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Topology Optimization for the Maximization of Frequency Separation Margin

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ABSTRACT

To avoid resonance problems in vibrating mechanical structures, it is desired to maximize the separation margin between natural frequencies in the operating frequency range. For this reason, optimization methods can be used to identify geometries that maximize the difference between two consecutive natural frequencies. In this work, a topology optimization method was used to maximize a given natural frequency gap and was tested on a two-dimensional linear elastic beam simply supported at each end. The algorithm implemented was BESO (Bidirectional Evolutionary Structural Optimization). As the evolutionary method progresses, some eigenvalues might approach the one being maximized, and the mode shift problem can emerge and disturb the convergence of the optimization algorithm. For this reason, a method based on multi-objective optimization is implemented and tested to control the mode shift problem. The multi-objective strategy used is based on the weighted sum method, where additional frequencies are added to the objective function. Since most studies show this problem often results in periodic topologies, a periodic constraint was implemented to test this behavior. Therefore, a strategy is proposed by combining these constraints and the method to control mode shift. Although this restraint did not produce topologies whose gap was larger, it allowed the removal of more elements, and thus, reducing environmental impact and its total weight.

Keywords: frequency gaps, periodic structures, natural frequencies, BESO

1. Introduction

Topology optimization method has become of great importance in engineering problems and is used when designing new products. One of its applications can be to control a structure's natural frequencies, in order to satisfy certain design restrictions. Thus, this method can be employed to separate two adjacent natural frequencies, to maximize the frequency range where vibration and propagation of elastic waves are inhibited. These resonance-free ranges are known as band gaps [1].

Ma et al [2] initially applied topology and shape optimization in order to modify a structure's natural frequencies. They presented a versatile objective function which can be used to control selected eigenvalues or to maximize the separation between two consecutive ones.

Xie and Steven [3] later analysed the optimization of one or multiple eigenvalues. For this, they applied, however, another algorithm, the Evolutionary Structural Optimization (ESO). This algorithm operates by removing elements at each iteration until the final prescribed volume is reached.

Yang et al [4] then approached this problem using the newly developed Bi-directional Evolutionary Structural Optimization (BESO), a bidirectional version of the ESO, which not only removes elements from the mesh, but can also add some at each iteration [5].

Huang et al [6] also addressed the optimization of a single eigenvalue but implemented a Soft-Kill method, by not completely removing void elements, but penalizing them. This makes the evolutionary process more stable, however, the presence of these void elements can cause local modes to appear. To avoid this problem, Huang et al proposed using an alternative interpolation scheme, as recommended by Pedersen [7].

Implementation of the Soft-Kill BESO method has since increased for optimizing natural frequencies, for example, Picelli et al. [8] extended the application of this method for the case of frequency optimization of coupled fluid-structure systems and Lopes H.N. [9] studied the application of Lagrange multipliers in BESO, which could be applied in natural frequency optimization.

Jensen and Pedersen [10] approached the eigenvalue separation problem for 1D and 2D cases. For the latter, they applied a topology optimization method for separating two consecutive natural frequencies of structures with free boundary conditions (no supports). It is noted that the resulting topologies present some kind of periodicity.

Also on this, Olhoff and Cheng [11] executed a shape optimization of Bernoulli-Euler beams for maximizing band gaps. They noted that the results were periodic, even though that wasn't assumed beforehand.

This way, executing an optimization while forcing periodicity in the final topology can be performed in order to analyse the viability of their solutions. Several studies have already implemented successfully periodic constraints on BESO and have resulted in optimal solutions [12, 13].

The goal of this work is to perform a topology optimization procedure to obtain a solution which maximizes the separation between two adjacent natural frequencies. Then, the procedure is repeated while forcing periodicity on the domain, in order to compare their results to the ones not considering any periodicity.

2. Methods

2.1 Topology Optimization

Topology optimization is a general structural optimization method in which, by altering the local design variable, it is possible to find the distribution of the material in a fixed initial design domain. In this work, the method BESO (Bidirectional Evolutionary Structural Optimization) is used [5]. The used topology optimization method is based on a sensitivity analysis, which are calculated by executing a finite element analysis. Figure 1 illustrates a hipotetical result from this method and Figure 2 is a flowchart that presents the algorithm of the BESO method.

