COBEM 2023 27TH INTERNATIONAL CONGRESS OF MECHANICAL ENGINEERING

ABCM Brazilian Society of Mechanical Sciences and Engineering
27th ABCM International Congress of Mechanical Engineering
December 04-08, 2023, Florianópolis-SC, Brazil

COB-2023-0773
AN ANALYSIS OF A 4-DOF KINEMATICALLY REDUNDANT PLANAR PARALLEL MECHANISM

Luan Meneghini
Henrique Simas
Universidade Federal de Santa Catarina, Rua Engeheiro Agronômico Andrei Cristian Ferreira, s/n - Trindade, Florianópolis - SC, Brasil
luan.meneghini@posgrad.ufsc.br, henrique.simas@ufsc.br

Roberto Simoni
Universidade Federal de Santa Catarina, Estrada Dona Francisca, 8300 - Bloco U - Zona Industrial Norte, Joinville - SC, Brasil
roberto.simoni@ufsc.br

Abstract. *A mobile platform and a fixed base must be connected by at least two serial kinematic chains to form a parallel manipulator (PM). PMs have many benefits over serial ones, including greater load capability, stiffness, accuracy, dexterity, inertia, and so on. Due to the wide application of PMs in industries, simulators, force sensors, machine tools and others, there have been significant advances in recent years. However, some challenges, such as its complex mathematical models, complex control and manufacturing, are inherent in this type of architecture. The main disadvantages of PMs are the workspace singularity regions and their limited workspace. It is known that the workspace of a parallel manipulator is typically smaller than that of a serial manipulator. Redundancy in PMs can be taken into account to reduce workspace constraints, improve rotational capabilities and avoid singularities. In this paper, redundancy in PMs is revised, focusing on singularity and workspace. Redundancy was employed by some authors to create a novel planar 4-DoF redundant parallel manipulator with unlimited rotational capabilities that was used as a case study for analysis and discussions. The parallel manipulator has a redundant leg that promotes singularity avoidance and contributes to the unlimited rotational capability of the movable platform. The positional and differential models are developed, as are the singularities, which are mapped by the Jacobian matrices. Also, the workspace is evaluated by means of a geometrical approach. Finally, a new proposal of the PM studied was established using self-aligning, facilitating the assembly of the prototype.*

Keywords: *parallel manipulator, redundancy, unlimited rotation.*

1. INTRODUCTION

Parallel manipulators are closed-loop mechanisms that consist of at least two serial chains connecting the fixed base to the movable platform. PMs have many advantages over serial chains, such as load capacity, stiffness, precision, inertia and suitable positional actuator arrangements. There are a several applications to PMs, including manufacturing, testing machines, simulators and pick-and-place manipulation (Briot and Bonev (2008)). However, they present limited workspace and complicated singularities, according Staicu (2009).

Redundant robots allow for additional tasks during their operation. Such robots have greater versatility and possibilities of movement in the execution of a task when compared with conventional robots, according Simas and Di Gregorio (2018).

Hunt (1983) proposed the 3-R$\underline{P}$R planar PM, where R denotes a revolute joint and P denotes a prismatic joint, and the underlined letter represents an actuated joint. This architecture has also been studied by other authors: kinematic design by Gosselin and Angeles (1988); workspace analysis by Gosselin and Jean (1996); direct kinematic by Merlet (1996); singularity analysis by Daniali *et al.* (1995); dynamics by Gosselin (1996) and others. This 3-R$\underline{P}$R planar PM has very good properties, however, some of the drawbacks of this architecture is its limited range of rotational motion and singularities.

So, Gosselin *et al.* (2015) proposed a new approach based on the 3-R$\underline{P}$R structure, introducing a kinematically redundant architecture, where completely avoiding singularities and providing an unlimited rotational range of motion. The mechanism obtained has four actuators and four degrees of freedom, 4-DoF.

With the new 4-DoF manipulator proposal by Gosselin *et al.* (2015), an analysis of mobility and constraints is made through mobility equations and Reshetov's method. The kinematic position and differential equations were deducted, an important singularity analysis and workspace evaluation were done, and discussions about them were held.

This paper is structured as follows: Section 2 presents a brief redundancy review and how it influences some properties. Section 3 shows discussions about a case study: kinematics, singularities and workspace evaluation of a 4-DoF redundant planar parallel mechanism. Section 4 exhibits the new proposal of the 4-DoF manipulator with self-aligning. Section 5 presents the conclusions of this work.

2. A BRIEF REVIEW OF REDUNDANCY OF PMS

In robotics, redundancy is frequently related only to kinematic redundancy, which is linked to the robot geometry and/or the task that needs to be executed. With the structures becoming more complex, like in PMs, the concept of actuation redundancy starts to be used. So, redundancy in parallel manipulators can be of two main types: kinematic redundancy (or mobility redundancy) and actuation redundancy (Notash and Huang (2003)).

