# COB-2023-1876
# CAPABILITY STUDY OF A MEASUREMENT INSTRUMENT THROUGH COEFFICIENTS CDL* AND CG AND CONTROL CHARTS

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**Abstract.** *This work presents a study about Cdl\* and Cg capability coefficients and control charts applied to the control of a dimensional measurement process. The Cdl\* capability coefficient was developed by the Tolerancing and Metrology research group of the Federal Institute of Pernambuco (IFPE). The mean and range Shewhart control charts were elaborated, so that there was the possibility to analyze the behavior of the capability coefficients before the control charts. Both studies started with a group of 20 samples, each containing five measurements/points, performed with a digital caliper with a nominal range of 0 – 150 mm. Measurements were made on a standard high speed steel block with a nominal dimension of 50 mm. It should be noted that, before generating the capability coefficients using the Cdl\* Web Software and the control charts using the Control Chart Software, the samples were tested for normality, using the Shapiro-Wilk, Cramér-Von Mises and Kolmogorov method. -Smirnov via ACIC Normality, and by an outlier detection, by the methods of Dixon, Grubbs, Chauvenet, via Analysis Outliers.*

*Keywords: standard high speed steel block, mean-rage control chart, capability index cg, capability index cdl\*, digital caliper.*

## 1. INTRODUCTION

Within the quality sector, there are types of capability coefficients that are applied to the process control of measurement systems. One of the capability coefficients used in measurement systems is the Cg (Touminen et al., 2008), (Bujara et al., 2007), (Jankovych et al., 2011), (Rahmati et al., 2015), (Polák et al., 2014) and is commonly used when no significant systematic errors are observed.

The objective of this study is to present the capability coefficient Cdl\*, developed by (Oliveira et al., 2021) can be applied to instruments and measurement systems and, furthermore, monitor both over time; he considers systematic and random components related to the measurement process, allowing the assessment of the capability of a measurement process to meet a specific standard or metrological reliability.

A major difference among the Cdl\* capability coefficient, in line with the others, is the relationship between the maximum allowable uncertainty (U) and the uncertainty obtained by combining the statistical uncertainty with the uncertainty inherited from the instrument (Ucal) or the measurement system (Oliveira et al., 2021). This encompasses the variables that influence the measurement result. Other coefficients focused on measurement systems and instruments, consider only the standard deviation as a criterion of variability.

A second and third distinguisher lie in presenting the variable l, which is a systematic component of the error, in other words, it is a sensitivity variable that informs us, how much the mean deviate from the nominal value. Finally, we have the consideration of the sample size, as it deals with the standard deviation of the mean, which represents the statistical uncertainty, also known as Type A.

In addition to capability coefficients, there is an important tool in quality management called control charts, developed and implemented by Walter A. Shewhart (ISO 7870-2: 2023). This quality tool is used to analyze whether a specific process is under statistical control, in order to improve the quality and productivity of services (Rodrigues et al., 2015). This work also presents the control charts for mean and range (from the data of the instrument in study), verifying the technical standards of (ISO 7870-2: 2023 parts 1 to 9), which refer to the special causes detected from the analysis of control charts.

## 2. MATERIALS AND METHODS

Initially this topic is divided into two parts. The first consists of calculating the Cg capability coefficient according to Eq. (1), (Andrejiová et al., 2014), (Polák et al., 2014), (Runje et al., 2017), and the Cdl\* coefficient according to Eq. (2), (Oliveira et al., 2021), (Neto et al., 2022) for a digital caliper with a nominal range of 0 - 150 mm. Finally, the control charts for the mean and range were plotted.

$$Cg = \frac{0,2*T}{6*s} \tag{1}$$

$$C_{dl}^* = \frac{U}{3*\sqrt{\left(\frac{s}{\sqrt{n}}\right)^2 + \left(\frac{U_{cal}}{K_{cal}}\right)^2}} * l \tag{2}$$

where T = tolerance; s = standard deviation; U = maximum allowed uncertainty in measurement; n = sample size, $U_{cal}$ = inherited uncertainty from the measurement system or instrument; kcal = coverage factor associated with uncertainty Ucal. and l = relationship between the mean of the values ($\bar{X}$) and the nominal value (VN), been assigned the value of $l_1$ by the Eq. (3) or $l_2$ according to Eq. (4). The Table 1 presents all the features of the digital caliper used in the experiment.