In the flowchart of the Figure 2, ER is the evolutionary rate, that is, the maximum volume variation per iteration, AR_{max} is the maximum percentage of added elements per iteration, p is the penalization exponent and r_{min} is the filter radius.

Also, since the algorithm implemented uses a Soft-Kill method, void elements are not completely removed from the domain, but their design variables' value are changed to x_{min} (e.g. 10^{-6}).

Applying a filter for sensitivities is necessary to avoid checkerboard patterns in the resulting topolo-

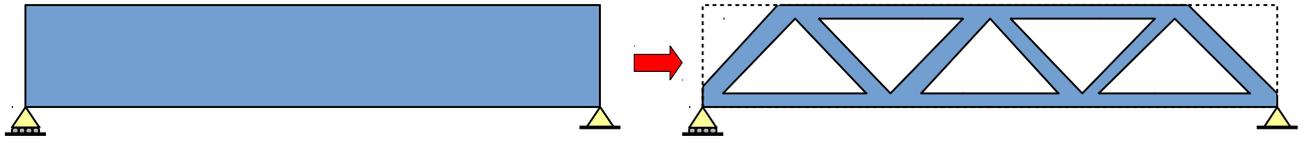


Figure 1. Illustration of a topology optimization procedure.

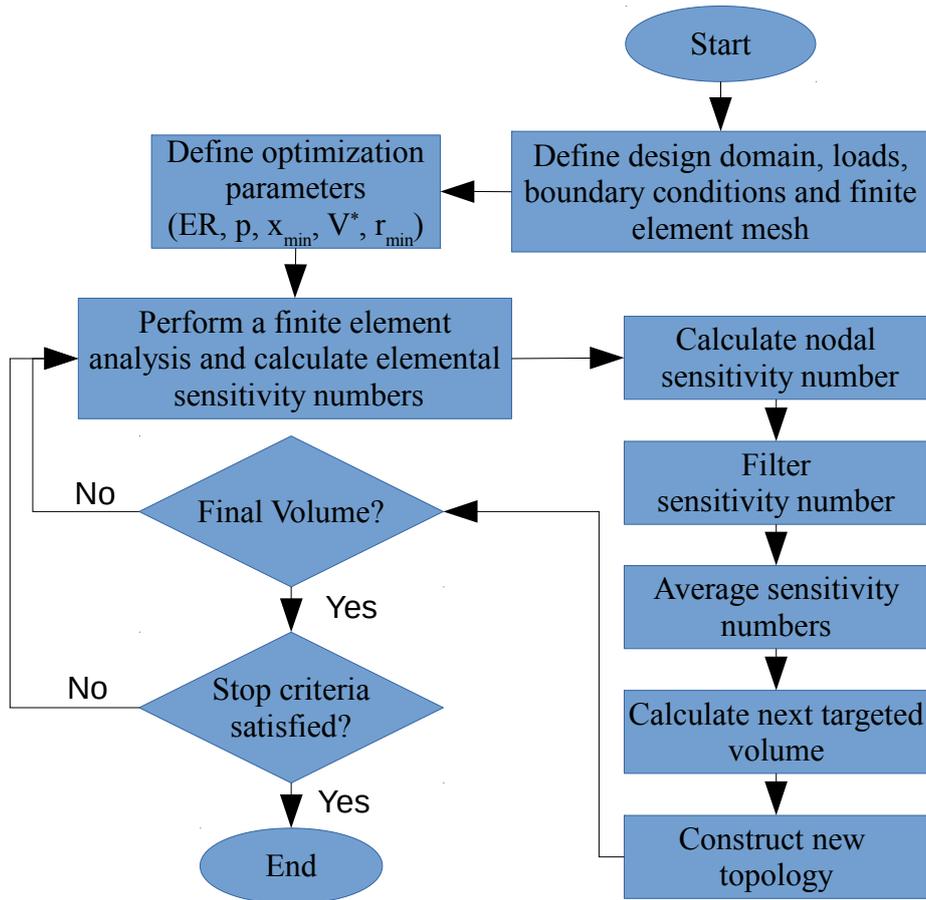


Figure 2. BESO algorithm.

gies and also to make the process independent to the mesh size, allowing refinement of the mesh.

