

## **Action plane fracture criterion applied to extended finite method: A framework for onset and crack propagation**

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### **ABSTRACT**

The concept of Extended Finite Element Method is reliable and useful for analyzing complex geometries in heavy nonlinear regimes since a delinking between mesh and geometric discontinuity is possible. Many authors have research in this field, but 3D XFEM models applied to non-standard composite parts, such as tapered structures or joints, are rarely found in the literature. In the present work, a framework for evaluating structural behavior of composite structures by using Extended Finite Element Method has been developed and implemented. A new criterion for initiation and crack propagating direction definition based on Puck's theory is presented in details. The proposed three-dimensional model for composites based on Extended Finite Method is implemented through Abaqus' subroutines, using optimization algorithms for computational efficiency. To verify the performance of the model to different stacking sequences and geometry, a sensibility study for the most important parameters of the model is also provided. Finally, experimental results from the literature are shown and used to verify the efficiency of the proposed approach

**Keywords:** 3D XFEM, Crack propagation, Puck's theory, composite joints, tapered structures

### **1 INTRODUCTION**

There was undoubtedly a huge advance on the analysis of the failure on composite materials in later years nevertheless the concept of damage tolerance in such structures is still a field wide open for researches and poses many issues already unsolved. Even with modern approaches involving fracture mechanics and Representative Volume Elements, the intrinsic structural behavior is too complex and, therefore, micro approaches bring challenges to be applied widely in an industry environment. Thus, one alternative consists on using meso-scale models. For example, meso-scale models based on the Puck's theory are easily found in the literature. One reason for this is because Puck's theory was one of the best evaluated in Worldwide Failure Exercise I and showed good results in WWFE II. "Exhibiting good predictive capability, none or one fundamental weakness and many

relatively minor weaknesses”, as stated by [1, 2]. WWFE II was the second edition of this benchmark, which tries to evaluate and compare the response of composite mechanics models.

On one hand, Worldwide Failure Exercises focused on plane stress problems. However, on the other hand, Puck’s theory was rarely evaluated for predicting translaminar failure, which involves 3D stress state like in tapered structures shown by Figure 1. These structures are widely used in rotorcraft industries in primary and vital parts. In fact, the shown type of lay-up is also very similar to the one found on joints and repairs.

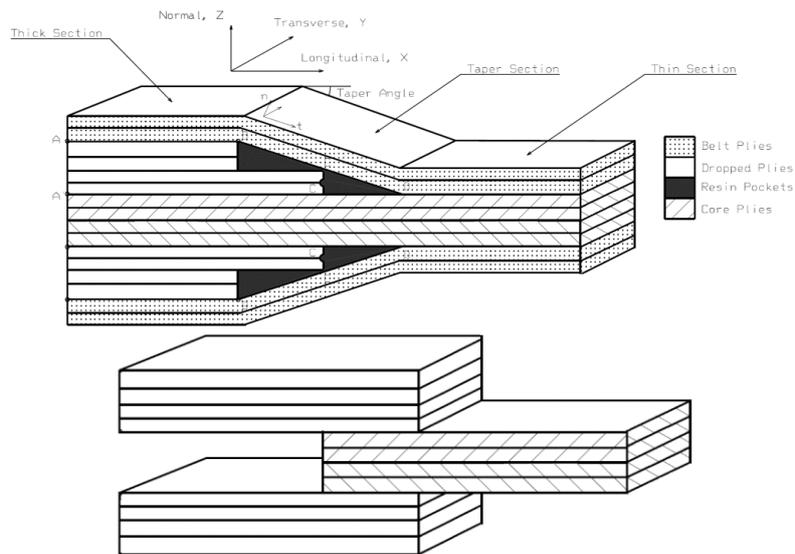


Figure 1 – Tapered structure and typical double lap joint (evident similarity)

Based on the scenario pointed above, the present work aims to propose a new framework for evaluating onset and propagation of failure (cracks) on aeronautical tapered structures made of composite materials by using XFEM and Puck’s failure theory. Thus, the computational model is composed by two main modules: (1) onset/initialization and (2) propagation of failure. Therefore, a methodology is developed in order to determine when the crack takes place, as well as the angle established for the propagation of the crack.

