27th ABCM International Congress of Mechanical Engineering
December 04-08, 2023, Florianópolis-SC, Brazil

COB-2023-0646
COMBINED SCREENING SENSITIVITY ANALYSIS APPLIED TO ONE-DIMENSIONAL MODEL OF A HIGH-PERFORMANCE S-CO2 CENTRIFUGAL COMPRESSOR

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Abstract. *The Enhanced Oil Recovery (EOR) is a process used to inject CO_2 into oil wells through high-performance supercritical centrifugal compressors to increase the extraction in the Tertiary Recovery. To achieve the best performance of the machine, a one-dimensional simulation is the first step on the development of centrifugal compressor devices. From that starting design point, it´s possible to reduce variables that are considered and have a good approximation for the most efficient geometry. Thus, saving time, money and achieving a better performance. Moreover, two mathematical methods were chosen to perform the screening Sensitivity Analysis (SA): Morris and Fractional Factorial (FF). Both working with the same variables that, in this case, are seven: Blade thickness (t), number of blades (Z_{fb}), splitter blade length (L_{sf}), shroud radius (r_{1s}), axial height (ΔZ), impeller exit height (b) and specific diameter (D_s). After ranges are stablished, each method's Design of Experiments (DoE) are computed in a python program (One-dimensional model) that returns, as outputs, the parameters to be analyzed: isentropic efficiency, consumed power, and pressure recovery. Then, all the data is stored and analyzed to identify which variable configuration returns the best efficiency. Furthermore, these two screening SA methods are combined, because the Morris method is unable to estimate the interactions between different factors due to its nature of sampling. Therefore, this work aims to determine if the FF can complement the screening analysis, i.e., identifing the two-factor interactions importance. Finally, the SA can find the most relevant variables for the system's efficiency, which can demonstrate the future focus of further studies. Thus, with these preliminary studies it is possible to have a reliable glance of the sample space behavior, and which variable is more relevant for the system. Showing the importance of a proper screening sensitivity analysis of models with high number of variables as a low-cost first step of centrifugal compressor's design.*
Keywords *One-dimensional model, S-CO2 Centrifugal Compressors, Combined Screening Sensitivity Analysis, Enhanced Oil Recovery*.

1. INTRODUCTION

High-performance supercritical CO_2 (S-CO_2) centrifugal compressors are used in the Enhanced Oil Recovery (EOR) process (Tertiary Recovery), and their function is to inject CO_2 into oil wells. This apparatus allows extraction to be extended (estimated increase of 30% to 60% of the reservoir capacity, when compared to the extraction rate in primary and secondary recoveries (20% to 40%), which points to a significant difference in extraction (MUGGERIDGE et al., 2014). Furthermore, this practice mitigates carbon dioxide from the atmosphere and injects it into reservoirs, which is one of the main greenhouse effect gases, contributing to environmental well-being. In addition, the use of this Centrifugal Compressor (CC) makes it possible to extract resources from the pre-salt layer, as it is an agent for optimizing oil exploration that reduces environmental impacts and even makes the activity more profitable and safer. (LOPES, F. et al. 2019)

For this compressor to be as efficient as possible, it is necessary to optimize its geometry. The most robust approach to perform this optimization procedure would be through a Computational Fluid Dynamics (CFD) analysis, which would provide a more reliable representation of fluid flow inside the equipment. However, this method is expensive and requires a large computational effort due to the number of variables inherent to its geometry. Thus, surrogate models emerged as a practical and efficient tool, as they would be faster, and comparatively cheaper, providing reliable results that would facilitate an optimization procedure. The same procedure is widely used in aerodynamic devices, generating positive results (IULIANO, E. *et al.*, 2011)

Moreover, numerical methods of Sensitivity Analysis (SA) are defined as statistical tools that studies how changes in the output of a model can be attributed to changes in input variables (Figure 1) (SALTELLI, 2008). Currently, several SA methods are known, quantitative (variance based) or qualitative (screening) (CAMPOLONGO, SALTELLI, et al., 2011). These SA methods can be used to provide proper space sample representation for surrogate model training (SALVIANO, L. *et al.,* 2021).