Finally, the stop criterion used in this process is based on the comparison between the values of the latest iterations. Thus, the evolutionary process continues until this criterion is satisfied

$$\left| \frac{\sum_{j=1}^5 f(x_i)_{k-j+1} - \sum_{j=1}^5 f(x_i)_{k-j-4}}{\sum_{j=1}^5 f(x_i)_{k-j+1}} \right| \leq \tau \quad (1)$$

where k is the current iteration, τ , the tolerance and $f(x_i)$, the objective function.

2.2 Periodicity constraint

In order to apply periodicity in the domain, first the mesh is divided in $m_1 \times m_2$ equal sections, as illustrated in Figure 3.

All sections are identical, so each element can be associated with a corresponding one in all other cells. At each iteration, after filtering, all their sensitivity numbers are replaced by the average. Also,

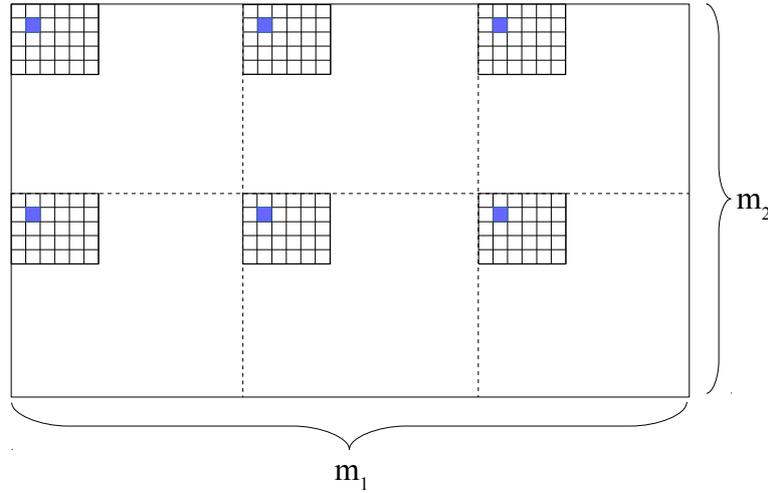


Figure 3. Representation of a periodic domain. The highlighted elements are the corresponding ones at each cell.

the number of elements removed is always divisible by the total amount of cells, in a way that when one of them is removed or added, so are all the other corresponding ones.

2.3 Problem statement

For free vibration analysis, the behavior of undamped structures can be determined by solving the eigenproblem

$$\left([K] - \omega_{n_j}^2 [M] \right) \{ \phi_j \} = \{ 0 \} \quad (2)$$

where $[M]$ is the mass matrix, $[K]$, the stiffness matrix, ω_{n_j} , the j th natural frequency and $\{ \phi_j \}$, the j th mode of vibration.

The Rayleigh quotient can be obtained by premultiplying Eq. 2 by $\{ \phi_j \}^T$ and isolating the eigenvalue, which results in

$$\omega_{n_j}^2 = \frac{\{ \phi_j \}^T [K] \{ \phi_j \}}{\{ \phi_j \}^T [M] \{ \phi_j \}} \quad (3)$$

The optimization problem for maximizing all natural frequencies above a certain threshold ω_0 , while prioritizing the lowest ones, can be described as

$$\begin{aligned} \text{Maximize : } & f(x_i) = \left[\sum_{j=n}^m \frac{1}{(\omega_{n_j}^2 - \omega_0^2)} \right]^{-1} \\ \text{Subject to : } & V^* - \sum_{i=1}^N V_i x_i = 0 \\ & x_i = 1 \text{ or } x_{min} \end{aligned} \quad (4)$$

here, are maximized natural frequencies from n th to m th. V^* is the final volume to be achieved, N , the total amount of elements and x_i , the design variable, which is equal to 1 when the element is solid and x_{min} when it is void, as mentioned previously. The parameter ω_0 is a value used to weight the natural frequencies, in other words, eigenvalues that are closer to this are prioritized in this process.

Another property of this function is that, since it takes into account more than just the maximized frequency, whenever a mode shift occurs, it adjusts smoothly, without sudden increases or decreases of either the function and its derivatives. Thus, it provides a more stable evolutionary process.

