^{xx}}{\partial x} + \tilde{v}\frac{\partial\mathbf{T}^{xx}}{\partial y} - 2\tilde{T}^{xy}\frac{\partial U}{\partial y} + \xi\tilde{T}^{xy}\frac{\partial U}{\partial y} - 2\mathbf{T}^{xy}\frac{\partial\tilde{u}}{\partial y} + \xi\mathbf{T}^{xy}\frac{\partial\tilde{u}}{\partial y} + T^{xx}\frac{\partial\tilde{v}}{\partial y} - T^{xx}\frac{\partial\tilde{u}}{\partial x} + 2\xi T^{xx}\frac{\partial\tilde{u}}{\partial x} + \xi T^{xy}\frac{\partial\tilde{v}}{\partial x}\right) = 2\frac{(1-\beta)}{Re}\frac{\partial\tilde{u}}{\partial x}, \tag{19}$$

For the equation of the non-Newtonian tensor $T^{xy}$, it follows that:

$$f(tr(T))\tilde{T}^{xy} + Wi\left(\frac{\partial\tilde{T}^{xy}}{\partial t} + U\frac{\partial\tilde{T}^{xy}}{\partial x} + \tilde{u}\frac{\partial\mathbf{T}^{xy}}{\partial x} + \tilde{v}\frac{\partial\mathbf{T}^{xy}}{\partial y} - \frac{\xi}{2}\tilde{T}^{xx}\frac{\partial U}{\partial y} + \frac{\xi}{2}\tilde{T}^{yy}\frac{\partial U}{\partial y} + \frac{\xi}{2}T^{xx}\frac{\partial\tilde{u}}{\partial y} - T^{yy}\frac{\partial\tilde{u}}{\partial y} + \frac{\xi}{2}T^{yy}\frac{\partial\tilde{u}}{\partial y} + \mathbf{T}^{yy}\frac{\partial\tilde{u}}{\partial y} + T^{xy}\frac{\partial\tilde{v}}{\partial y} + \tilde{T}^{xy}\frac{\partial U}{\partial x} - T^{xx}\frac{\partial\tilde{v}}{\partial x} + \frac{\xi}{2}T^{xx}\frac{\partial\tilde{v}}{\partial x} + \frac{\xi}{2}T^{yy}\frac{\partial\tilde{v}}{\partial x}\right) = \frac{(1-\beta)}{Re}\left[\frac{\partial\tilde{v}}{\partial x} + \frac{\partial\tilde{u}}{\partial y}\right], \tag{20}$$

For the equation of the non-Newtonian tensor $T^{yy}$, it follows that:

$$f(tr(T))\tilde{T}^{yy} + Wi\left(\frac{\partial\tilde{T}^{yy}}{\partial t} + U\frac{\partial\tilde{T}^{yy}}{\partial x} + \tilde{v}\frac{\partial\mathbf{T}^{yy}}{\partial y} + \xi\tilde{T}^{xy}\frac{\partial U}{\partial y} + \xi T^{xy}\frac{\partial\tilde{u}}{\partial y} - \mathbf{T}^{yy}\frac{\partial\tilde{v}}{\partial y} + 2\xi\mathbf{T}^{yy}\frac{\partial\tilde{v}}{\partial y} + T^{yy}\frac{\partial\tilde{u}}{\partial x} - 2T^{xy}\frac{\partial\tilde{v}}{\partial x} + \xi T^{xy}\frac{\partial\tilde{v}}{\partial x}\right) = 2\frac{(1-\beta)}{Re}\frac{\partial\tilde{u}}{\partial x}. \tag{21}$$

The solution of the normal modes is considered as follows

$$\tilde{u}(x,y,t) = \overline{u}(y)e^{i(\alpha x-\omega t)},$$

where $i=\sqrt{-1}$, $\alpha$ is the wave number in the $x$ direction and $u, v, p$ and $T$ are the amplitudes of the disturbances. Where $\omega$ is the frequency with which the disturbances, of wavelength $\lambda = 2\pi/\alpha$ and wave speed $c = \frac{\omega}{\alpha}$, propagate. We also consider $\overline{u}$, $\overline{v}$ and $\overline{p}$ as the amplitudes of the disturbances. Assuming that these equations make up a solution to the simplified system, the complex conjugates also make up a possible solution to the system in question. For a solution belonging to the set of real numbers, a linear combination of the solutions is taken as the solution. Substituting these linear combinations into the equations obtained for the perturbations, we get:
Continuity:

$$i\alpha\overline{u} + \overline{v}' = 0, \tag{22}$$

For the momentum equation in the $x$ direction, we get

$$Re(i\alpha U - i\omega)\overline{u} - \beta(\overline{u}'' - \alpha^2\overline{u}) + Re\overline{v}U' = i\alpha(\overline{T}^{xx} - \overline{p}) + \overline{T}'^{xy}. \tag{23}$$

