

## **INFLUENCE OF LIMIT STATES ON THE OPTIMIZATION OF REINFORCED CONCRETE BEAMS**

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**Abstract:** Most current structural design codes are based on the concept of limit states, that is, when a structural element fails to meet one of its purposes, it is said that it has reached its limit state. According to the Brazilian code ABNT NBR 6118:2014, in the design of reinforced concrete structures, the following limit states must be checked: A. Ultimate Limit State (ULS), related to collapse or any other way of structural ruin that results in the discontinuation of the structure use; B. Serviceability Limit State (SLS), related to the structures durability, appearance and performance and comfort of the user. However, there are not many studies in the literature that point out the influence of such limit states on the optimal design of a structural element. In view of this, this paper presents an optimization scheme for reinforced concrete structures, based on the lowest cost, considering two groups of constraints: the first is related to the ULS and the second to the SLS. The optimal characteristics of a structural element subject to each group of constraints, individually, as well as to both groups simultaneously, are discussed. The methodology is applied to simply supported beams using genetic algorithms to solve the optimization problem, since this problem involves several local minima. The objective function employed corresponds to the cost of the structural element, which includes the consumption of concrete, steel and formwork. The limit states are formulated according to the recommendations of the ABNT NBR 6118:2014. As a result, predominant characteristics were observed in the optimal sections, according to the limit states: for ULS, the sections presented more steel reinforcement; in the SLS predominance of concrete occurred; in ULS+SLS the optimal factors remained within the limit formed by ULS and SLS.

**Keywords:** optimization, reinforced concrete, limits states, genetic algorithms.



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## ABSTRACT

Most current structural design codes are based on the concept of limit states, that is, when a structural element fails to meet one of its purposes, it is said that it has reached its limit state. According to the Brazilian code ABNT NBR 6118:2014 [1], in the design of reinforced concrete structures, the following limit states must be checked: A. Ultimate Limit State (ULS), related to collapse or any other way of structural ruin that results in the discontinuation of the structure use; B. Serviceability Limit State (SLS), related to the structures durability, appearance and performance and comfort of the user. However, there are not many studies in the literature that point out the influence of such limit states on the optimal design of a structural element. In view of this, this paper presents an optimization scheme for reinforced concrete structures, based on the lowest cost, considering two groups of constraints: the first is related to the ULS and the second to the SLS. The optimal characteristics of a structural element subject to each group of constraints, individually, as well as to both groups simultaneously, are discussed. The methodology is applied to simply supported beams using genetic algorithms to solve the optimization problem, since this problem involves several local minima. The objective function employed corresponds to the cost of the structural element, which includes the consumption of concrete, steel and formwork. The limit states are formulated according to the recommendations of the ABNT NBR 6118:2014 [1]. As a result, predominant characteristics were observed in the optimal sections, according to the limit states: for ULS, the sections presented more steel reinforcement; in the SLS predominance of concrete occurred; in ULS+SLS the optimal factors remained within the limit formed by ULS and SLS.

**Keywords:** optimization, reinforced concrete, limit states, genetic algorithms

## 1. INTRODUCTION

Due to the development of new technologies and the increase of market competitiveness, the search for more efficient and lower-cost designs has increased. At the same time, many reinforced concrete structures have been designed and built around the world, so that this construction system has become one of the most used. In this scenario, the importance of studies related to the design concept of reinforced concrete structures is valid.

To ensure the safety of a structure, the engineer must choose a design option which meets the normative checks related to its purpose. However, due to the large number of variables involved in the design of reinforced concrete structures, there are several different configurations that can meet

the required conditions, with different costs and performances. Usually, the choice of a configuration is not simple, which makes it difficult to obtain an optimal design from traditional methods, that is, a design that is safe with the lower cost possible.

Several studies in the field of optimization of reinforced concrete structures have been developed in the last decades, with the objective of obtaining designs with optimal parameters, generally related to the cost of the structures ([2–9]). In this sense, many researchers have used genetic algorithms to optimize reinforced concrete structures ([10–18]).

However, there is a lack in studies that discuss such optimal characteristics and their relationships with normative verifications. Most current codes, including the Brazilian ABNT NBR 6118:2014 [1], are based on the concept of limit states. According to these codes, the structure must satisfy certain limit states simultaneously. Within this context, an analysis of the influence of such limit states on what would be the optimal configuration of reinforced concrete structures needs to be better evaluated, with the purpose of assisting the studies in project design.

Conceptually, a limit state is given by the situation (limit) from which a structural element no longer meets one of its design goals, in other words, when a structure fails to satisfy any of the purposes of its construction. The current Brazilian code establishes that the following limit states must be considered: Ultimate Limit State (ULS) and Serviceability Limit State (SLS). The ULS are related to any situation that determines the interruption, in whole or in part, of the use of the structure. The SLS are characterized by situations that, due to their occurrence, repetition or duration, generate structural effects that do not meet the conditions specified for the normal use of the structure, or indicate impairment of its durability [19].