Since the goal of this work is to maximize the difference between two consecutive natural frequencies, this objective function was extended to maximize the n th eigenvalue and ones above it and minimize the other ones below it. Therefore, the optimization is described as

$$\begin{aligned} \text{Maximize : } f(x_i) &= \left[\sum_{j=n}^m \frac{1}{(\omega_{n_j}^2 - \omega_0^2)} \right]^{-1} + \left[\sum_{j=1}^{n-1} \frac{1}{(\omega_0^2 - \omega_{n_j}^2)} \right]^{-1} \\ \text{Subject to : } V^* - \sum_{i=1}^N V_i x_i &= 0 \\ x_i &= 1 \text{ or } x_{min} \end{aligned} \quad (5)$$

here ω_0 is chosen as a frequency between the n th natural frequency and the one directly below it. For all examples in this work, the average of these eigenvalues is used, in order to equalize the maximization and the minimization processes.

2.4 Sensitivity analysis

In order to carry out the optimization, it is necessary to calculate the sensitivity of each element, that is, the first derivative of the objective function in respect to the design variable of the i th element. Assuming that the modes of vibration are normalized in respect to the global mass matrix, and using Eq. 3 to find the derivative of $\omega_{n_j}^2$, the sensitivity becomes

$$\begin{aligned} \frac{df(x_i)}{dx_i} &= \left[\sum_{j=n}^m \frac{1}{(\omega_{n_j}^2 - \omega_0^2)} \right]^{-2} \sum_{j=n}^m \frac{1}{(\omega_{n_j}^2 - \omega_0^2)^2} \{\phi_j\}^T \left(\frac{d[K]}{dx_i} - \omega_{n_j}^2 \frac{d[M]}{dx_i} \right) \{\phi_j\} + \\ &\left[\sum_{j=1}^{n-1} \frac{1}{(\omega_0^2 - \omega_{n_j}^2)} \right]^{-2} \sum_{j=1}^{n-1} \frac{1}{(\omega_0^2 - \omega_{n_j}^2)^2} \{\phi_j\}^T \left(\frac{d[K]}{dx_i} - \omega_{n_j}^2 \frac{d[M]}{dx_i} \right) \{\phi_j\} \end{aligned} \quad (6)$$

In order to obtain the derivatives of the stiffness and mass matrices, a material interpolation model must be chosen. Thus, using the material model proposed by [6] we can state that

$$E(x_i) = \left[\frac{x_{min} - x_{min}^p}{1 - x_{min}^p} (1 - x_i^p) + x_i^p \right] E \quad (7)$$

$$\rho(x_i) = x_i \rho \quad (8)$$

This interpolation assures that the ratio between stiffness and mass remains constant when $x_i = x_{min}$, which guarantees that the void elements will have little effect in the solid elements' vibration.

Therefore, the derivative of the two matrices become

$$\frac{d[K]}{dx_i} = \frac{1 - x_{min}}{1 - x_{min}^p} p x_i^{p-1} [K^0] \quad (9)$$

$$\frac{d[M]}{dx_i} = [M^0] \quad (10)$$

where $[K^0]$ and $[M^0]$ are the stiffness and mass matrices of the i th element considering $x_i = 1$.

Therefore, by substituting the derivatives in Eq. 6 by the values in Eqs. 9 and 10, the sensitivity is obtained as

$$\alpha_i = \frac{1}{\rho} \frac{df(x_i)}{dx_i} = \left[\sum_{j=n}^m \frac{1}{(\omega_{n_j}^2 - \omega_0^2)} \right]^{-2} \sum_{j=n}^m \frac{1}{(\omega_{n_j}^2 - \omega_0^2)^2} \{\phi_j\}^T \left(\frac{1-x_{min}}{1-x_{min}^p} x_i^{p-1} [K^0] - \frac{\omega_{n_j}^2}{\rho} [M^0] \right) \{\phi_j\} + \left[\sum_{j=1}^{n-1} \frac{1}{(\omega_0^2 - \omega_{n_j}^2)} \right]^{-2} \sum_{j=1}^{n-1} \frac{1}{(\omega_0^2 - \omega_{n_j}^2)^2} \{\phi_j\}^T \left(\frac{1-x_{min}}{1-x_{min}^p} x_i^{p-1} [K^0] - \frac{\omega_{n_j}^2}{\rho} [M^0] \right) \{\phi_j\} \quad (11)$$

3. Results

In order to analyse this method, a simply supported beam at both ends with dimensions of 10m x 1m x 1m is studied. This structure is shown in Figure 4.