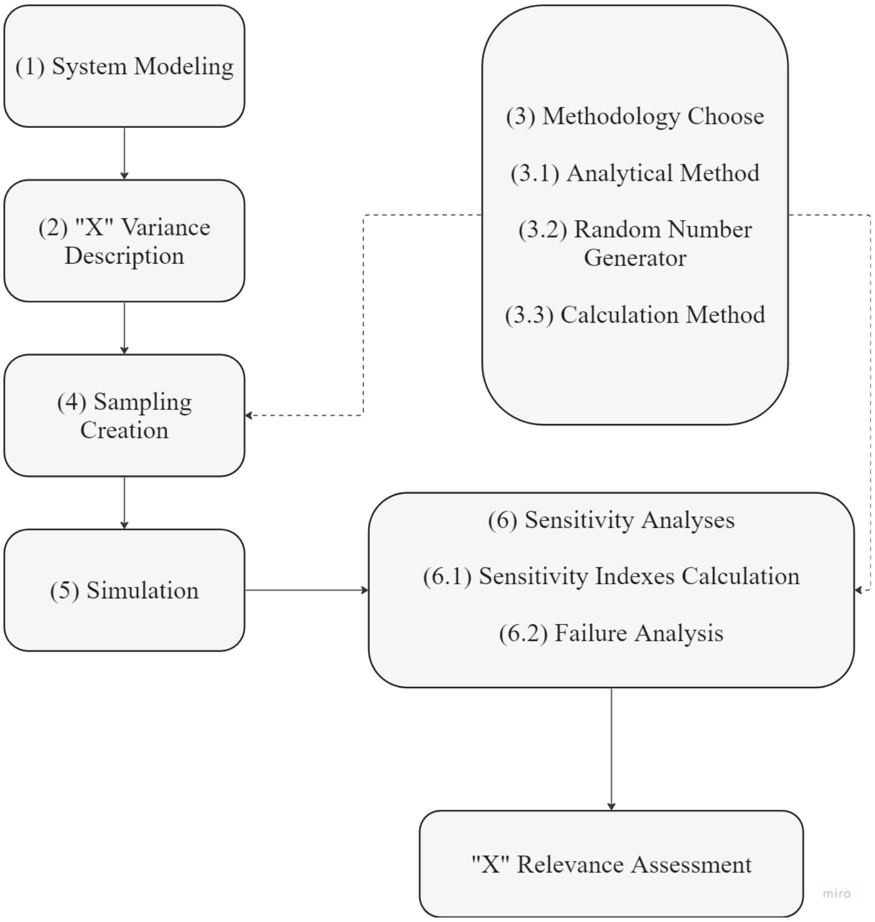

Figure 1 – Procedure of a global sensitivity analysis.

Considered as good practice in qualitative SA, the Elementary Effects (EE) method introduced by Morris (1991) and refined by Campolongo (2007) offers a good result in models with high computational cost. Being a screening method, it seeks to identify input variables that have low influence on the result. However, this method is not able to identify the interaction between two factors due to the nature of its sampling ('one-at-a-time') (SALTELLI et al., 2019). To get around this limitation, Fractional Factorial Sampling (FF) can be used in conjunction with the Morris method, as it identifies the interaction between two factors. The modeling provides a quick verification and a preliminary estimate of the interaction between the parameters, complementing the analysis of the main effects of the EE method at a low cost (DAS, TESFAMARIAM, 2022).

Thus, the main objective of the study is to apply the SA methods in the development of high performance supercritical centrifugal compressors for oil extraction. In addition, verify whether the combination between the EE and FF methods provides accuracy in identifying the main effects and interactions between the variables. And yet, if the strategy returns good results, apply the methodology to expensive computational models (CFD), to reduce processing time and generate faster results with greater reliability. In order to confirm this thesis, the combination of methods (FF and EE) will be compared to another numerical model, the Smoothing Spline ANOVA (SS-ANOVA), that returns a quantitative result to the discussion, if the combination of elementary effects and fractional factorial converges with the results off SS-Anova the study it's able to confirm that the combined methods are reliable and can be used to complex and expensive models.