$$l_1 = \frac{(\bar{X})}{VN}, se\ (\bar{X}) < VN \tag{3}$$

$$l_2 = \frac{VN}{(\bar{X})}, se\ (\bar{X}) \geq VN \tag{4}$$

Table 1. Features of the digital caliper.

| Digital Caliper | Value |
|---|---|
| Nominal value, mm | 0 -150 |
| Smallest division, mm | 0.01 |
| Expanded measurement uncertainty, mm | 0.01 |
| Coverage factor | 2 |
| Maximum allowed uncertainty, mm | 0.04 |

In this procedure, a standard block with a nominal dimension of 50 mm was used as the measurement object for the measuring instrument. Twenty measures were performed, each containing five points. The samples were subjected to normality tests using the following methods: Shapiro-Wilk, Cramer von Mises and Kolmogorov Smirnov (Shapiro et al., 1965), (Sheskin, 2003), (Kapur, 2014) using the ACIC NORMALIDADE software at 95% confidence level. In addition, the Anderson-Darling test (Mohd et al., 2011) was applied using Minitab 19 software with the same confidence level.

After the normality verification of the samples, the data were pass into the ANALYSIS OUTLIERS software in order to identify the presence of outliers. The methods used were Dixon, Grubbs and Chauvenet tests, at a confidence level of 95% (Rorabacher, 1991), (Grubbs, 1969), (Vuolo, 1996).

From the flowchart Figure 1, we can observe the steps that the data went through before performing the calculation of the Cg and Cdl*capability coefficients in the Cdl* Web Software.

After the generation of capability coefficients, the procedure for generate the control charts for mean-range began. This type of chart was chosen due to the sample size being inferred as 10. According to (Oliveira, 2016) in order to create this type of graph/chart, the process must be in normal operation, meaning it should be in the best common operating conditions.

For the development of the mean control chart, we have the following Eq. (5), (6), (7).

$$LSC = \bar{\bar{x}} + A_2\bar{R} \tag{5}$$

$$LC = \bar{\bar{x}} \tag{6}$$

$$LIC = \bar{\bar{x}} - A_2\bar{R} \tag{7}$$

where **LSC**, **LC**, **LIC**, $\bar{\bar{x}}$, $A_2$, $\bar{R}$, are as respectively the upper control limit, central control limit, lower control, mean of the means, statistical factor located at Table 2, range mean.
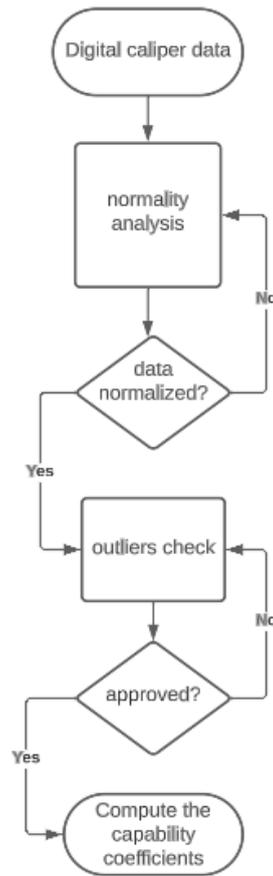
Figure 1. Operation flowchart for generating Cg and Cdl* capability coefficients.