For the momentum equation in the $y$ direction, we have:

$$Re(i\alpha U - i\omega)\overline{v} - \beta(\overline{v}'' - \alpha^2\overline{v}) = i\alpha\overline{T}^{xy} + \overline{T}'^{yy} - \overline{p}. \tag{24}$$

For the extra stress tensors $T^{xx}$, $T^{xy}$ and $T^{yy}$ components, we have

$$f(tr(T))\overline{T}^{xx} + Wi\left(-i\omega\overline{T}^{xx} + i\alpha U\overline{T}^{xx} + \overline{v}\mathbf{T}^{xx} - 2\overline{T}^{xy}\frac{\partial U}{\partial y} + \xi\overline{T}^{xy}\frac{\partial U}{\partial y} - 2\mathbf{T}^{xy}\frac{\partial\overline{u}}{\partial y} + \xi\mathbf{T}^{xy}\frac{\partial\overline{u}}{\partial y} + T^{xx}\frac{\partial\overline{v}}{\partial y} - i\alpha\overline{u}T^{xx} + 2i\alpha\xi\overline{u}T^{xx} + i\alpha\xi\overline{v}T^{xy}\right) = -\frac{2(1-\beta)}{Re}\overline{v}, \tag{25}$$

$$f(tr(T))\overline{T}^{xy} + Wi\Bigg(-i\omega\overline{T}^{xy} + i\alpha U\overline{T}^{xy} + \overline{v}\mathbf{T}^{xy} + \frac{\xi}{2}\overline{T}^{xx}\frac{\partial U}{\partial y} - \frac{\xi}{2}\overline{T}^{yy}\frac{\partial U}{\partial y} + \frac{\xi}{2}\overline{T}^{yy}\frac{\partial U}{\partial y} + \frac{\xi}{2}T^{xx}\frac{\partial \overline{u}}{\partial y} - \mathbf{T}^{yy}\frac{\partial \overline{u}}{\partial y} + \frac{\xi}{2}T^{yy}\frac{\partial \overline{u}}{\partial y} + \mathbf{T}^{xy}\frac{\partial \overline{v}}{\partial y} + i\alpha\overline{u}T^{xy} - i\alpha\overline{v}T^{xx} + i\alpha\frac{\xi}{2}\overline{v}T^{xx} + i\alpha\frac{\xi}{2}\overline{v}T^{yy}\Bigg) = \frac{(1-\beta)}{Re}\left(i\alpha\overline{v} + \frac{i}{\alpha}\overline{v}\right), \tag{26}$$

$$f(tr(T))\overline{T}^{yy} + Wi\Bigg(-i\omega\overline{T}^{yy} + i\alpha U\overline{T}^{yy} + \overline{v}\mathbf{T}^{yy} + \xi\overline{T}^{xy}\frac{\partial U}{\partial y} + \xi T^{xy}\frac{\partial \overline{u}}{\partial y} - \mathbf{T}^{yy}\frac{\partial \overline{v}}{\partial y} + 2\xi\mathbf{T}^{yy}\frac{\partial \overline{v}}{\partial y} + i\alpha T^{yy}\overline{u} - 2i\overline{v}\alpha T^{xy} + i\alpha\xi\overline{v}T^{xy}\Bigg) = \frac{2(1-\beta)}{Re}\overline{v}. \tag{27}$$

### 2.3 Base flow

An explicit solution was obtained using the numerical solution seen in equations (22)-(27) for the base flow for the LPTT fluid, seen in ARAUJO *et al.* (2022). The (22)-(27) equations for the LPTT model were used together with the Matlab/Octave tool for implementation.