In this sense, the present paper proposes to analyze the optimal configuration of reinforced concrete beams, through the minimization of its costs, considering two groups of constraints: the first is related to the ULS and the second to the SLS. The optimal characteristics of this structural element when subject to each group of constraints, individually, as well as to both groups, simultaneously, are discussed.

The main motivation of this paper is to expand the studies on the design of reinforced concrete structures through an analysis of the influence of the normative constraints on the optimal characteristics of this structures, in addition to contribute to the development of research in optimization of reinforced concrete structures by the development of a computational tool capable of optimizing their costs.

## **2. METHODOLOGY AND PROBLEM FORMULATION**

The proposed problem consists in minimizing the cost of simply supported, rectangular section beams subjected to uniformly distributed loading. The solutions of the structures subject to the constraints of the ULS and the SLS, individually, are analyzed, as well as the structure subject to the constraints of both limit states simultaneously. Besides the constraints related to the limit states, in all cases, constructive constraints are also considered, that is, the optimum sections should check the following requirements: minimum longitudinal and transverse reinforcement rates; ductility conditions; concentrated stress in the center of gravity of the longitudinal reinforcements. All calculations follow the recommendations of the Brazilian code ABNT NBR 6118:2014 [1]. Figure 1 shows the configuration of the beams studied and the nomenclatures used.

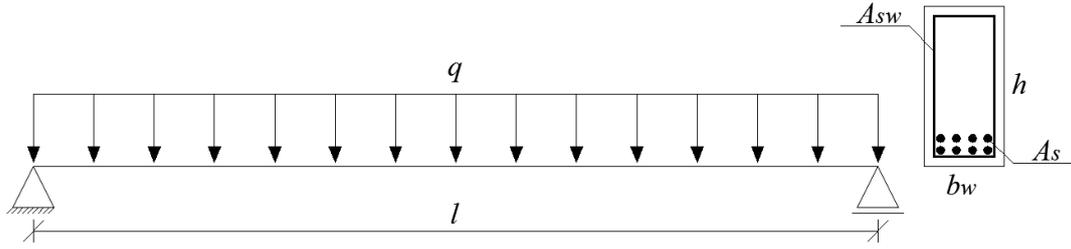


Figure 1. Beam design

## 2.1 Design variables

The design variables are the unknowns of the objective function, that is, the cross-sectional dimensions of the rectangular beam: width,  $b_w$ , and height,  $h$ ; and the number of bars of the longitudinal,  $N_s$ , and transverse,  $N_{sw}$ , reinforcement. The variables are represented by the vector  $\mathbf{x}$  described in Equation 1.

$$\mathbf{x} = [b_w, h, N_s, N_{sw}] \quad (1)$$

## 2.2 Objective function

In this paper, the objective function employed corresponds to the cost of the structural element, based on the cost of concrete volume, longitudinal and transverse reinforcement mass and formwork area. The unit costs adopted include materials and labor to construct the structure. The Equation 2 is the objective function used in the problem.

$$f(\mathbf{x}) = b_w \cdot h \cdot l \cdot C_c + A_s \cdot N_s \cdot l \cdot \rho_s \cdot C_s + A_{sw} \cdot N_{sw} \cdot l_{sw} \cdot \rho_{sw} \cdot C_{sw} + (2 \cdot h + b_w) \cdot l \cdot C_f \quad (2)$$

Where:

$\mathbf{x}$  = Vector of design variables;

$C_c$  = Unit cost of concrete (R\$/m<sup>3</sup>);

$C_s$  = Unit cost of longitudinal reinforcement (R\$/kg);

$C_{sw}$  = Unit cost of transverse reinforcement (R\$/kg);

$C_f$  = Unit cost of formwork (R\$/m<sup>2</sup>);

$b_w$  = Beam width (m);

$h$  = Beam height (m);

$l$  = Beam length (m);

$A_s$  = Cross-sectional area of one longitudinal reinforcement bar (m<sup>2</sup>);

$N_s$  = Number of longitudinal reinforcement bars;

$\rho_s$  = Unit mass of steel of the longitudinal reinforcement (kg/m<sup>3</sup>);

$A_{sw}$  = Cross-sectional area of one transverse reinforcement bar (m<sup>2</sup>);

$N_{sw}$  = Number of transverse reinforcement bars;

$l_{sw}$  = Length of one bar of the transverse reinforcement (m);

$\rho_{sw}$  = Unit mass of steel of the transverse reinforcement (kg/m<sup>3</sup>).

## 2.3 Constraints

The constraints imposed on the problem are represented by the vector  $\mathbf{g}$  described by Equation 3, where  $n$  is the number of constraints  $g_i(\mathbf{x})$  of the problem, with  $i = 1, \dots, n$ . The restrictions were applied in the most requested sections, which in this case are in the middle or in the supports of the beam. The constraints adopted in this paper are described in the following.

$$\mathbf{g} = [g_1(\mathbf{x}), \dots, g_n(\mathbf{x})] \quad (3)$$

### 2.3.1 ULS constraints

The ULS constraints are related to the strength capacity of the structural element section. In view of the applied load, the structure must withstand the design bending moment ( $M_{Sd}$ ) and the design shear force ( $V_{Sd}$ ). The internal forces are obtained from the normal ultimate combination of applied loads.