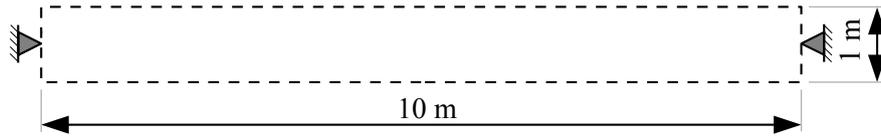


Figure 4. Simply supported beam whose natural frequencies are analysed.

Its material is assumed to have a specific mass of $\rho = 1 \text{ kg/m}^3$, Young's modulus of $E = 10 \text{ MPa}$, and Poisson's ratio of $\nu = 0.3$.

This domain was discretized in a 420x42 mesh of identical quadrilateral elements.

3.1 Gap between 2nd and 3rd natural frequencies

In this first example, the separation between the third and second natural frequencies was the desired result. In order to achieve this, the BESO process was carried out with the following parameters: $V^* = 60\%$, $ER = 2\%$, $AR_{max} = 1\%$, $p = 5$, $x_{min} = 10^{-6}$, $r_{min} = 75 \text{ mm}$ and $\tau = 0.01\%$.

The optimization process stopped after 38 iterations. The final topology is shown in Figure 5 and the evolution of natural frequencies, in Figure 6. In addition, the value of these natural frequencies are shown in Table 1.



Figure 5. Topology that separates second and third frequencies.

The gap between these two natural frequencies is 102.54 Hz. It can also be noted that, although no periodicity was imposed, the structure's shape resembles one with three cells.

This way, the same optimization was repeated with three cells, this time, however, it was possible to reduce the final volume to $V^* = 40\%$. It took 53 iterations for the procedure to end.

The topology obtained is shown in Figure 7, and the evolution of frequencies, in Figure 8. Once again, the natural frequencies are displayed in Table 2.