The concept of kinematic redundancy is presented by many authors from different perspectives, there isn't one unique definition in the literature. Conkur and Buckingham (1997) defines the concept of kinematic redundancy in robots according to the topology and the dimensions of task space. When the mobility of the mechanism is greater than the dimension of the coordinate system, there is kinematic redundancy. This redundancy can be quantified by the difference between mobility and the coordinate system.

According to definitions by Dasgupta and Mruthyunjaya (1998), if a manipulator possesses degrees of mobility that outnumber the dimensions of the task space, the redundancy thus obtained is called kinematic redundancy. On the other hand, if the number of actuators is larger than mobility, the resulting redundancy is called actuation redundancy.

Parallel manipulators whose number of actuators is greater than that of the mobility are classified as parallel manipulators with redundant actuation Cheng *et al.* (2001). Actuation redundancy makes the design of the parallel structure more complicated and introduces challenges in controlling closed-chain mechanisms.

Redundancy can be improved some parameters of parallel manipulators, like stiffness, singularity avoidance, and increase of workspace.

Wu *et al.* (2007) presented the relationship between the singular configuration and the stiffness of the parallel manipulator. Compared to the corresponding non-redundant parallel manipulator without the redundant link, the redundantly actuated parallel manipulator has better dexterity and higher stiffness. The stiffness of a robot deteriorates near the robot's singular poses. Redundancy can decrease or eliminate parallel singularities and therefore enhance the stiffness of the robot, in accordance with Mostashiri *et al.* (2018). Cheng *et al.* (2001) also confirm that redundancy can be used in PMs to avoid, reduce, or eliminate singular configurations.

Zanganeh and Angeles (1994) proposed the earliest mechanism that took advantage of kinematic redundancy to increase the PM workspace. A 6-DoF $3-\underline{RP}RR$ redundant PM was obtained by adding an active revolute joint to all three legs of a $3-\underline{P}RR$ mechanism by Ebrahimi *et al.* (2008). The results obtained by Ebrahimi *et al.* (2008) shows that the dexterous workspace is particularly improved on the redundant mechanism when compared to its non-redundant counterpart.

There are a lot of redundantly parallel manipulators in literature with different approaches. However, the smart trick in the grip function proposed by Gosselin in 4-DoF planar PM attracted attention. So, this manipulator was studied in this paper and their parameters were discussed.

3. CASE STUDY: 4-DOF REDUNDANT PLANAR PARALLEL MANIPULATOR

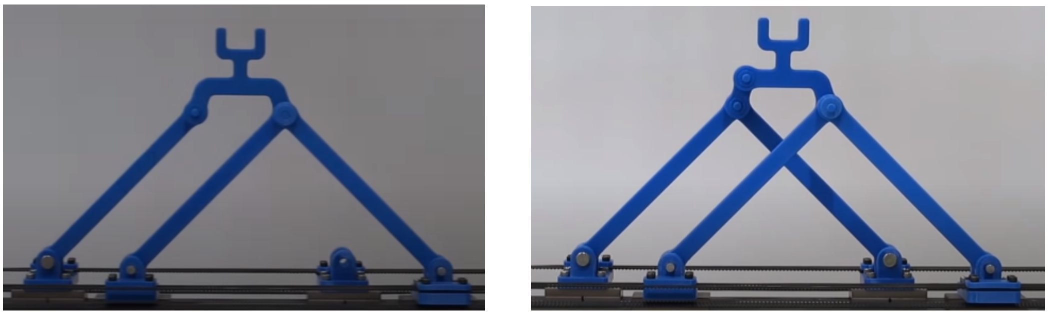
In the Figure 1(a) it is possible to observe the 3D printed prototype of the planar PM ($\lambda = 3$) 3$\underline{P}$RR manipulator. This manipulator has linear actuators and only revolute passive joints with $n = 8$ links and $j = 9$ joints. Considering the $\lambda = 3$, applying the Modified Grübler–Kutzbach criterion (Equation 1) the mobility is three, $M = 3$, as can be shown in Equation 2. In this planar case, there are not redundant constraints. In Figure 1(b) it's possible to observe a modification in the structure of Figure 1(a). This modification was added one leg connecting the base to the end-effector, as proposed by Gosselin *et al.* (2015). There are three links and four kinematic pairs additional, so applying the Equation 1 can be obtained four degrees of freedom in the planar space, Equation 3.

$$M = \lambda(n - j - 1) + \sum f_i + C_N \tag{1}$$

$$M = 3(8 - 9 - 1) + 9 + 0 = 3 \tag{2}$$

$$M = 3(11 - 13 - 1) + 13 + 0 = 4 \tag{3}$$

To determine the constraints and extra mobilities, it is possible to apply the method proposed by Reshetov. The goal of the Reshetov's analysis is to eliminate the redundant constraints by increasing the degrees of freedom of a parallel mechanism to improve design while avoiding overconstraints. More information can be found in Meneghini *et al.* (2021).