- Load-bearing capacity of a reinforced beam cross-section in bending:

The section of the element is safe with respect to the bending moment if it satisfies the constraint given by the Equation 4, where  $M_{Rd}$  is the ultimate (design) moment.

$$g_1(\mathbf{x}) = M_{Sd} - M_{Rd} \leq 0 \quad (4)$$

The  $M_{Rd}$  is calculated from Equation 5, obtained from the equilibrium of forces acting on the cross section, where  $f_{yd}$  is the design yield strength of longitudinal steel reinforcement,  $d$  is the effective depth (the distance from the extreme compression fibers of the concrete to the centroid of the reinforcement),  $\lambda$  is a parameter depending on the characteristic compressive strength of concrete ( $f_{ck}$ ) e  $x$  is the neutral axis.

$$M_{Rd} = A_s \cdot N_s \cdot f_{yd} \cdot (d - 0.5 \cdot \lambda \cdot x) \quad (5)$$

The value of  $x$  is obtained from Equation 6, where  $\alpha_c$  is a parameter of reduction of the compressive strength of concrete and  $f_{cd}$  is the design compressive strength of concrete.

$$x = \frac{A_s \cdot N_s \cdot f_{yd}}{\lambda \cdot \alpha_c \cdot f_{cd} \cdot b_w} \quad (6)$$

- Load-bearing capacity of a reinforced beam cross-section in shear:

The strength of the section with respect to the shear force is guaranteed if the Equations 7 and 8 are checked, where  $V_{Rd2}$  and  $V_{Rd3}$  are, respectively, the design shear strength of compressive diagonals of concrete and the design shear strength of traction diagonals (supplied by concrete and transverse reinforcement). Model I of the code is used to obtain shear strengths.

$$g_2(\mathbf{x}) = V_{Sd} - V_{Rd2} \leq 0 \quad (7)$$

$$g_3(\mathbf{x}) = V_{Sd} - V_{Rd3} \leq 0 \quad (8)$$

The value of  $V_{Rd2}$  is obtained from the Equation 9, where  $\alpha_{v2}$  is a parameter which depends on the  $f_{ck}$ .

$$V_{Rd2} = 0.27 \cdot \alpha_{v2} \cdot f_{cd} \cdot b_w \cdot d \quad (9)$$

The value of  $V_{Rd3}$  is obtained from Equation 10, where  $f_{ctd}$  is the design tensile strength of concrete,  $s$  is the spacing between the transverse reinforcements and  $f_{ywd}$  is the design tension in the transverse reinforcement.

$$V_{Rd3} = 0.6 \cdot f_{ctd} \cdot b_w \cdot d + \frac{A_{sw}}{s} \cdot 0.9 \cdot d \cdot f_{ywd} \quad (10)$$

### 2.3.2 SLS constraints

In this paper, the limit state of excessive deformation is considered. It is represented by Equation 11, where  $a$  is the calculated deflection and  $a_{lim}$  is the maximum deflection allowed by the code.

$$g_4(\mathbf{x}) = a - a_{max} \leq 0 \quad (11)$$

In the evaluation of deflection, linear elastic behavior is considered for concrete and steel. Thus, the deflection in the beam is given by Equation 12, where  $\alpha_f$  is a factor which considers the creep of concrete,  $F_{SLS}$  is the load obtained with the quasi-permanent load combination and  $(E \cdot I)_{eq}$  is the equivalent flexural rigidity, which considers a weighting of the inertia of stages I and II, as well as the homogenized section.

$$a = (1 + \alpha_f) \cdot \frac{5 \cdot F_{SLS} \cdot l^4}{384 \cdot (E \cdot I)_{eq}} \quad (12)$$

## 3. OPTIMIZATION

Genetic algorithms are search algorithms, based on the mechanism of natural selection and natural genetics, developed in the 1970s by John Holland at the University of Michigan [20]. The algorithm can be applied to solve problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, nondifferentiable or highly nonlinear.

These algorithms use some terms in analogy to natural genetics: the individual is a solution, which may or may not be viable; the population is the set of solutions; the chromosome is the coding that represents the individual; the gene is the coding that represents the variable; the allele is the value that the variable can assume [21].

In general, the algorithm repeatedly modifies a population of individual solutions. At each step, the genetic algorithm selects individuals from the current population and uses them as parents to produce the children for the next generation. Over successive generations, the population "evolves" toward an optimal solution.

Genetic algorithms are composed of three main operators [16]:

- **Reproduction (selection):** This operator is used to select the set of individuals that will reproduce to generate the next population. This selection is made based on the fitness of each individual, determined from the objective function. Individuals with higher fitness are more likely to be chosen. The suitability of an individual is affected by violation of the constraints of the problem, and to introduce viability into the fitness of a solution, penalties are taken when a solution does not meet some constraint.

- **Crossover:** This operator is used to propagate the positive characteristics of the fittest individuals of the population (selected by the reproduction operator), by means of the exchange of segments of information between them, in order to originate new individuals.