## 2 METHODS

To correctly reproduce the behavior of a tapered structure, it is necessary to model with good accuracy matrix behavior as well as off-the-plane stresses. In order to proper model it, the present work invokes Mohr’s assumption, where the collapse of the matrix is dependent on the stress state acting on the plane in which failure occurs.

On the other hand, fiber failure mechanisms are very brittle and abrupt. Since the fibers are the main responsible for providing stiffness in a composite structure, their failure strongly modifies the stress/strain field and quickly reduces its strength, which implies stress concentration and convergence problems on FEA (Finite Element Analysis).

In order to overcome singularities and computational issues, which appear near the failure (crack), solution via eXtended Finite Element Method (XFEM) was applied with Matzenmiller's theory, which was used to simulate mechanisms in the longitudinal direction, and Puck's theory, which was used to simulate failure in the transverse and shear directions [3]. In addition, a non-local approach was adopted to avoid premature stops in calculus due to convergence problems when fiber-failure, following modifications on the work proposed by [4]. The material model was previously implemented into Abaqus UMAT subroutine, and full details can be seen in [5, 6]) Initiation and propagation routines are organized as shown by the analysis flowchart in Figure 2. In fact, in the present work, it was evaluated a part of the new framework for simulating onset and propagation of failure (cracks) on aeronautical tapered structures made of composite materials.

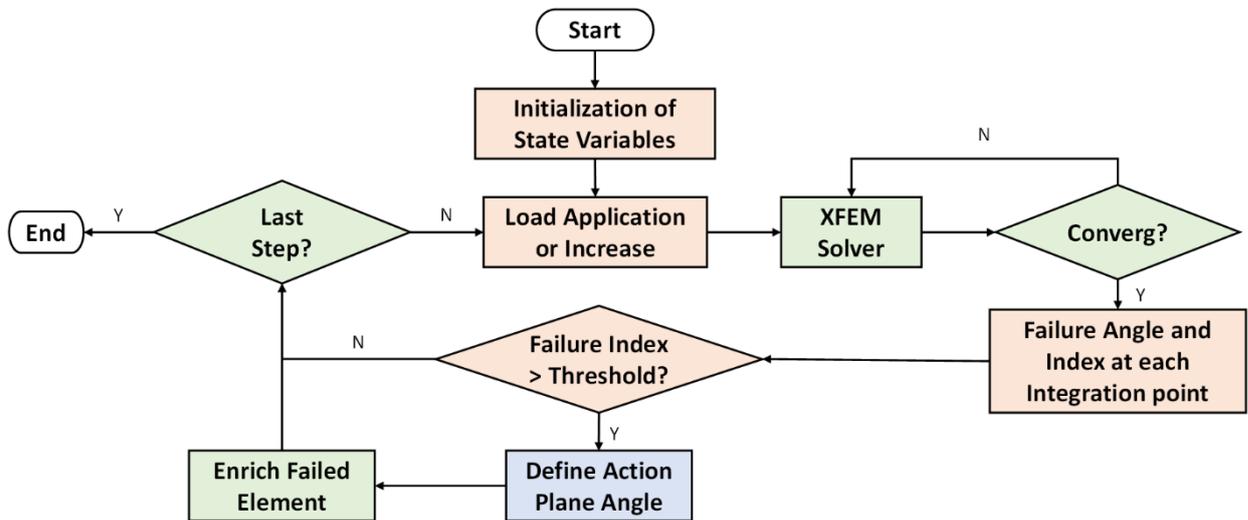


Figure 2 – Analysis flowchart – in red: UMAT, in blue: UDMGINI, in green: Abaqus Default

## 2.1 Crack Initiation

The main difficult for implementing Puck's theory is the proper determination of the action plane. The action plane is the basis of Puck's theory and is defined as the plane in which the Puck's stresses are maximum. In this plane, the failure index from the criterion assumes its maximum value. An example of this plane is shown in Figure 3.

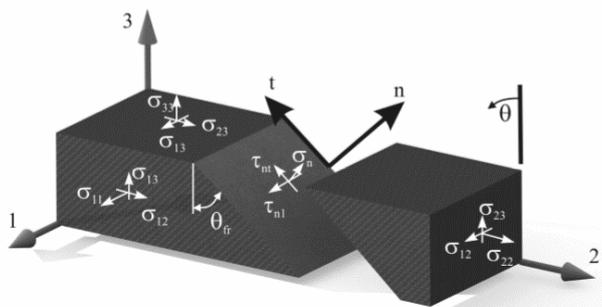


Figure 3 – Puck's theory: Action plane with its angle [7]

The action plane inclination in 2D problems may be done analytically [8], but for 3D problems, it is necessary to use numerical methods to determine the angle. In the present work, routines were implemented into Abaqus – User Defined Damage Initiation (UDMGINI). According to these routines, the user must provide the angle and threshold for crack onset [9].