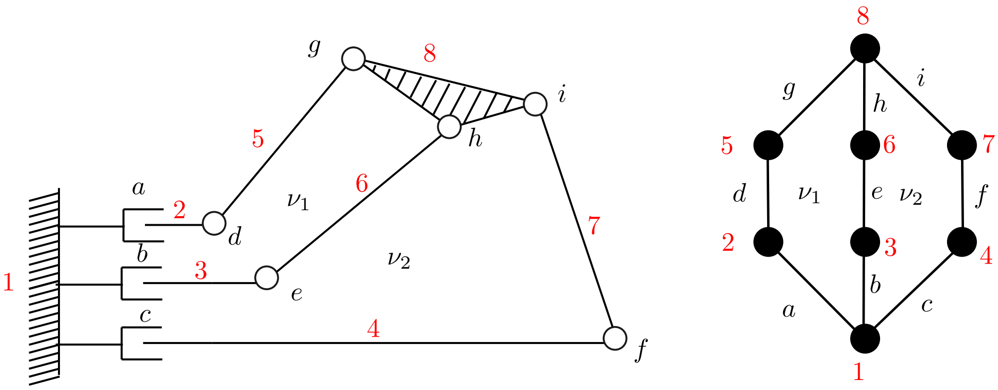
(a) (b)

Figure 1. Prototypes of Planar PMs by Gosselin *et al.* (2015):
(a) 3-DoF. (b) 4-DoF

Reshetov is a method where the freedoms and constraints of a mechanical system are organized into a table. The table is composed of two columns that correspond to rotational and translation freedoms. Each column is composed of three lines with axes x, y and z. Distributing the freedoms of the system in the table is possible given the mobilities and constraints of mechanism. To replace a linear freedom, it must be on an axis perpendicular to the axis of the rotating freedom that can be used. Once substitutions for missing translations are made, the number of missing freedoms indicates the number of redundant constraints. The number of extra freedoms, after satisfying the first condition, indicates the mobility of the mechanism.

There are other methods to identify the redundant constraints, such as Davies's Method. The Reshetov's Method is a systematic and simple method. The mechanisms are represented in a planar coordinate system, which is expected to identify three constraints in each loop. These constraints are the freedoms that don't belong to the planar coordinate system: rotation around axes x and y and translation in axle z. In Section 4, a proposal for a self-aligning mechanism is presented and discussed.

The structural representation of a 3-DoF manipulator is shown in Figure 2(a) and the graph representation is shown in Figure 2(b). Note that the mechanism has two loops, we considered the joints a, b, d and g belongs to the loop ν_1 and the joints c, f, i, h belongs to loop ν_2. Observing that with Reshetov's method, it is possible to conclude that the mobility of the mechanism is correct and there are only 3 constraints per loop, which are the constraints of the spatial coordinate system $\lambda = 6$ that aren't present in the planar coordinate system $\lambda = 3$. The analysis is shown in the table in Figure 3(a). The 3D model of a 4-DoF PM presented in Figure 1(b) has its structural representation shown in Figure 4(a) and it's graph representation in Figure 4(b). So, the same Reshetov's analysis was applied to 4-DoF mechanism and is shown in Figure 3(b). The method was applied considering: ν_1 formed by joints a, j, i, e, ν_2 by f, k, l, g, b and ν_3 by h, m, c, d. So, in this analysis, it is possible to confirm the mobility of the mechanism and see the planar constraints.



(a) (b)

Figure 2. 3-DoF planar PM: (a) Structural representation. (b) Graph representation.

3.1 Differential Kinematic

Kinematics studies the motion of bodies without consideration of the forces or moments that cause the motion. Robot kinematics refers to the analytical study of the motion of a manipulator. Formulating suitable kinematics models for a robot is crucial for analyzing the behaviour of the manipulator. The robot's kinematics can be divided into forward kinematics and inverse kinematics. The forward kinematics problem is straightforward, and there is no complexity in

(a)

$M = 2$

R_x		T_x	a, b
R_y		T_y	e
R_z	d, g, e	T_z	

$C_N = 3$

$M = 1$

R_x		T_x	c
R_y		T_y	h
R_z	f, i, h	T_z	

$C_N = 3$

(b)