- **Mutation:** The mutation operator is used to bring about random changes in the population. At this stage, characteristics of individuals are altered, thus adding variety to the population. This operation is carried out in order to search for unexplored areas and avoid premature convergence in an optimal local solution. At the same time, a higher frequency of this operator can destroy the important information contained in the population. Thus, the mutation occurs only in a small part of the population.

In this paper, a preprogrammed genetic algorithm routine, contained in the MatLab software, is used. Figure 2 presents a flowchart of the algorithm.

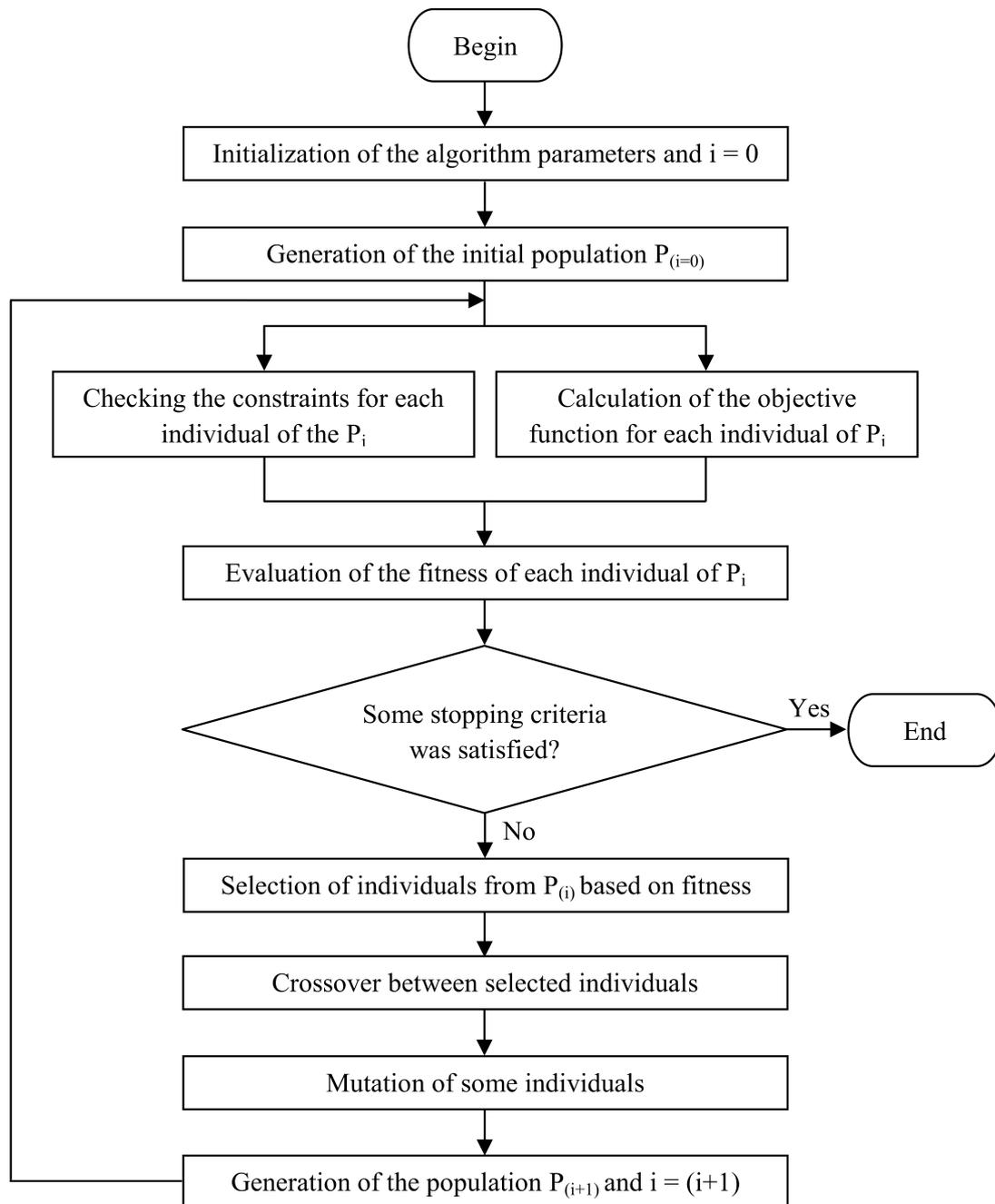


Figure 2. Flowchart of the optimization algorithm

#### 4. NUMERICAL EXAMPLE

The example consists of a beam from a residential design, presented highlighted ( $B_4$ ) in Figure 3. The input data used in the example is shown in Table 1 and the values of unit costs are taken from [22]. In this example, four different loading levels are considered: 40%, 60%, 80% e 100% of the total load applied to the beam ( $F_{ULS}$  e  $F_{SLS}$ ). It should be noted that the load due to the weight of the beam ( $Pp$ ) is also considered, that is, in addition to the loads already mentioned, at each iteration the weight of the structure is updated. The variables  $b_w$  and  $h$  are said to be continuous within the intervals [0.12 m; 0.30 m] and [0.20 m; 0.60 m], respectively. The variables  $N_s$  and  $N_{sw}$  are discretized in sets [2 3 4 5 6 7 8] and [51 35 26 21 18], respectively.