### 2.1.1 Action plane inclination: Puck’s approach

The action plane angle may be determined through “brute force”, i.e. a sweep made through all possible angles and the index of Inter-Fiber Failure (IFF) is calculated for each of them. The highest value is used to determine the critical angle and the inclination angle, considering the respective failure index. This method is extremely computationally inefficient, even though it is assured that the right angle will be reached. To increase precision, it is enough to take smaller steps for sweeping the angles, taking into account that higher number of steps and amount of calculations are required.

Figure 4 displays the failure index plot against the failure angle. A sweep from  $-90^\circ$  to  $+90^\circ$  is performed taking  $1^\circ$  step. As analysis evolves, failure indexes assume higher values. In this analysis, there is no crack and results are shown up to 90% of complete analysis.

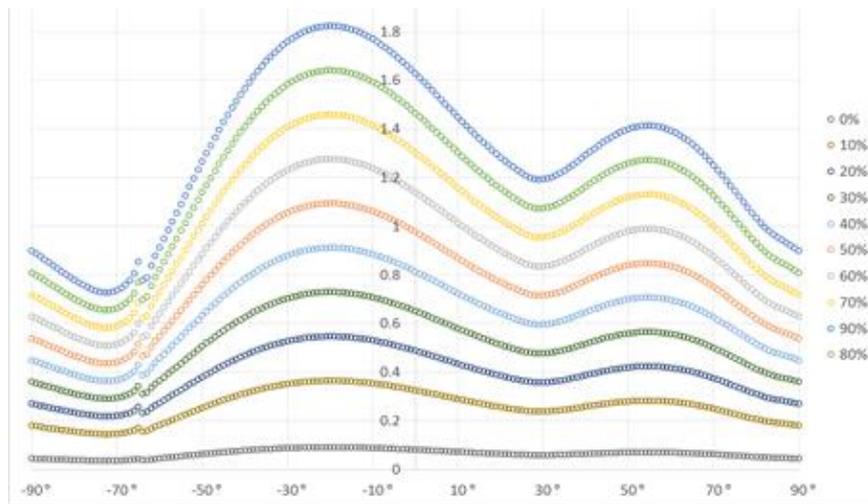


Figure 4 – Example of calculation of action plane inclination as proposed by Puck’s Theory. Index of failure (non-dimensional) behavior against angle of inclination (degrees) along analysis completion (up to 90%)

### 2.1.2 Action plane inclination: Proposed approach

As shown by Figure 4, there are multiple points of maxima and minima locally. Therefore a robust approach was proposed in order to increase efficiency and avoid this problem. Following the works of [7], it was implemented the approach based on the so called “golden section algorithm”. Thus, the interval of interest, in this case  $[-90,90]$ , was subdivided as per the golden ratio shown in Eq. (1):

$$\varphi = \frac{1 + \sqrt{5}}{2} \quad (1)$$

The subdivision is performed by the pseudo-algorithm shown by Figure 5.

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Define  $\theta_1 = -90$  and  $\theta_2 = 90$ 

Define  $\theta_3 \mid \frac{\theta_2 - \theta_3}{\theta_3 - \theta_1} = \varphi$ 

Define  $\theta_4 \mid \frac{\theta_3 - \theta_1}{\theta_4 - \theta_3} = \varphi$ 

Calculate  $IFF(\theta_4)$  and  $IFF(\theta_3)$ 

If  $IFF(\theta_4) > IFF(\theta_3)$ ,  $\theta_1 = \theta_3$  and repeat

Else  $\theta_2 = \theta_4$  and repeat

Iterate until stop criterion is reached
    
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Figure 5 – Pseudo-algorithm

The golden section method was expanded taking into account the hypothesis from [10], which states that no extrema appear closer than  $25^\circ$  from each other. Taking into account this hypothesis, the whole interest interval is divided in  $9 \times 20^\circ$  and, for each subinterval, the golden section method was applied. Nine values were obtained and compared. Hence, the maximum among them is considered the global maximum as shown by Figure 6.