$M = 1$

R_x		T_x	a
R_y		T_y	i
R_z	e, i	T_z	

$C_N = 3$

$M = 2$

R_x		T_x	b
R_y		T_y	g
R_z	f, k, l, g	T_z	

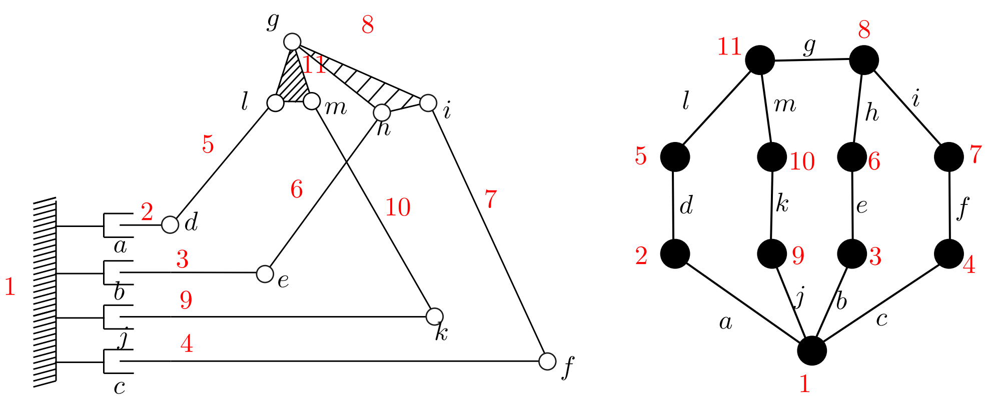
$C_N = 3$

$M = 1$

R_x		T_x	c, d
R_y		T_y	m
R_z	h, m	T_z	

$C_N = 3$

Figure 3. Reshetov's Method: (a) 3-DoF mechanism. (b) 4-DoF mechanism.



(a) (b)

Figure 4. 4-DoF planar redundant PM: (a) Structural representation. (b) Graph representation.

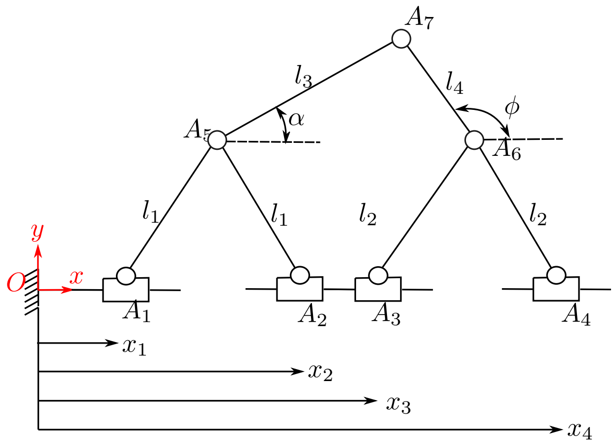


Figure 5. Nomenclature adopted in 4-DoF PM to analysis.

deriving the equations. Hence, there is always a forward kinematics solution for a manipulator. Inverse kinematics is a much more difficult problem than forward kinematics. The solution of the inverse kinematics problem is computationally expansive and generally takes a very long time in the real-time control of manipulators. Singularities and non-linearities that make the problem more difficult to solve (Kucuk and Bingul (2006)).

According Kucuk and Bingul (2006), two main techniques for the inverse kinematic problem are commonly used: analytically according to configuration data and based on the numerical techniques. There are two approaches yo the analytical method: geometric and algebraic solutions. Geometric approach is applied to the simple robot structures, such as 2-DoF planar manipulator or less-DoF. For the manipulators with more links and whose arms extend into three dimensions or more, geometry gets much harder. In this case, the algebraic approach is more beneficial for the inverse kinematics solution, as cited by Kucuk and Bingul (2006).

There are some difficulties in solving the inverse kinematics problem when the kinematic equations are coupled and multiple solutions and singularities exist. Mathematical solutions for an inverse kinematic problem may not always correspond to the physical solution, and the method of its solution depends on the robot structure, as reported by Kucuk and Bingul (2006).

The Figure 5 describes the nomenclature adopted in this work. The variables x_1, x_2, x_3 and x_4 represent the displacement of prismatic joints in the direction of the x axle to points of revolute joints A_1, A_2, A_3 and A_4, respectively. There are other revolute joints in A_5, A_6 and A_7. The lengths of links are denoted by: l_1, l_2, l_3 and l_4. The angles shown are α and ϕ, correspond to the orientation of movable platform and the fourth degree of freedom of the manipulator.

Heron defines the area of any triangle with sides a, b and c, in Equation 4, where $s = (a + b + c)/2$. If we consider the lengths $a = b = l$, the Equation 4 can be rewritten as Equation 5. The most known formulation to calculate the area of a triangle is $bh/2$, where b is the base and h is the height of the triangle. If we relate this formulation to Equation 5 and consider $b = c$ we find the Equation 6.