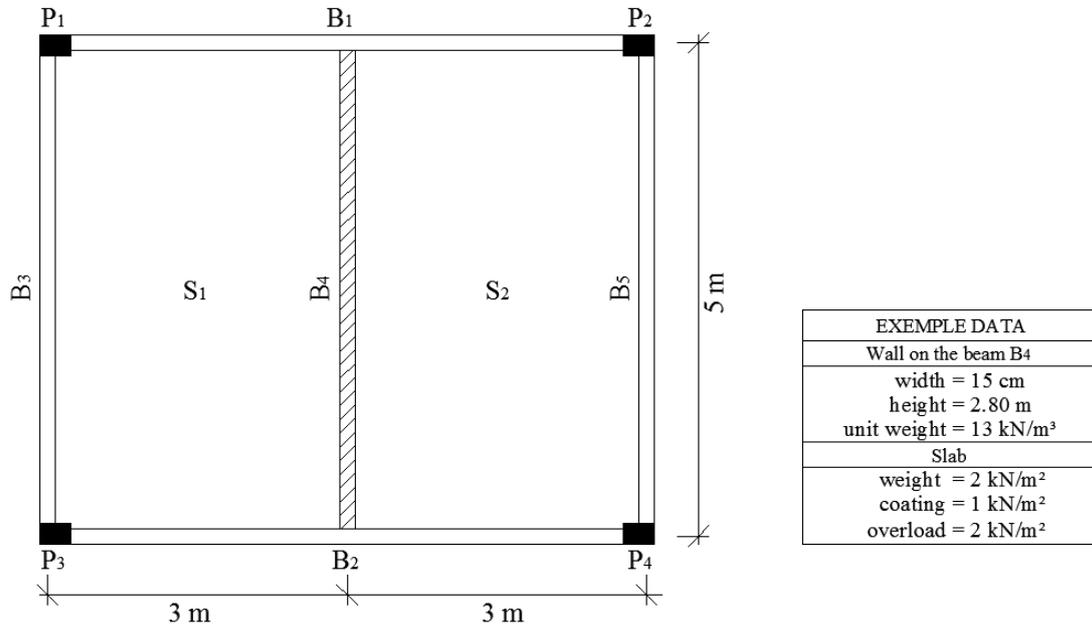


Figure 3. Part of the design with emphasis on the analyzed beam ( $B_4$ )

Table 1. Input data

| Data        |   | Value        | Unit               |
|-------------|---|--------------|--------------------|
| $E_s$       | Modulus of elasticity of longitudinal steel reinforcement         | 210,000      | MPa                |
| $\rho_s$    | Unit mass of steel of the longitudinal reinforcement              | 7,850        | kg/m <sup>3</sup>  |
| $\rho_c$    | Unit weight of reinforced concrete                                | 25           | kN/m <sup>3</sup>  |
| $f_{yk}$    | Characteristic yield strength of longitudinal steel reinforcement | 500          | MPa                |
| $f_{ck}$    | Characteristic compressive strength of concrete                   | 25           | MPa                |
| $c$         | Cover to reinforcement  | 2.5          | cm                 |
| $F_{ULS}$   | Load - Normal ultimate combination                                | 28.66 + $Pp$ | kN/m               |
| $F_{SLS}$   | Load - Quasi-permanent load combination                           | 16.87 + $Pp$ | kN/m               |
| $\phi_s$    | Nominal diameter of longitudinal reinforcement bar                | 12.5         | mm                 |
| $\phi_{sw}$ | Nominal diameter of transverse reinforcement bar                  | 6.3          | mm                 |
| $C_c$       | Unit cost of concrete   | 401.60       | R\$/m <sup>3</sup> |
| $C_s$       | Unit cost of longitudinal reinforcement                           | 5.59         | R\$/kg             |
| $C_{sw}$    | Unit cost of transverse reinforcement                             | 6.08         | R\$/kg             |
| $C_f$       | Unit cost of formwork   | 53.89        | R\$/m <sup>2</sup> |

Note: The properties of the steel of the transverse reinforcement are the same as those of the steel of the longitudinal reinforcement.

#### 4.1 Optimization results

Table 2 presents the results of the optimization for all loading cases. These results can be visualized in Figure 4.