2. METHODOLOGY

This section presents the combined strategy of EE and FF screening methods and the 1D model of centrifugal compressor used. As presented in Figure 2, the 1D model was parameterized considering seven variables: number of blades (Zfb), blade thickness (t1), splitter blade length (Lsf), shroud radius (r1s), axial height (deltaZ), impeller exit height (b) and specific diameter (Ds). The chosen interest outputs were isentropic efficiency, consumed power and pressure recovery of the centrifugal compressor, which provides insight of variables that highly influences performance of the machine. In addition, both screening Sensitivity Analyses (EE and AFF) are computed in parallel (as elucidated in Figure 2) and after they are compared with SS-ANOVA.

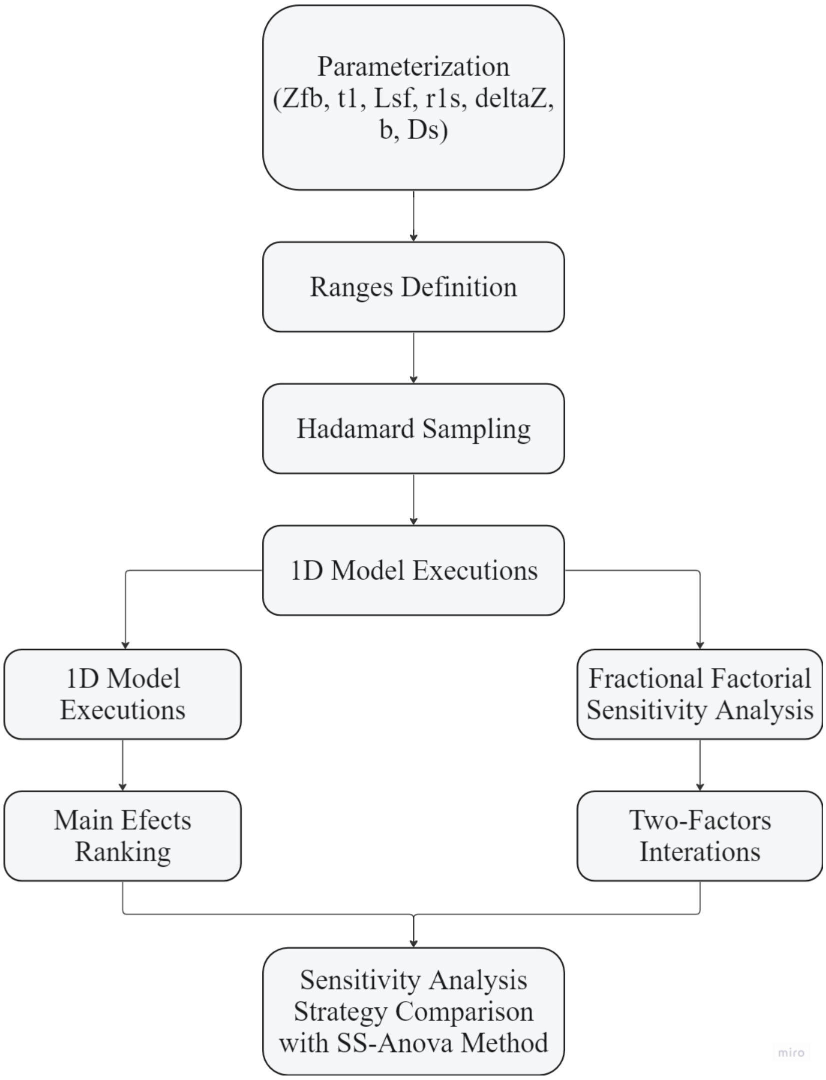

Figure 2 – Methodology flowchart for low-cost interaction identification through SA.

2.1 1D model for S-CO2 Centrifugal Compressors

The centrifugal compressor's 1D model developed for this research was based on Augier (2000) and Monje *et al.* (2014). The entire model consists on the evaluation of continuity, energy conservation, velocity triangle and Equation of State (EOS) (R.SPAN; W.WAGNER, 1994) until convergence. The loop of these governing equations is performed on impeller leading and trailing edges, vaneless diffuser and volute, providing a full preliminary assessment of fluid flow behavior in the machine and avoiding high Mach numbers, condensation and gas-like behavior for supercritical CO2. A more detailed description of this model can be found at (GASPARIN, E. *et al.,* 2023).