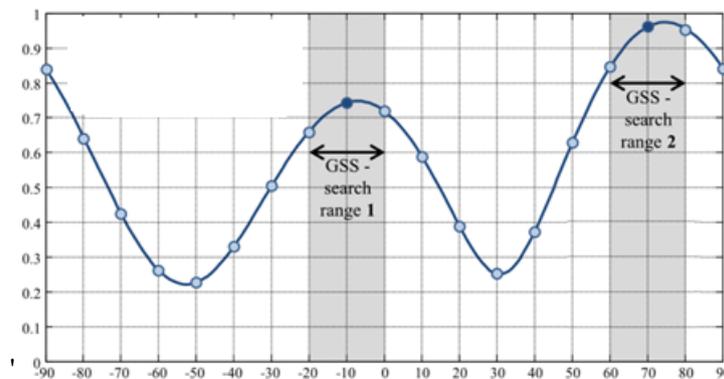


Figure 6 – Schematic representation of the enhanced golden section method

In order to exemplify the performance enhance by using the proposed approach, Table 1 shows results for each level of precision.

Table 1 – Performance comparison.

Approach	Precision Level		
	1.0°	0.5°	0.1°
Puck (reference)	<b>1.000</b>	2.000	10.00
Golden Section	0.061	0.072	0.089
Enhanced Golden Section	0.550	0.650	0.444

### 3 RESULTS AND DISCUSSION

In order to evaluate a part of the new framework for simulating onset and propagation of failure (cracks), two case studies were developed.

### 3.1 Case Study 1: 90° layers under tension and compression

When compression transversal loading is applied on unidirectional composite, according to experimental results presented by [11], it is expected that the angle for failure occurs around 54° [3]. On the other hand, when the same structure is loaded under tension transversal loading, it is expected that the angle for failure occurs at 0°, i.e. parallel to the fibers. Therefore, a computational model was developed in order to simulate those tests. Figure 7 shows the model for tension transversal loading, which was simulated by using hexahedral elements (C3D8 – Abaqus).

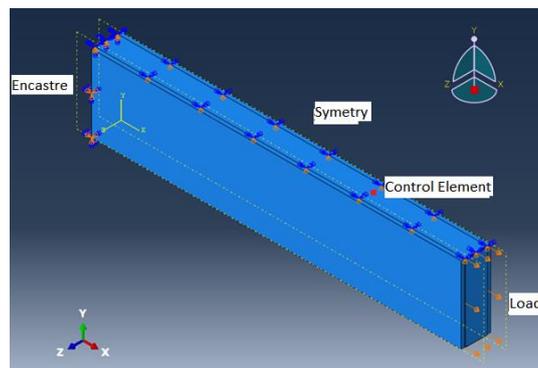


Figure 7 – Model under tension transversal loading.

The details for computational models are shown described in Table 2.

Table 2 – Details of computational analyses for Case Study 1

Stacking (XY plane, X as reference)	Loading	Expected Behavior
[[0°] <sub>2</sub> , [90°] <sub>6</sub> ] <sub>s</sub>	Tension	Failure angle at 0°
[[0°] <sub>2</sub> , [90°] <sub>6</sub> ] <sub>s</sub>	Compression	Failure angle around 54°

Results obtained at the control element for the compression analyses are shown in Figure 8.