In all cases, it is noticed that the higher the load applied, the higher the cost of the structure, as expected, and that in cases where the limit states are considered individually, the optimal costs are lower than in those cases where both limit states are considered simultaneously. Also in terms of costs, the predominant input in all cases (ULS, SLS and ULS+SLS) is the formwork, which reaches 60% of the total optimum cost, followed by the steel reinforcement for the ULS and the concrete for

Table 2. Results

| Limit State      | Cost [R\$]         |                    |                    |                  | Optimum Parameters |            |       |          |
|------------------|--------------------|--------------------|--------------------|------------------|--------------------|------------|-------|----------|
|                  | Concrete           | Steel              | Formwork           | Total            | $b_w$<br>[m]       | $h$<br>[m] | $N_s$ | $N_{sw}$ |
| Loading Case I   |                    |                    |                    |                  |                    |            |       |          |
| ULS              | 83.04<br>(19.79%)  | 118.73<br>(28.29%) | 217.93<br>(51.92%) | 419.70<br>(100%) | 0.12               | 0.34       | 3     | 35       |
| SLS              | 100.21<br>(22.65%) | 87.38<br>(19.75%)  | 254.81<br>(57.60%) | 442.40<br>(100%) | 0.12               | 0.41       | 2     | 26       |
| ULS + SLS        | 95.37<br>(21.01%)  | 112.94<br>(24.88%) | 245.55<br>(54.10%) | 453.86<br>(100%) | 0.12               | 0.40       | 3     | 26       |
| Loading Case II  |                    |                    |                    |                  |                    |            |       |          |
| ULS              | 93.52<br>(19.72%)  | 139.27<br>(29.37%) | 241.46<br>(50.91%) | 474.25<br>(100%) | 0.12               | 0.39       | 4     | 26       |
| SLS              | 118.49<br>(23.81%) | 85.46<br>(17.18%)  | 293.61<br>(59.01%) | 497.56<br>(100%) | 0.12               | 0.48       | 2     | 21       |
| ULS + SLS        | 112.43<br>(22.16%) | 111.17<br>(21.92%) | 283.65<br>(55.92%) | 507.25<br>(100%) | 0.12               | 0.47       | 3     | 21       |
| Loading Case III |                    |                    |                    |                  |                    |            |       |          |
| ULS              | 115.14<br>(21.18%) | 138.79<br>(25.53%) | 289.64<br>(53.29%) | 543.57<br>(100%) | 0.12               | 0.48       | 4     | 21       |
| SLS              | 133.58<br>(24.57%) | 84.17<br>(15.48%)  | 325.99<br>(59.95%) | 543.74<br>(100%) | 0.12               | 0.54       | 2     | 18       |
| ULS + SLS        | 119.41<br>(21.38%) | 139.91<br>(25.05%) | 299.29<br>(53.58%) | 558.61<br>(100%) | 0.12               | 0.50       | 4     | 21       |
| Loading Case IV  |                    |                    |                    |                  |                    |            |       |          |
| ULS              | 119.76<br>(20.41%) | 166.93<br>(28.45%) | 300.16<br>(51.15%) | 586.85<br>(100%) | 0.12               | 0.50       | 5     | 21       |
| SLS              | 144.51<br>(24.62%) | 87.04<br>(14.83%)  | 355.53<br>(60.56%) | 587.08<br>(100%) | 0.12               | 0.60       | 2     | 18       |
| ULS + SLS        | 126.18<br>(20.72%) | 168.58<br>(27.68%) | 314.34<br>(51.61%) | 609.09<br>(100%) | 0.12               | 0.52       | 5     | 21       |

the SLS. It is emphasized that the reuse of the formwork is not considered, which explains the great impact of this input on the total cost of the structure.

For the optimization that considers only the constraints of the ULS, it is seen that the optimal value of  $N_s$ , in all loading cases, is equal to or greater than the optimal values of the other two cases of optimization (SLS and ULS+SLS). The application of the constraint  $g_1$  causes an increase of longitudinal reinforcement, which can be understood by the analysis of the Equation 5. Note that  $N_s$  is proportional to  $M_{Rd}$ , that is, the larger the total longitudinal steel area, the greater the resistance of the section in relation to the bending moment. The value of the variable  $N_{sw}$  also tends to be higher in the ULS, given the constraint  $g_3$ , where the value of  $V_{Rd3}$  increases as the amount of transverse reinforcement increases. Other variables also influence the constraints  $g_1$ ,  $g_2$  and  $g_3$ , such as  $h$ , where the higher its value, the greater are  $M_{Rd}$ ,  $V_{Rd2}$  and  $V_{Rd3}$ . Therefore, it is understood that the configuration of the optimal section for this limit state seeks to find an equilibrium among  $h$ ,  $N_s$  and  $N_{sw}$ , so that the final cost is minimal and meets the constraints. However, relative to the other two cases of optimization (SLS and ULS+SLS), it is observed that the optimal sections of the ULS tend