Table 2. Statistical factor for mean and range control chart.

| Statistical Factor for Control Charts | | | |
|---|---|---|---|
| Subgroup Size (n) | $A_2$ | $D_3$ | $D_4$ |
| 2 | 1.880 | - | 3.267 |
| 3 | 1.023 | - | 2.574 |
| 4 | 0.729 | - | 2.282 |
| 5 | 0.577 | - | 2.114 |
| 6 | 0.483 | - | 2.004 |
| 7 | 0.419 | 0.076 | 1.924 |
| 8 | 0.373 | 0.136 | 1.864 |
| 9 | 0.337 | 0.184 | 1.816 |
| 10 | 0.308 | 0.223 | 1.777 |
| 15 | 0.223 | 0.347 | 1.653 |
| 25 | 0.153 | 0.459 | 1.541 |

For the range control chart, we have the Eq. (8), (9), (10):

$$LSC = D_4 \bar{R} \tag{8}$$

$$LC = \bar{R} \tag{9}$$

$$LIC = D_3 \bar{R} \tag{10}$$

where $D_4$ and $D_3$ are both the statistical factor Table 2.

The data were passed into a normality test using ACIC NORMALIDADE software and outliers were identified using ANALYSIS OUTLIERS software before being added into Control Chart Software (Control Chart Software, 2022) as shows in the flowchart in Figure 2.
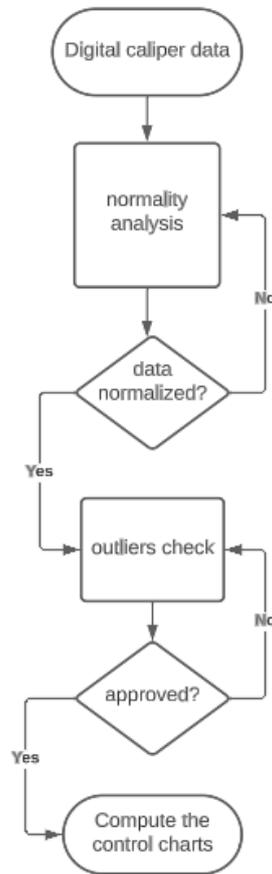


Figure 2. Operation flowchart for generating the mean-range control chart.

## 3. RESULTS AND DISCUSSIONS

For the calculation of the $C_g$ capability coefficient, the following parameters were used as are showed in the Table 3. Tough Table 4 presents the samples that were used to generate the capability coefficients and control charts.

Table 3. Cg input parameters.

| Cg Input Parameters | Value |
|---|---|
| Percentage of tolerance, % | 0.2 |
| Tolerance, mm | 0.1 |
| Number of standard deviations | 4 |

As we can see  Figure 3. The $C_g$ chart generated, shows that many points exceeded 1.33. However, samples 2, 9, 16, 18, and 19 remained below 1, indicating that the process is not capable. However, if they were between 1 and 1.33, we could consider the instrument reasonably capable.

Table 4. Data from the digital caliper measurements.

| Samples | X1, mm | X2, mm | X3, mm | X4, mm | X5, mm |
|---|---|---|---|---|---|
| 1 | 50.02 | 50.01 | 50.02 | 50.02 | 50.02 |
| 2 | 50.01 | 50.00 | 49.99 | 50.01 | 50.02 |
| 3 | 50.01 | 50.02 | 50.00 | 50.00 | 50.00 |
| 4 | 50.01 | 50.01 | 50.01 | 50.00 | 50.02 |
| 5 | 50.00 | 50.01 | 50.00 | 50.00 | 50.01 |
| 6 | 50.00 | 50.00 | 50.00 | 50.01 | 50.00 |
| 7 | 50.02 | 50.01 | 50.01 | 50.02 | 50.02 |
| 8 | 50.01 | 50.02 | 50.03 | 50.02 | 50.01 |
| 9 | 50.00 | 50.02 | 50.00 | 49.99 | 50.01 |
| 10 | 50.01 | 50.01 | 50.01 | 50.01 | 50.02 |
| 11 | 50.02 | 50.02 | 50.01 | 50.02 | 50.00 |
| 12 | 50.01 | 50.03 | 50.02 | 50.02 | 50.02 |
| 13 | 50.00 | 50.01 | 50.00 | 50.01 | 50.00 |
| 14 | 50.02 | 50.01 | 50.01 | 50.00 | 50.00 |
| 15 | 50.03 | 50.01 | 50.01 | 50.03 | 50.02 |
| 16 | 50.00 | 50.02 | 50.03 | 50.03 | 50.03 |
| 17 | 50.02 | 50.02 | 50.03 | 50.02 | 50.01 |
| 18 | 50.03 | 50.02 | 50.02 | 50.02 | 50.00 |
| 19 | 50.02 | 50.01 | 49.99 | 50.02 | 50.01 |
| 20 | 50.01 | 50.01 | 50.03 | 50.01 | 50.02 |