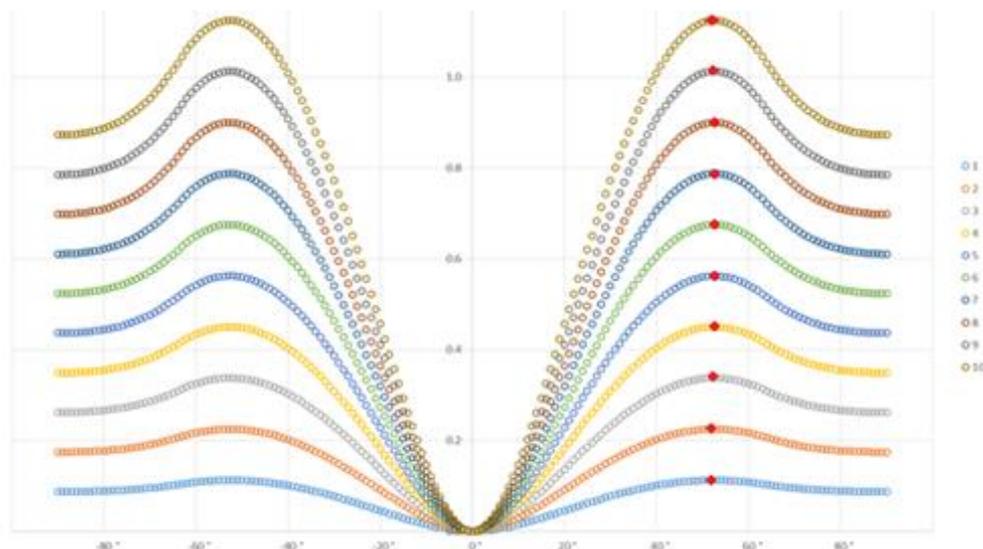


Figure 8 – Failure index for each angle: compression analyses. Failure index (non-dimensional) behavior against angle (degrees) for each of the ten steps of the analysis.

In Figure 8, the red dots represent values obtained by the proposed approach compared to the curves, which present the results for Puck’s approach. It can be noticed that there are two global maxima, which were found to be around  $53^\circ$  as well as in symmetric position, which was not identified by the proposed approach. This occurs since the routine UDMGINI takes only one value, and the positive one was adopted. In a similar way for instability in bulking, the answer is independent of the choice of the path (symmetrical behaviors). Figure 9 shows the results for the tension analyses.

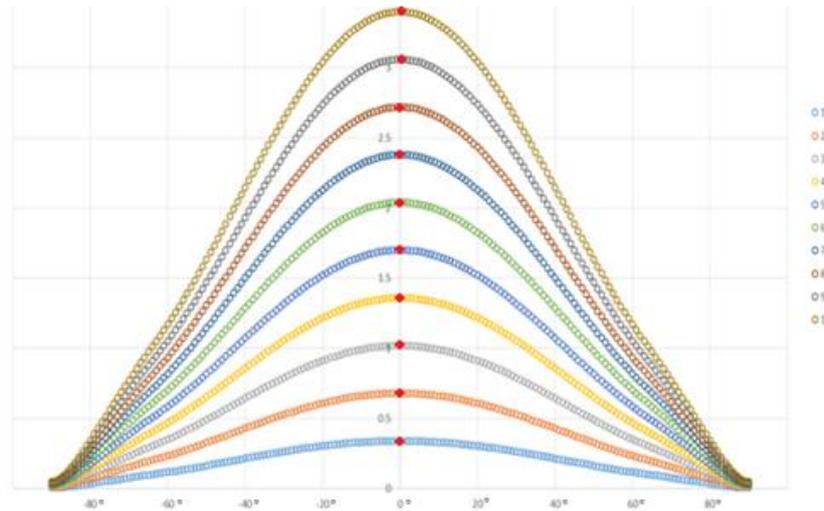


Figure 9 – Failure index for each angle: tension analysis. Failure index (non-dimensional) behavior against angle (degrees) for each of the ten steps of the analysis.

Again, the results are exactly as expected. Failure occurs at  $0^\circ$  i.e. in a plane parallel to the fibers and perpendicular to the loading. Red dots show the results obtained by the proposed approach in opposition to the curve, again obtained through Puck’s approach

### 3.2 Case Study 2: Compact tension specimen

Finally, to check the implementation of the initiation routine in the XFEM framework with material model, a compact tension specimen with displacement imposed was simulated. Three different layups were considered and are detailed in Table 3.

Table 3 – Details of computational analyses for Case Study 2

Stacking (XY plane, Y as reference)	Loading	Expected Behavior
$[0^\circ]_{16}$	Tension	Failure along the fibers direction at $0^\circ$
$[90^\circ]_{16}$	Tension	Failure along the fibers direction at $90^\circ$
$[45^\circ]_{16}$	Tension	Failure along the fibers direction at $45^\circ$

Since the strength in fiber direction is much higher than in transversal, it is expected the failure to go through the structure following the fibers as it can be seen in Figure 10(A), (B) and (C).