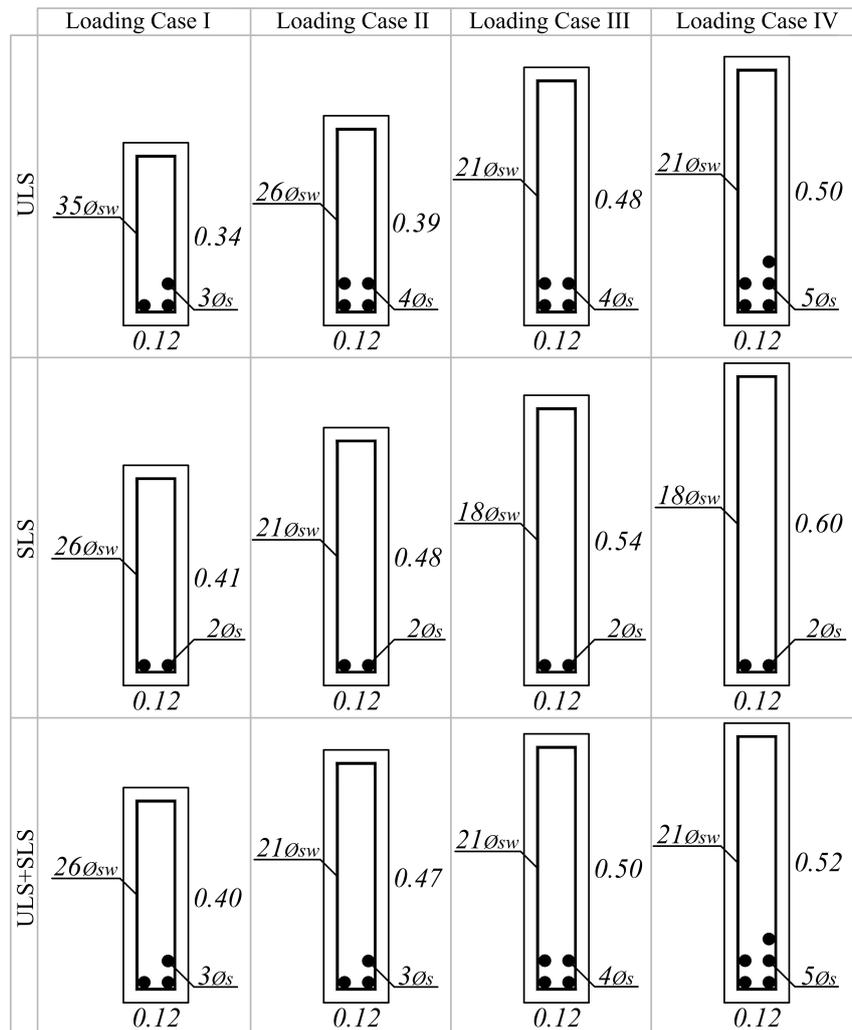


Figure 4. Detailing the optimal sections

to be more reinforced, with a smaller area of concrete.

In reference to the optimization that considers only the constraints of the SLS, it is noted that the optimal value of  $h$ , in all loading cases, is greater than in the other two cases of optimization (ULS and ULS+SLS). The constraint  $g_4$ , which governs this limit state, has great influence on the inertia of the cross-section of the beam. This fact can be understood from Equation 12, in which the only parameter that changes according to the design variables is the inertia of the section. Since  $h$  is an influential design variable in inertia, that is, the higher the section the greater its inertia, the optimum section for the SLS tends to have height as much as is necessary to counter the deflection of the beam. Despite of the fact that the inertia adopted consider the homogenized section, which takes into account the amount of longitudinal reinforcement, which in its turn depends on  $N_s$ , the optimum section found is predominantly concrete, because in SLS, the optimal value of  $N_s$  is the minimum allowed. Therefore, the optimum sections of the SLS tend to have less steel and more concrete when compared to the optimal sections of the ULS and ULS+SLS.

The values of the variables for the optimal sections that consider both limit states remained within the limits between the optimal of the ULS and the SLS. The values of  $h$  are higher than those of the ULS, due to the insertion of the maximum deflection constraint. Given this increase in height, the values of  $N_s$  and  $N_{sw}$  in the ELU+ELS, in some loading cases, decreased in comparison to the ULS.

The optimal width of the beam was similar in all cases studied, approximately 12 cm. It does not

have as much influence on the strength and inertia of the cross section as height, and the increase of its value increases the cost of formwork and concrete, which justifies its minimum value.

For better visualization of the analyzes, Figures 5 and 6 are presented. Note that for all loading cases, the optimal cost of the beam increases from the ULS to the SLS, until the higher cost is obtained when considering both limit states simultaneously. It is also noticed that, for lower loads, the optimal height of the ULS+SLS section is similar to that of the SLS, and for higher loads, the optimum height approaches the ULS.