$$A = \sqrt{s(s-a)(s-b)(s-c)} \tag{4}$$

$$A = \sqrt{\frac{1}{16}4l^2c^2 - c^4} \tag{5}$$

$$h^2 = l^2 - \frac{c^2}{4} \tag{6}$$

So this base formulation presented above is the way to find the relations between input and output parameters. Considering:

$$A_{5x} = \frac{x_1 + x_2}{2} \tag{7}$$

$$A_{5y}^2 = l_1^2 - \left(\frac{x_2 - x_1}{2}\right)^2 \tag{8}$$

$$A_{6x} = \frac{x_3 + x_4}{2} \tag{9}$$

$$A_{6y}^2 = l_2^2 - \left(\frac{x_4 - x_3}{2}\right)^2 \tag{10}$$

Note that point A_7 can be obtained according to points A_5 and A_6, respectively, as shown in Equation 11 and Equation 12.

$$A_7 = A_5 + l_3 \begin{bmatrix} cos(\alpha) \\ sin(\alpha) \end{bmatrix} \tag{11}$$

$$A_7 = A_6 + l_4 \begin{bmatrix} cos(\phi) \\ sin(\phi) \end{bmatrix} \tag{12}$$

If we decoupled Equation 11 and Equation 12 in components of the x and y axes, we have:

$$A_{6x} + l_4 cos(\phi) = A_{5x} + l_3 cos(\alpha) \tag{13}$$

$$A_{6y} + l_4 sin(\phi) = A_{5y} + l_3 sin(\alpha) \tag{14}$$

By deriving the Equations 7, 8, 13 and 14 and organizing them in matrix form, it is possibly to find the Jacobian's matrices of the PM. The point of the end-effector was considered to correspond to joint A_5.

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{x_2-x_1}{4} & -\frac{x_2-x_1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{x_4-x_3}{4} & -\frac{x_4-x_3}{4} \end{bmatrix} \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & A_{5y} & 0 & 0 \\ 1 & 0 & -(A_{6y} - A_{5y}) + l_3 sin(\alpha) & -(A_{6y} - A_{5y} \\ 0 & A_{6y} & A_{6y}((A_{6x} - A_{5x}) - l_3 cos_{\alpha}) & A_{6y}(A_{6x} - A_{5x}) \end{bmatrix} \begin{bmatrix} \dot{A_{5x}} \\ \dot{A_{5y}} \\ \dot{\phi} \\ \dot{\alpha} \end{bmatrix} \tag{15}$$

3.2 Singularity analysis

The study of the kinematics of mechanical systems leads inevitably to the problem of singular configurations. These configurations are defined as those in which the Jacobian matrices are involved. Those matrices, relating the input speeds with the output speeds, become rank deficient. Gosselin *et al.* (1990) says that singularities correspond to configurations of the system that are usually undesirable, since the degree of freedom of the system changes instantaneously.

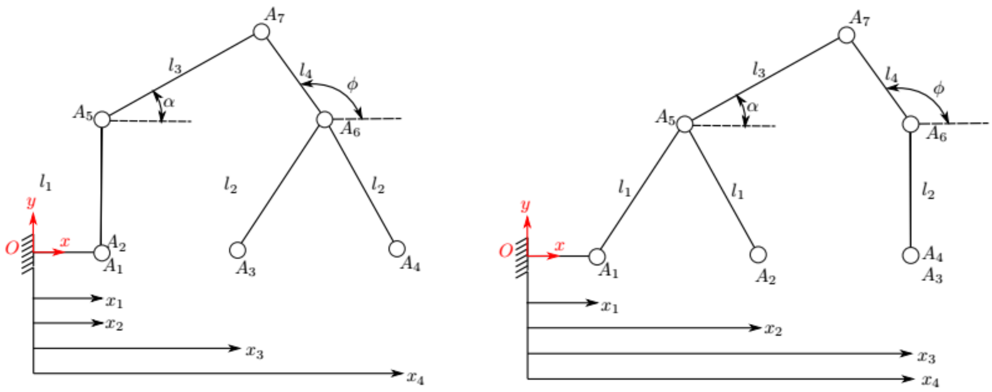
In engineering, a singularity is a configuration of a system in which the subsequent behavior cannot be predicted. The corresponding motions, forces, or other physical parameters modeled by these equations become undetermined Gogu (2008).

In a singular configuration, infinitesimal changes in input/output velocities or forces can produce huge variations in outputs/inputs. In singular configurations, the mechanism loses the ability to transmit resolute motion and force and becomes uncontrollable; the kinematic and static behavior of the mechanism change dramatically; the mechanism gains extra degrees of freedom and loses its stiffness. Gogu (2008) confirm that the actuator forces may become very intense, this might lead to robot failure or permanent damage to the robot and surrounding equipment, including a breakdown of the mechanism.

Singularities of PMs fall into two basic classes (Kong and Gosselin (2001)): the first one is called a stationary, kinematic, twist, or inverse kinematic singularity, in which the moving platform loses one or more degrees of freedom. The second one is called uncertainly, static, wrench or forward kinematic singularity, in which the moving platform may undergo infinitesimal or finite motion when all inputs are locked.