The Cg  chart generated at Figure 3, can be verified that many points exceeded 1.33. However, samples 2, 9, 16, 18, and 19 remained below 1, indicating that the process is not capable. However, if they were between 1 and 1.33, we could consider the instrument reasonably capable.
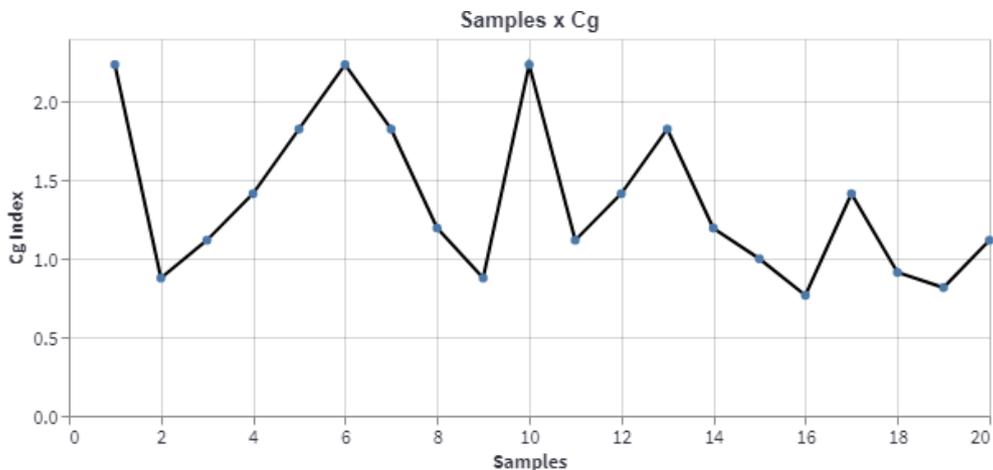


Figure 3. Cg capability graphic.

For the graphic Figure 4 of Cdl* capability coefficient, were used the following input parameters as showed in Table 3.

Table 3. Cdl* input parameters.

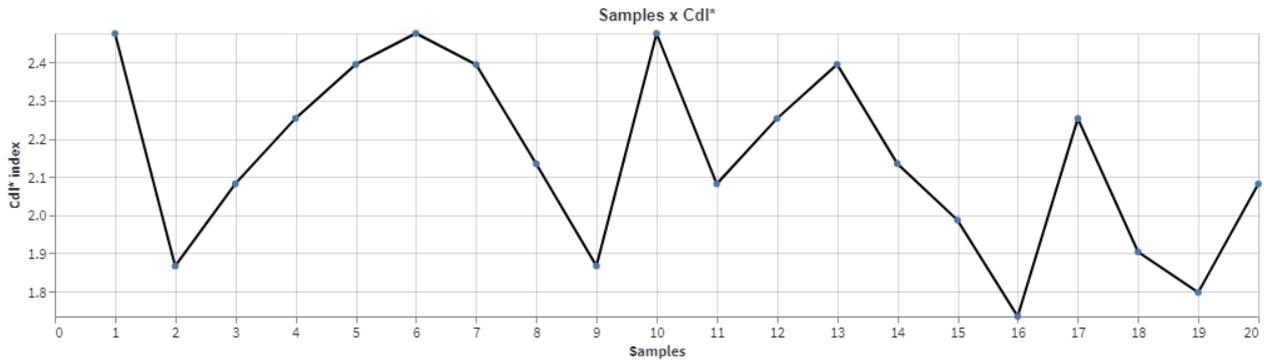| Cdl* Input Parameters | Value |
|---|---|
| Maximum allowed uncertainty, mm | 0.04 |
| Inherited uncertainty from the instrument, mm | 0.01 |
| Coverage factor | 2 |
| Sample size | 5 |
| Nominal value, mm | 50 |

Figure 4. Cdl* capability graphic.