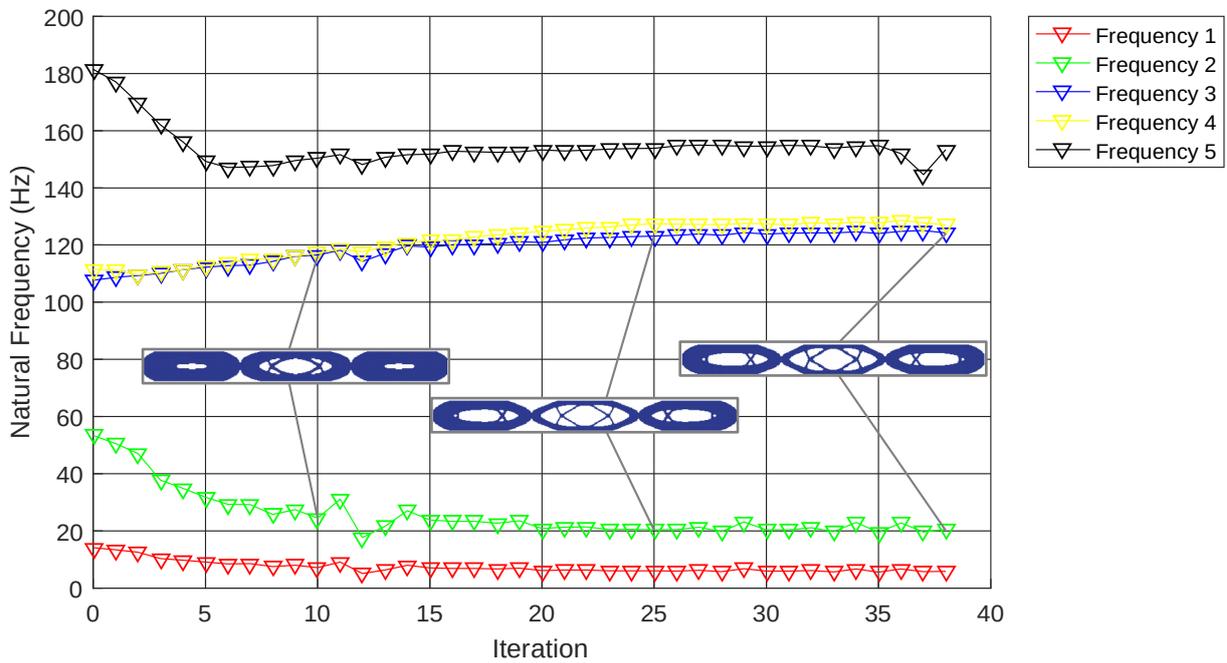


Figure 6. Evolution of the first five natural frequencies during the separation of the second and third ones. $\omega_0 = 506.24$ rad/s.

Table 1. Final topology's natural frequencies.

i	f_i (Hz)
1	5.647
2	20.13
3	122.7
4	126.4
5	154.6



Figure 7. Topology with three cells that separates second and third frequencies.

Table 2. Final topology's natural frequencies.

i	f_i (Hz)
1	5.078
2	18.63
3	111.4
4	123.2
5	130.6

It is possible to note in Figure 8, that there was an instability on the third iteration due to the

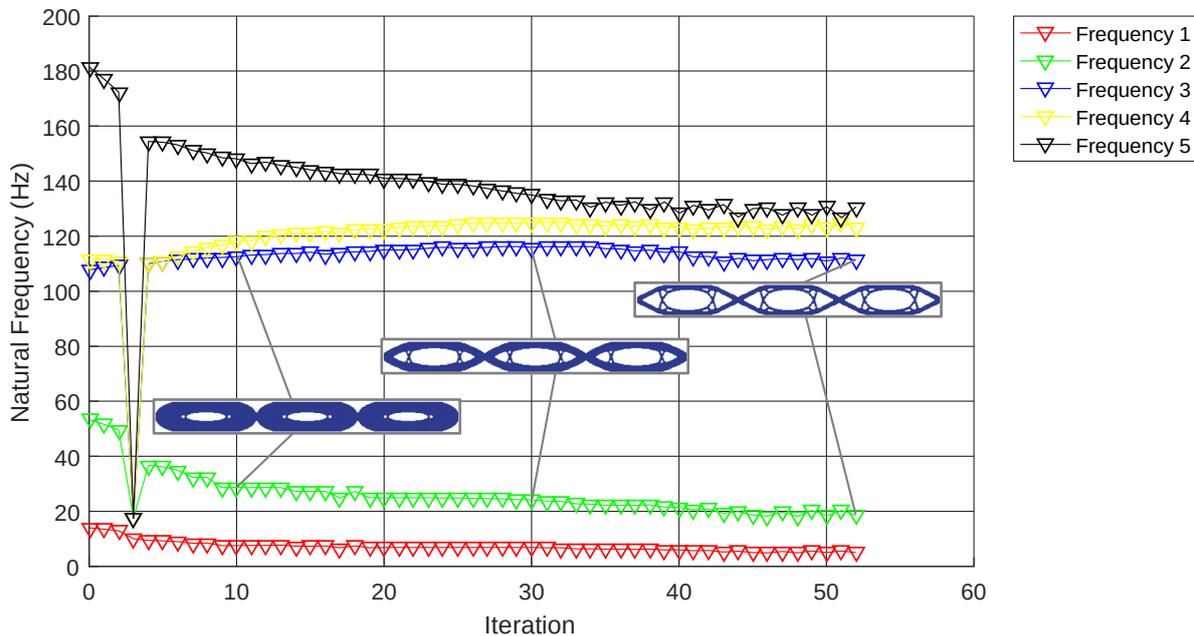


Figure 8. Evolution of the first five natural frequencies during the separation of the second and third ones while imposing periodicity. $\omega_0 = 506.24$ rad/s.

appearance of a local mode, however, the algorithm quickly corrected that problem and, on the fourth iteration, it is absent.

In the end, the final topology presents a gap of 92.74 Hz.

The periodic problem resulted in a lower gap than the unperiodic one, and also required more iterations to complete. However, it was possible to obtain a lighter and cheaper topology, since it has 40% of the original domain's mass, while the unconstrained one has 60%.