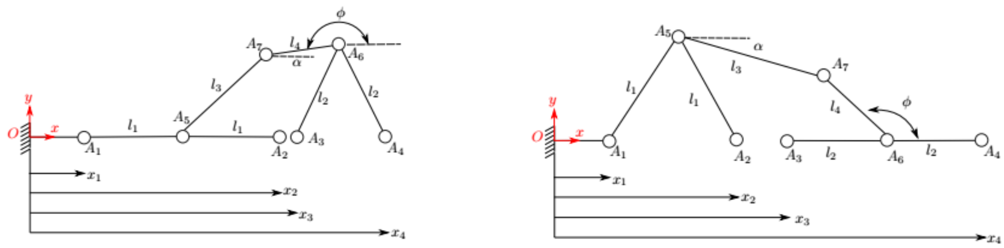
Based on the Jacobian matrix, when its determinant is zero, the manipulator is in singular position. So, if we observe the elements J_{21}, J_{22}, J_{43} and J_{44} in Equation 15, we note that when $x_1 = x_2$ and $x_3 = x_4$ the determinant tends to zero, so the manipulator is in a singular position. The Figure 6 can exemplify this position.

Analyzing the inverse Jacobian matrix, we can observe that the PM will be in a singular configuration if A_{5y} and A_{6y} is null. So if this situation is satisfied, we can conclude that correspond to $x_2 = x_1 + 2l_1$ and $x_4 = x_3 + 2l_2$. The Figure 7 exemplifies this situation.



(a) (b)

Figure 6. Forward kinematic singularities: (a) $x_1 = x_2$. (b) $x_3 = x_4$.



(a) (b)

Figure 7. Forward kinematic singularities: (a) $x_1 = x_2$. (b) $x_3 = x_4$.

3.3 Workspace

Kumar (1992) defines the reachable workspace as the volume or space within a reference point on the hand or end-effector that can be made to coincide with any point in space.

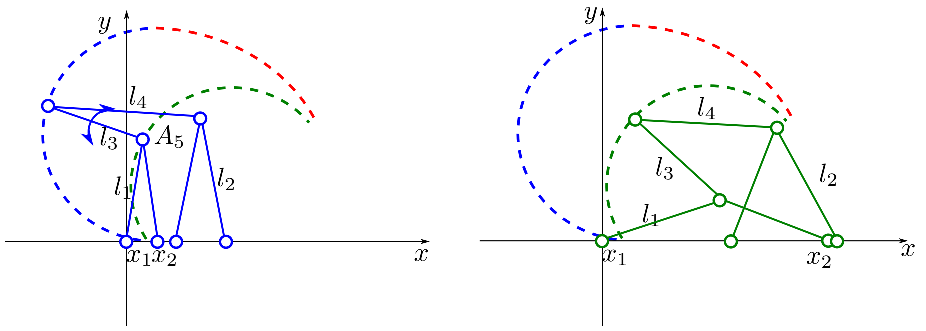
For a parallel manipulator, workspace limits are due to the bounded range of linear actuators, mechanical limits on passive joints, and interference between links. Merlet (1997) says that one important step in the design of a parallel manipulator is to define its geometry according to the desired workspace.

In line with Agrawal (1991), the most common technique for determining the reach of a reference point chosen on the end-effector of an in-parallel system is direct numerical simulation. The independent joint variables in the system are incremented in steps, and using the forward kinematics, the position of the end-effector reference point is determined. Such a method is usually iterative and does not require any special understanding of the system.

The workspace of a 4-DoF redundant PM is shown in the figures below. The boundaries of the workspace are defined by three semi-arcs, the first semi-arc is formed by the dotted blue line by the radius of length of link l_3 around the joint A_5 when the prismatic joints x_1 and x_2 are in the minimum displacement, as shown in the same Figure 8.a. The other boundary is formed by a dotted green semi-arc by a radius of length of link l_3 also around joint A_5, but when the prismatic joint x_2 is in the maximum displacement, as shown in Figure 8.b. The last semi-arc that defines the workspace is the dotted red semi-arc, which is formed by the radius of length of aligned links l_1 and l_3 rotated around the base of the leg l_1, as shown in Figure 9.a. But if the prismatic joints displace to the right of the coordinate system, a rectangular area will be formed into the workspace, as shown in Figure 9.b. So it is possible to observe a complex surface that defines the workspace of this planar parallel manipulator.