## 3. NUMERICAL METHOD AND RESULTS

A 2D temporal LST code was used to analyze the temporal stability of channel flow for the LPTT viscoelasticity model. In the case of a temporal analysis, $\omega$ is used to analyze the stability of a disturbance given a real angular frequency, $\alpha$. By solving the eigenvalues of this system, a spectrum of eigenvalues is obtained, where the search is made for the most unstable eigenvalue, which corresponds to the stable or unstable data set depending on the value of the imaginary part of $\omega$. Initially, all spurious modes are eliminated, and then the search is made for the imaginary $\omega$ with the smallest value within the obtained spectrum.

### 3.1 Neutral stability curves of the LPTT model using LST

Checking the influence of the constant $\beta$:

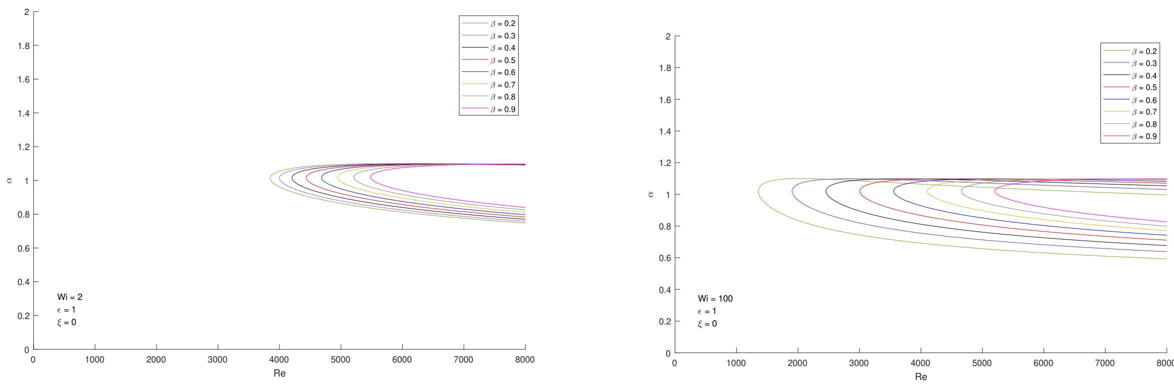


Figure 1. Temporal Neutral Stability Diagram.

In figure 1, it is observed that in the neutral stability curves of the LPTT model, as the value of $\beta$ increases, the value of $\alpha$ increases. It is also noted that as the value of $\beta$ decreases, the neutral stability curve shifts to the left, where it is notable that the flow becomes unstable for lower values of the Reynolds number. Increasing the value of $Wi$ the curves also move away to the left.

Checking the influence of the Weissenberg number (Wi):

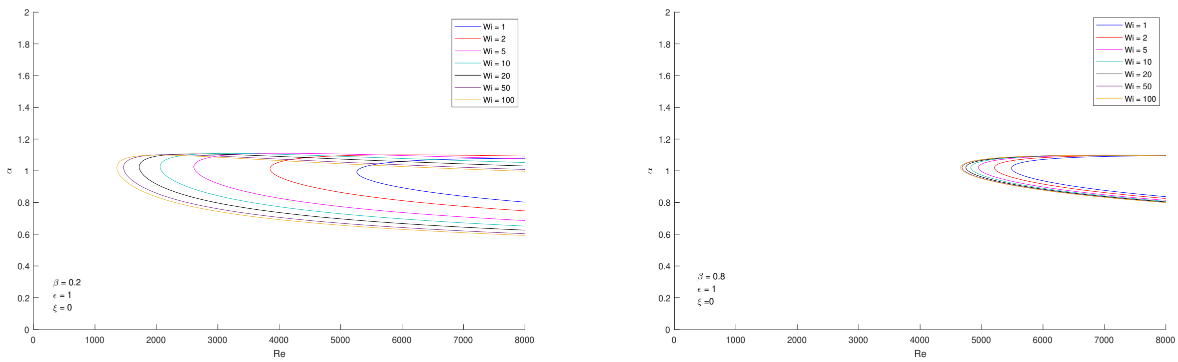


Figure 2. Temporal Neutral Stability Diagram.

In figure 2, in the neutral stability curves of the LPTT model as the value of $Wi$ increases, there is a shift of the neutral curve to the left, indicating that the flow becomes unstable for smaller values of the Reynolds number. Increasing the value of the constant $\beta$ to $0.8$ (closer to the Newtonian fluid), it can be seen that the neutral stability curves shift to the right, indicating that the flow becomes unstable for higher values of the Reynolds number.