The preliminary geometry obtained through the 1D model is the first step in the centrifugal compressor design, which is followed by a Computational Fluid Dynamics (CFD) model, as an already established strategy for achieving high machine performances (XIA *et al.,* 2021).

2.2 Elementary Effects

The EE method (Morris, 1991) is recognized as a cheap and effective method for screening a model, recognizing the most important input parameters among the many that can be contained in a model. If a model has k independent inputs, then X_i (i=1,…,k), the Elementary effect is defined as shown in Equation (1).

$$EE_i = \frac{Y(X_1, X_2, \ldots, X_i + \Delta, \ldots, X_k) - Y(X_1, X_2, \ldots, X_k)}{\Delta} \quad (1)$$

in which Δ is the step for the discretized input space. The sampling strategy suggests that r elementary effects are computed for each input variable. Each one of the sample space trajectories contains k+1 points. If r=10 is chosen and seven variables are considered according to table 1, the total cost of the method is: r(k+1) = 80 model executions, which is relatively cheap in comparison with quantitative methods.

In order to properly rank the input variable importance, a main effect sensitivity measure is proposed as the average of each input EE distribution on absolute values (μ*), Equation (2), which assesses the overall influence of the variable (CAMPOLONGO; CARIBONI, 2007), being sufficient to provide a reliable ranking of variables.

$$\mu_i = \frac{1}{r}\sum_{j=1}^{r} EE_i^j \quad (2)$$

The sampling strategy maximizes the r trajectories dispersion in the sample space. First, it generates a high number of trajectories (*M = 500*), and then it selects r trajectories with the highest ‘spread’, based on the definition of ‘distance’ (d_{ml}) between trajectories, following the Equation (3).

$$d_{ml} = \begin{cases} \sum_{i=1}^{k+1}\sum_{j=1}^{k+1}\sqrt{\sum_{z=1}^{k+1}\left[X_i^m(z) - X_j^l(z)\right]^2}, & m \neq l \\ 0, & otherwise \end{cases} \quad (3)$$

2.3 Fractional Factorial

The Fractional Factorial method is a screening method based on the orthogonality of Hadamard matrices, which produces a useful Design of Experiment (DoE) for two-factors interaction identification (SALTELLI *et al.,* 2008). The Hadamard matrices are composed by -1 and +1, indicating the lower and the upper bounds of the variable range, that were defined in table 1, you are analyzing, respectively. Its order (*n*) must be higher than the number of variables (*k*) in model and is always power of 2. In order to further improve the properties of the FF DoE, its construction uses the proper Hadamard matrix and its opposite, as elucidated in Equation (4).

$$Mn = \frac{Hn}{-Hn} \quad (4)$$

In such a Mn design, an equal number of simulations are allocated to each corner of the sample space, which is helpful in determining which combinations of parameters are working together to influence the value of Y (SALTELLI et al., 2008). Moreover, the sensitivity measure used to account for the effect of a parameter or two-factor combination in FF analysis, $ME_r(Y)$, is presented in Equation (5).

$$MEr(Y) = \frac{1}{2n}\sum_{j=1}^{2n} XjrYjr \quad (5)$$

In which x_{jr} is the Hadamard matrix column of the specific input variable and y_{jr} is the response vector after the DoE is assessed by the model in analysis.

At last, the combination of both methods can be useful in determining the importance of two-factors interaction in comparison with main effects. The EE method is responsible for providing the overall main effect influence on the model and FF provides interactions importance, completing each other.

2.4 SS-ANOVA

The Smoothing Spline ANOVA method is a sensitive analyses method based on classical study of variance (ANOVA), it works with the decomposition of main effects and the two-factors interactions measuring the relevance of the variable and its combinations to the global model (RIGONI; RICCO, 2011), defined by Ratto and Pagano (2010) as:

$$f(X) = f_0 + \sum_{j=1}^{n} f_j(x_j) + \sum_{j<i}^{n} f_{j,i}(x_j, x_i) \tag{6}$$

Moreover, the procedure developed by Kim and Gu (2004) is combined, in which a more scalable computation of smoothing spline regression is used. This method can be descried as a minimization of the usual least square functional subjected to the constraint J. The contribution of each factor (π_k) is defined as:

$$\pi_k = \frac{f_k{}^T \cdot \sum_{j=1}^{n} f_j}{\left(\sqrt{\sum_{j=1}^{n} f_j \cdot \sum_{j=1}^{n} f_j}\right)^2} \tag{7}$$

In this context, $f_k{}^T$ is a column vector where each row corresponds to the decomposition of individual variables and $\sum_{j=1}^{n} f_j$ represents the cumulative decomposition of all sampled points. π_k can be understood as the proportion of the model's variance attributed to a specific factor. The disparity between the sum of individual contributions and unity indicates the aggregation of all interaction effects, as discussed by RIGONI and RICCO in 2011.