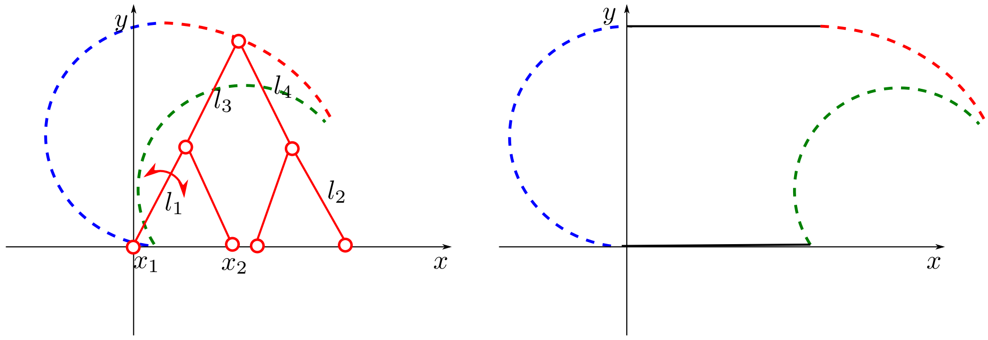
4. 4-DOF SELF-ALIGNING PROPOSAL MECHANISM

In this section, a new proposal for a 4-DoF self-aligning PM is presented. As it was shown, the mechanism in planar coordinate system presents three constraints per loop. However, it is known that the prototype mechanism is built in the spatial coordinate system. The fabrication process will never be perfect, like the designed parts. There are a lot of errors that could be generated by warped links. These kinds of problems affect the assembly of the mechanism. So, if more freedom is given in certain joints, it is possible to assemble the mechanism without generating internal forces that can be harmful to the correct operation of the machine. These extra freedoms don't change the mobility of the mechanism if it's well positioned.



(a) (b)

Figure 8. 4-DoF planar redundant PM: (a) Structural representation. (b) Graph representation.



(a) (b)

Figure 9. 4-DoF planar redundant PM: (a) Structural representation. (b) Graph representation.

If we analyze the Reshetov table in Figure 3, it can be seen that there are 9 constraints in the mechanism. So it is recommended that these constraints be replaced by freedoms. These freedoms are added by changing the mechanism's joint type. By changing a rotating joint to a spherical one, for example, we've reduced the number of constraints on this joint from five to just three. It was proposed to replace the revolute joints d, e, h, l by spherical joints and joint f was replaced by a universal joint. With this action, it is possible to guarantee that the mechanism will be assembled without problems, even though there are warped links.

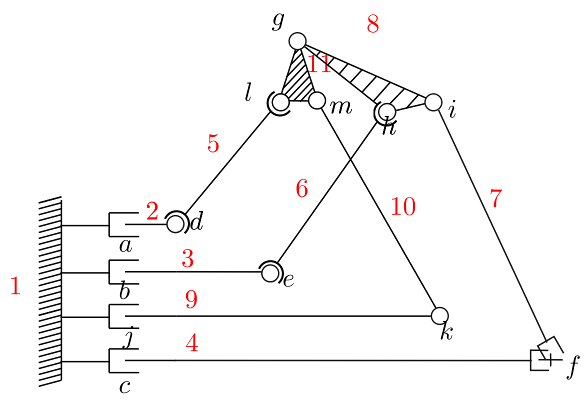


Figure 10. Self-Aligning 4-DoF PM.

5. CONCLUSION

This paper presented a brief review of redundancy in parallel manipulators. Usually there are two kinds: kinematic redundancy and actuated redundancy. In both cases, the application of redundancy in mechanisms brings many advantages, like: singularity avoidance, better stiffness, increased workspace, and so on.

A geometrical approach was applied to develop a kinematic analysis of kinematic redundant PM proposed by Gosselin Gosselin *et al.* (2015). The mechanism has a redundant leg that allows improvements in singularity control, providing unlimited rotational capabilities. The proposed approach allows modeling the redundant leg to provide additional rotational capability and to grip objects as an end-effector.

The mobility of the PM is analyzed as well as its differential kinematics. The Jacobian matrix is obtained and their singular positions are determined. Its workspace is also evaluated using a graphical tool, and it was possible to verify his complex workspace geometry.

To complete this work, some changes in the structure of the 4-DoF mechanism studied were presented. With these changes, it is possible to have a self-aligning mechanism, which improves the assembly process. With the self-aligning version, it is feasible that errors in the manufacturing process of the links don't harm the spatial prototype tests.

6. ACKNOWLEDGMENTS

This work has been developed at Laboratory of Applied Robotics of the Federal University of Santa Catarina, supported by CNPq—Conselho Nacional de Desenvolvimento Científico e Tecnológico (National Council for Scientific and Technological Development) Project 307249/2021-2, Brazil.

7. REFERENCES

Agrawal, S.K., 1991. "Workspace boundaries of in-parallel manipulator systems". In *Fifth International Conference on Advanced Robotics' Robots in Unstructured Environments*. IEEE, pp. 1147–1152.

Briot, S. and Bonev, I.A., 2008. "Accuracy analysis of 3-dof planar parallel robots". *Mechanism and Machine Theory*, Vol. 43, No. 4, pp. 445–458.