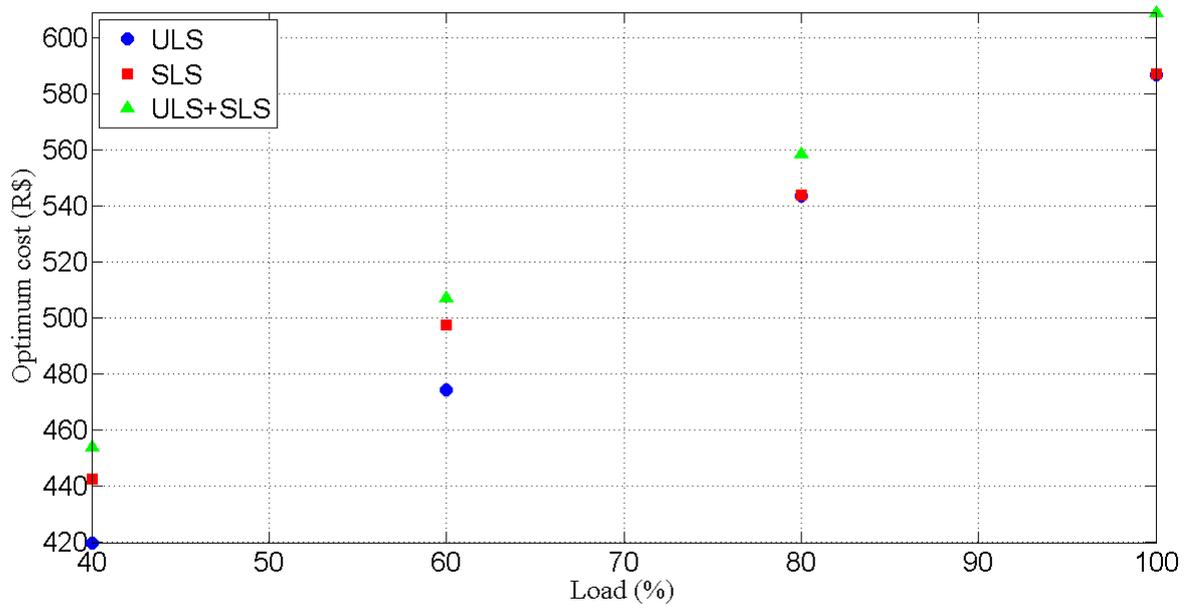


Figure 5. Relation between the applied load and the optimum cost

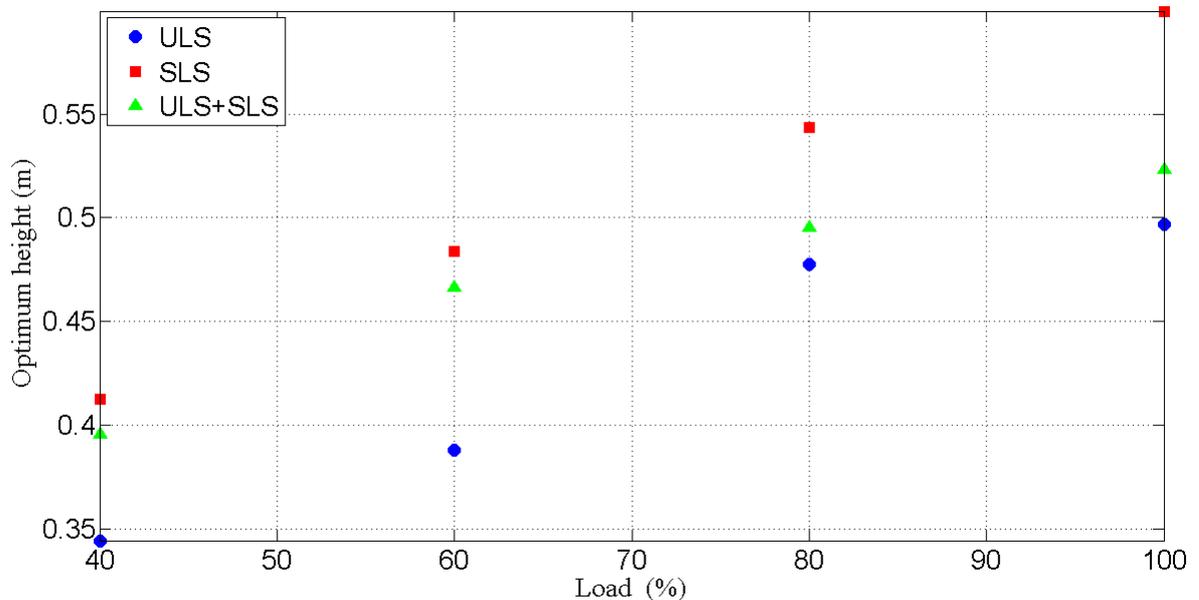


Figure 6. Relation between applied load and optimum height

## 5. CONCLUSIONS

This paper presented an optimization scheme for reinforced concrete beams, based on the ultimate and serviceability limit states, in which genetic algorithms were applied. The purpose of the paper was to verify the optimal characteristics of the structure for each limit state individually and for both limit states simultaneously. Four loading cases were considered and the optimum sections presented predominant characteristics when comparing the three cases of optimization (ULS, SLS and ULS+SLS): the optimal sections for the ULS were more reinforced and with smaller areas of concrete; in the SLS case, the optimal sections presented the smaller amount of bars and larger heights, which results in sections that were less reinforced and with greater area of concrete; when both limit states were considered, an intermediate optimal solution between the ULS and SLS was found. It should be noted that no optimal section for ULS met the SLS constraints, the reverse being true as well, so that the cost of ULS+SLS is the highest in the four loading cases studied. The input with preponderant cost in all cases was the formwork, followed by the steel reinforcement for the ULS and the concrete for the SLS, which agrees with the preponderant characteristics of the optimal sections of each limit state. It is concluded that the optimal factors of the sections for each limit state are distinct and conflicting with each other, which results in predominant characteristics for the beams, according to the limit states that must be verified.

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