Regarding the Cdl* capability coefficient, all samples above 1.33 have been returned, considering the instrument capable. In Figure. 5 and 6, we have the control charts for the mean and range, respectively. In the face of the mean control chart Figure 5, we have an upper limit of 50.02 mm and a lower limit of 50.001 mm. The charts presented are divided into three regions (three standard deviations) both above and below, where the measured samples are distributed. Upon analyzing the mean chart Figure 5, we observe that the instrument does not meet the additional criteria for Shewhart analysis (ISO 7870-2, 2013). Out of the eight criteria, the chart does not pass rule 5 which consider "2 or 3 consecutive points in zone A, on the same side of the centerline". These points refer to samples 15, 16, and 17, thus rendering the measurement instrument used in this procedure inadequate.
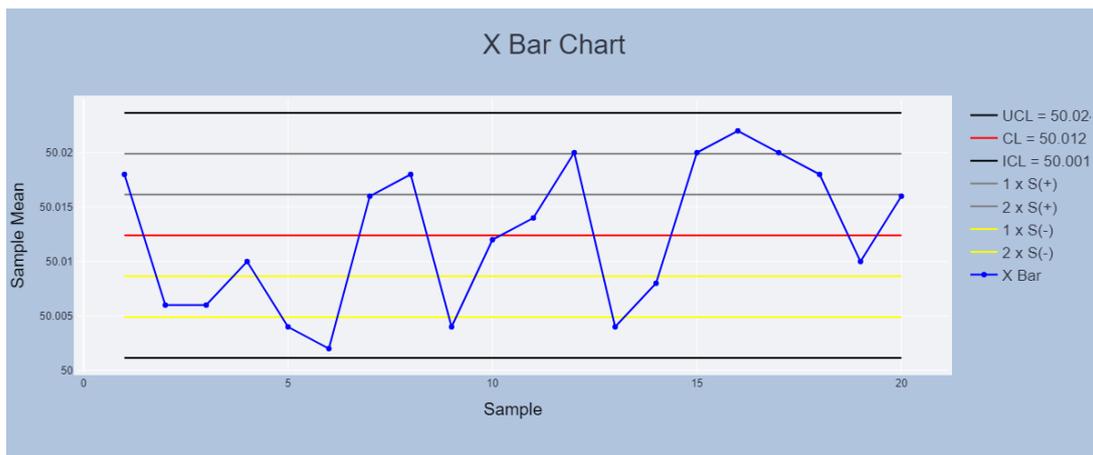


Figure 5. Mean control chart.



Figure 6. Range control chart.

Analyzing the behavior of the range control chart in Figure 6, we verify that the instrument is considered approved, as it does not violate any rules defined by (ISO 7870-2, 2013).

## 4. CONCLUSIONS

In view of the presented study, the instrument failed since it was only deemed capable based on the Cdl* capability coefficient and the range control chart. It is important to emphasize that this capability coefficient relates the maximum allowable uncertainty to the uncertainty obtained through the combination of the instrument's inherited uncertainty (Type B) and statistical uncertainty (Type A), characterized by the standard deviation. Furthermore, it also includes the variable l, which serves the purpose of considering the influence of how much the sample mean deviates from the nominal value. This is different from the Cg coefficient, which only considers the standard deviation with a variability criterion.

When analyzing the mean chart, the samples violate the rule-5 defined from (ISO 7870-2, 2013). We observed that the graphs in Figures 3-4, when there are oscillations both upwards and downwards, behave similarly. On the other hand, the oscillations of Cdl* return values above 1.33, whereas those of the Cg coefficient return values both above 1.33 and between 1.33 and 1, as well as below 1.

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