Cheng, H., Liu, G., Yiu, Y.K., Xiong, Z. and Li, Z., 2001. "Advantages and dynamics of parallel manipulators with redundant actuation". In *Proceedings 2001 IEEE/RSJ International Conference on Intelligent Robots and Systems. Expanding the Societal Role of Robotics in the the Next Millennium (Cat. No. 01CH37180)*. IEEE, Vol. 1, pp. 171–176.

Conkur, E.S. and Buckingham, R., 1997. "Clarifying the definition of redundancy as used in robotics". *Robotica*, Vol. 15, No. 5, pp. 583–586.

Daniali, H.M., Zsombor-Murray, P. and Angeles, J., 1995. "Singularity analysis of planar parallel manipulators". *Mechanism and Machine Theory*, Vol. 30, No. 5, pp. 665–678.

Dasgupta, B. and Mruthyunjaya, T., 1998. "Force redundancy in parallel manipulators: theoretical and practical issues". *Mechanism and Machine Theory*, Vol. 33, No. 6, pp. 727–742.

Ebrahimi, I., Carretero, J.A. and Boudreau, R., 2008. "Kinematic analysis and path planning of a new kinematically redundant planar parallel manipulator". *Robotica*, Vol. 26, No. 3, pp. 405–413.

Gogu, G., 2008. *Structural synthesis of parallel robots*, Vol. 930. Springer.

Gosselin, C. and Angeles, J., 1988. "The optimum kinematic design of a planar three-degree-of-freedom parallel manipulator".

Gosselin, C., Angeles, J. *et al.*, 1990. "Singularity analysis of closed-loop kinematic chains." *IEEE transactions on robotics and automation*, Vol. 6, No. 3, pp. 281–290.

Gosselin, C., Laliberté, T. and Veillette, A., 2015. "Singularity-free kinematically redundant planar parallel mechanisms with unlimited rotational capability". *IEEE Transactions on Robotics*, Vol. 31, No. 2, pp. 457–467.

Gosselin, C.M. and Jean, M., 1996. "Determination of the workspace of planar parallel manipulators with joint limits". *Robotics and Autonomous Systems*, Vol. 17, No. 3, pp. 129–138.

Gosselin, C., 1996. "Parallel computational algorithms for the kinematics and dynamics of planar and spatial parallel manipulators".

Hunt, K., 1983. "Structural kinematics of in-parallel-actuated robot-arms".

Kong, X. and Gosselin, C.M., 2001. "Uncertainty singularity analysis of parallel manipulators based on the instability analysis of structures". *The International Journal of Robotics Research*, Vol. 20, No. 11, pp. 847–856.

Kucuk, S. and Bingul, Z., 2006. *Robot kinematics: Forward and inverse kinematics*. INTECH Open Access Publisher.

Kumar, V., 1992. "Characterization of workspaces of parallel manipulators".

Meneghini, L., Noal Artmann, V., Simoni, R. and Simas, H., 2021. "Mobility analysis and self-alignment of a novel asymmetric 3t parallel manipulator". In *Multibody Mechatronic Systems: Papers from the MuSMe Conference in 2020 7*. Springer, pp. 94–101.

Merlet, J.P., 1996. "Direct kinematics of planar parallel manipulators". In *Proceedings of IEEE international conference on robotics and automation*. IEEE, Vol. 4, pp. 3744–3749.

Merlet, J.P., 1997. "Designing a parallel manipulator for a specific workspace". *the International Journal of Robotics research*, Vol. 16, No. 4, pp. 545–556.

Mostashiri, N., Dhupia, J.S., Verl, A.W. and Xu, W., 2018. "A review of research aspects of redundantly actuated parallel robotsw for enabling further applications". *IEEE/ASME Transactions on Mechatronics*, Vol. 23, No. 3, pp. 1259–1269.

Notash, L. and Huang, L., 2003. "On the design of fault tolerant parallel manipulators". *Mechanism and Machine Theory*, Vol. 38, No. 1, pp. 85–101.

Simas, H. and Di Gregorio, R., 2018. "Adaptive extended jacobian can improve the global conditioning index of redundant robots". In *International Design Engineering Technical Conferences and Computers and Information in Engineering*

Conference. American Society of Mechanical Engineers, Vol. 51814, p. V05BT07A015.
Staicu, S., 2009. “Inverse dynamics of the 3-prr planar parallel robot”. *Robotics and Autonomous Systems*, Vol. 57, No. 5, pp. 556–563.
Wu, J., Wang, J., Wang, L. and Li, T., 2007. “Dexterity and stiffness analysis of a three-degree-of-freedom planar parallel manipulator with actuation redundancy”. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, Vol. 221, No. 8, pp. 961–969.
Zanganeh, K.E. and Angeles, J., 1994. “Instantaneous kinematics and design of a novel redundant parallel manipulator”. In *Proceedings of the 1994 IEEE International Conference on Robotics and Automation*. IEEE, pp. 3043–3048.

8. RESPONSIBILITY NOTICE

The authors are solely responsible for the printed material included in this paper.