In addition, the computational cost of the model is usually k^3 , being k the number of model variables, however in order to encompass a better sample space instead of 350 cases it was used 500 to contemplate all the possibilities of factors combinations.

Table 1. Ranges of variation of all factors

Variables	Low	Up
t1, t2 (Blade thickness)	0.001	0.0042
Zfb (Number of lades)	11	14
Lsf (Splitter blade length)	0.5	1
r1s (Shroud radius)	0.044	0.052
DeltaZ (Axial height)	0.035	0.06
B (Impeller exit height)	0.005	0.018
Ds (Specific diameter)	7.4	8

3. RESULTS

This section is intended to show the results of the research, which is divided in three main parts: the Fractional Factorial, the Elementary Effects, and lastly the SS-Anova method analyses, showing how the sensitivity analyses were defined by each interest output, and comparing each other

3.1 Fractional Factorial Results

The Hadamard matrix DoE was executed, using 16 model runs, and the FF single and two-factor effects were computed, as presented in Figure 3. The methodology identified as “most influent” variable to isentropic efficiency the blade thickness.

Analyzing these results is possible to conclude the interactions between the parameters are not important to achieve the best result for isentropic efficiency at the compressor.

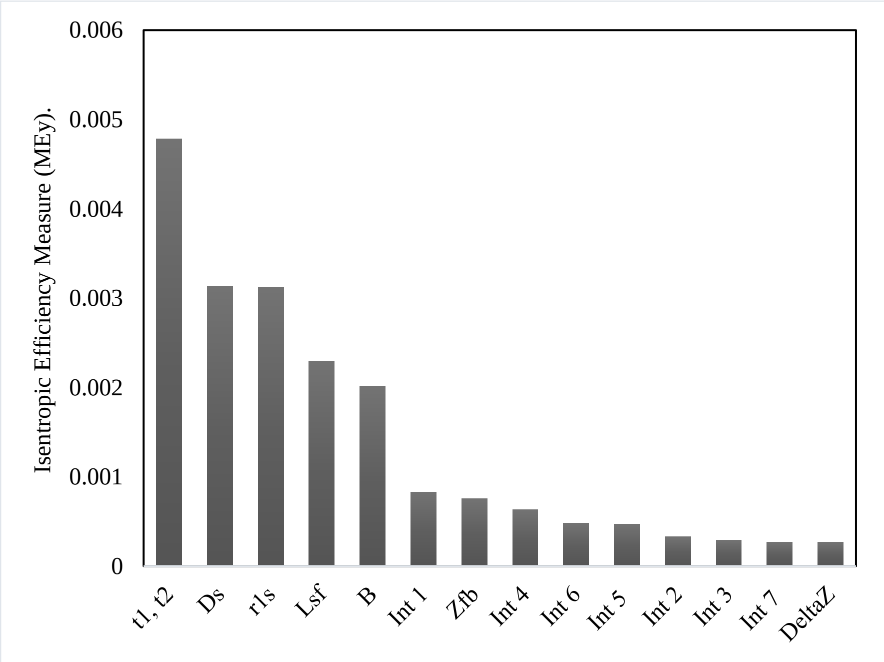

Figure 3 – Fractional factorial method for isentropic efficiency.

Moreover, for consumed power, the most relevant variable was also the blade thickness, as show in Figure 4, and the interactions between factors were relatively neutral, yet relevant to achieve a great result, so these interactions must be considered for consumed power. Now, analyzing the pressure recovery, the most relevant factor was the specific diameter, shown in Figure 5.