3.2 Gap between 9th and 10th natural frequencies

In this other case, the desired result was the separation of the ninth and tenth natural frequencies. Initially, no periodicity was imposed and the BESO was run with the following parameters: $V^* = 80\%$, $ER = 1\%$, $AR_{max} = 1\%$, $p = 5$, $x_{min} = 10^{-6}$, $r_{min} = 75$ mm and $\tau = 0.01\%$.

This process stopped after 47 iterations. Figure 9 shows the resulting structure, Figure 10, the evolution of certain frequencies. Their values, until the 15th one, are shown in Table 3.

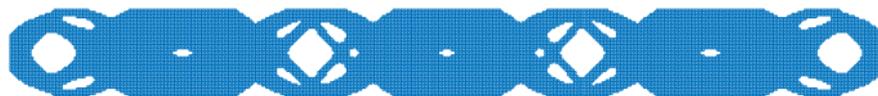


Figure 9. Final topology that separates ninth and tenth frequencies.

As displayed by Figure 10 and Table 3, the frequency gap is $\Delta f = 199.80$ Hz.

Similar to what was done in the last case, this optimization was repeated, but imposing periodicity. Since a 3 cell periodicity was used for maximizing the 3rd transversal mode, the number of cells used here will be consistent to the 10th mode. It is important to note that both axial and transversal frequencies are calculated here, in a way that the 10th mode is equivalent to the 7th transversal one.

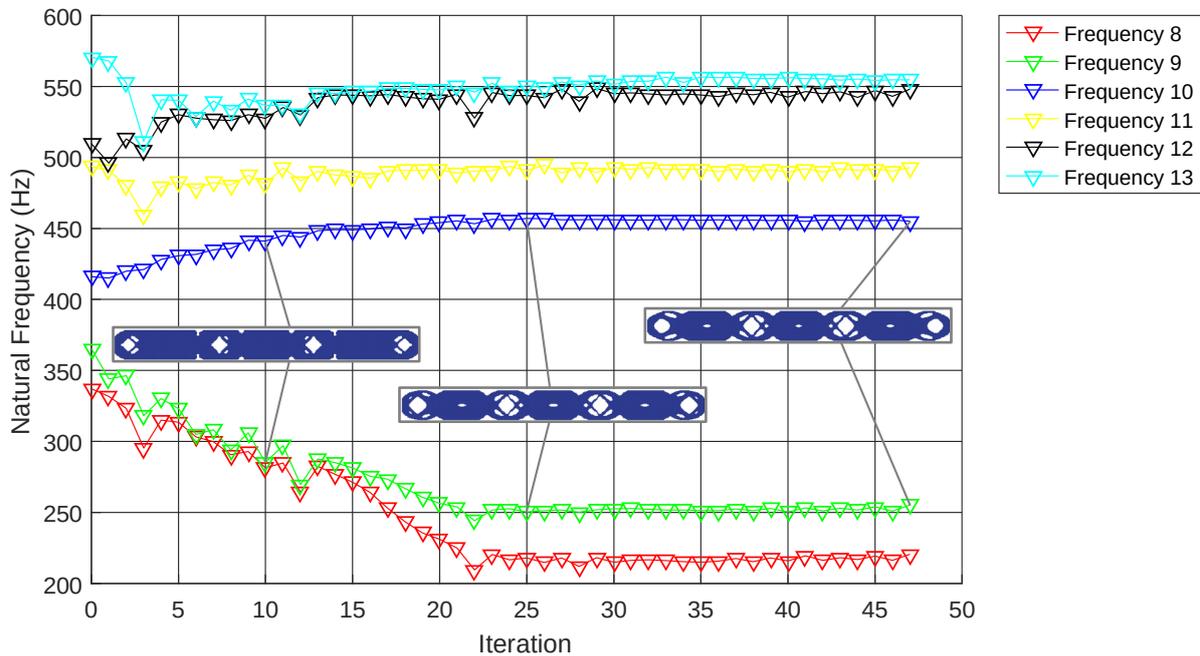


Figure 10. Evolution of some natural frequencies during the separation of the ninth and tenth ones.
 $\omega_0 = 2455.5$ rad/s.