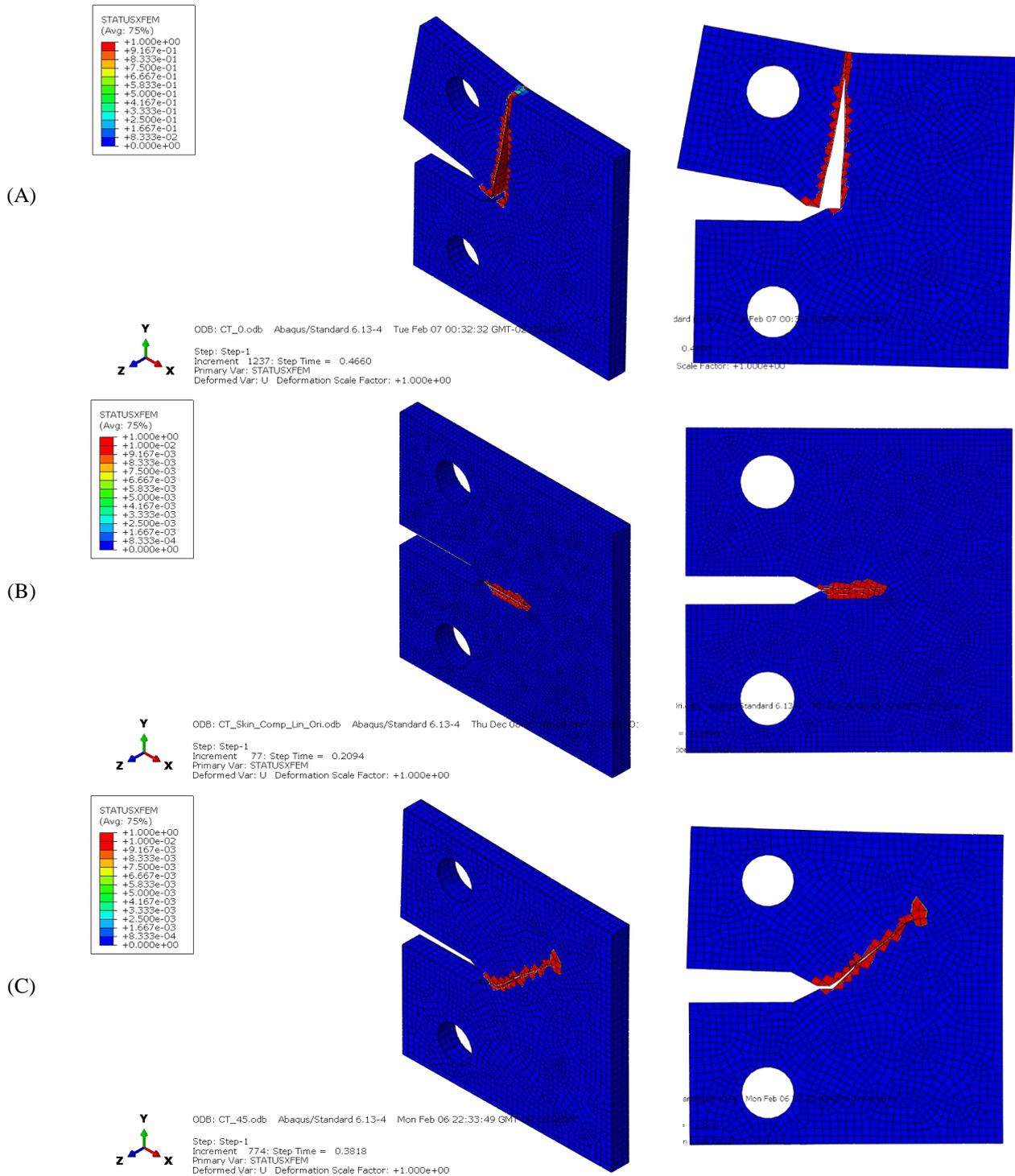


Figure 10 – Status XFEM compact tension results: (A) 0°, (B) 90° and (C) 45° specimen. Non dimensional Boolean indicating enrichment status

## 4 CONCLUSION

From the obtained results, the Puck's action plane theory used to determine initiation and direction angle of propagation in XFEM framework shows itself promising. Computational analyses held in Abaqus subroutines using the proposed methodology are consistent to the literature results. The computational effort required by the new approach is up to 100 times smaller than when using classical approach. Experimental results are already being used and validation is promising.

## 5 ACKNOWLEDGEMENTS

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