Checking the influence of the constant $\epsilon$:

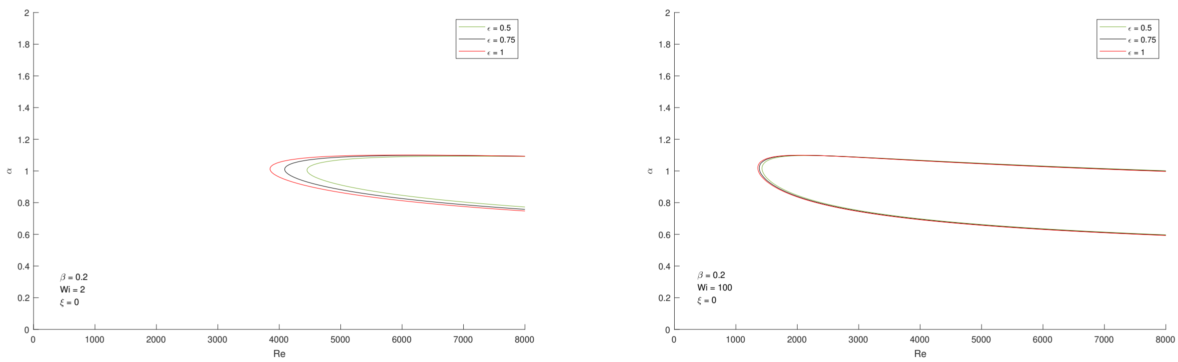


Figure 3. Temporal Neutral Stability Diagram.

Analyzing Figure 3, the neutral stability curves of the LPTT model as the value of $\epsilon$ increases, the neutral curve shifts to the left. This deviation is more noticeable in the Figure on the left, showing that the flow becomes unstable for lower values of the Reynolds number. In the Figure on the right it is observed that when the Weissenberg number $(Wi)$ increases, the neutral curve shifts to the left, but the spacing between the neutral curves is less pronounced.

Checking the influence of the constant $\xi$:

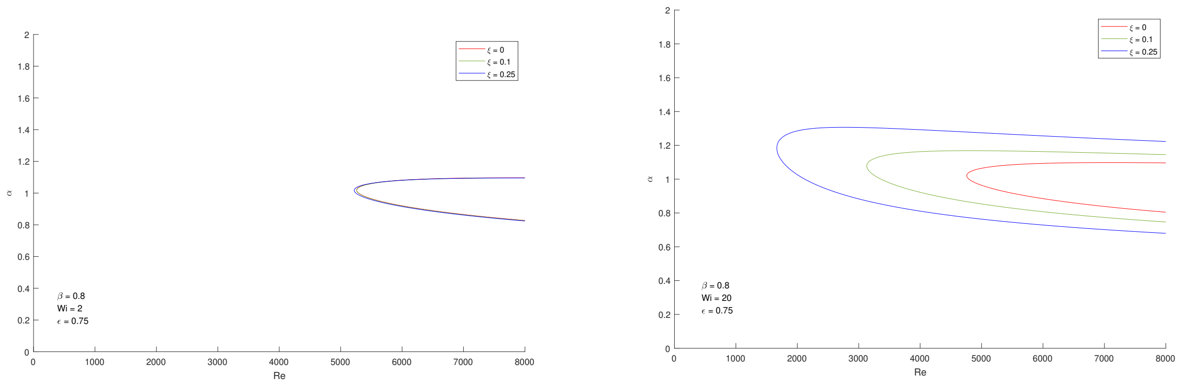


Figure 4. Temporal Neutral Stability Diagram.

It can be seen in Figure 4 that the neutral stability curves of the LPTT model as the value of $Wi$ and $\xi$ increases, there is a shift of the neutral curve to the right, indicating that the flow becomes unstable for lower values of the Reynolds number.

## 4. CONCLUSIONS

The two-dimensional, isothermal and incompressible flow equations for a non-Newtonian viscoelastic fluid were presented. The viscoelastic model adopted was the Linear Phan Thien Tanner (LPTT) model. Temporal analysis was used to investigate the stability of viscoelastic fluid flows using the Linear Stability Theory through neutral stability curves. The neutral stability curves were evaluated only through two-dimensional perturbations for different values of the model's dimensionless parameters. The numerical results obtained by the LST technique were satisfactory in analyzing the stability of viscoelastic flows.

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