Table 3. Final topology's natural frequencies.

i	f_i (Hz)	i	f_i (Hz)	i	f_i (Hz)
1	11.20	6	157.5	11	492.8
2	42.41	7	205.1	12	548.1
3	80.64	8	220.6	13	554.7
4	88.72	9	255.3	14	583.4
5	151.43	10	455.1	15	608.03

That way, the number of cells chosen was 7.

For this analysis, the new final volume could be reduced to $V^* = 60\%$. After 45 iterations, the topology obtained is shown in Figure 11. Natural frequencies' evolution is displayed in Figure 12 and their values, in Table 4.



Figure 11. Periodic topology that separates ninth and tenth frequencies.

In this optimization, the gap is $\Delta f = 106.46$ Hz. Once again, the periodic procedure resulted in a shorter gap than the unconstrained one, however, it was possible to achieve an optimal topology with less volume.

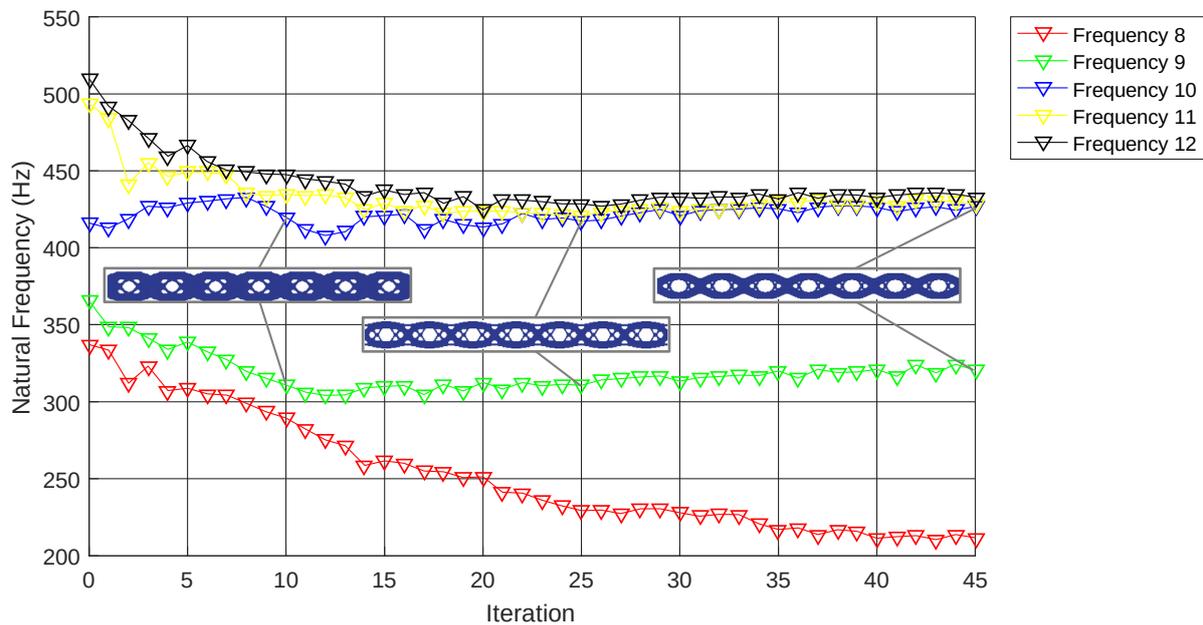


Figure 12. Evolution of some natural frequencies during the separation of the ninth and tenth ones while imposing periodicity. $\omega_0 = 506.24$ rad/s.

Table 4. Final topology's natural frequencies.

i	f_i (Hz)	i	f_i (Hz)	i	f_i (Hz)
1	9.427	6	165.0	11	428.1
2	35.63	7	204.6	12	432.9
3	73.79	8	211.8	13	486.9
4	104.9	9	320.4	14	529.2
5	118.7	10	426.8	15	531.9

4. Conclusion

In this work a topology optimization method for maximizing separation of consecutive frequencies was applied and was able to, successfully, obtain topologies that achieved this goal.

Although periodicity is often obtained in these types of problems, the unconstrained results were able to maximize this gap more than the periodic ones. However, by forcing this constraint, it was possible to obtain topologies with lower volumes, thus reducing the environmental impact and the total weight. It is also important to note that these shapes are also manufacturable, since they have smooth boundaries between each cell.

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