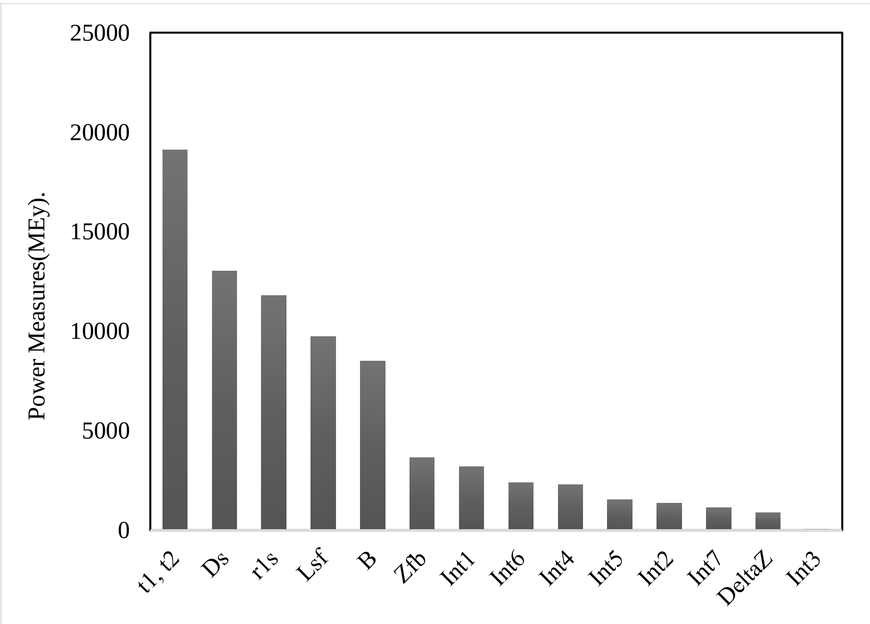

Figure 4 – Fractional factorial method for consumed power.

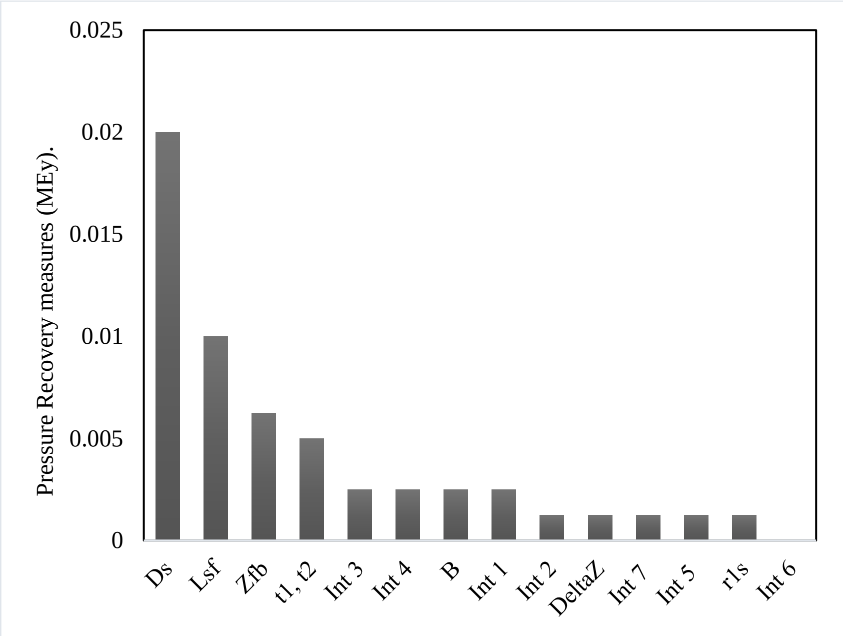

Figure 5 – Fractional factorial method for pressure recovery

3.2 Morris' Elementary Effects Results

First, the sampling used 80 model executions in Morris' method setup, allowing computation of Morris' EE for all the seven variables, presented in Figure 6. For isentropic efficiency the most important variable were again the blade thickness, showing a pattern. Now, analyzing the consumed power of the machine, the "strongest" factor was again the blade thickness followed by the specific diameter. As shown in the Figure 7. Which make sense when compared to the literature. Finally, confirming the fractional factorial method, the most relevant variable for pressure recovery were the specific diameter, as shown in Figure 8.

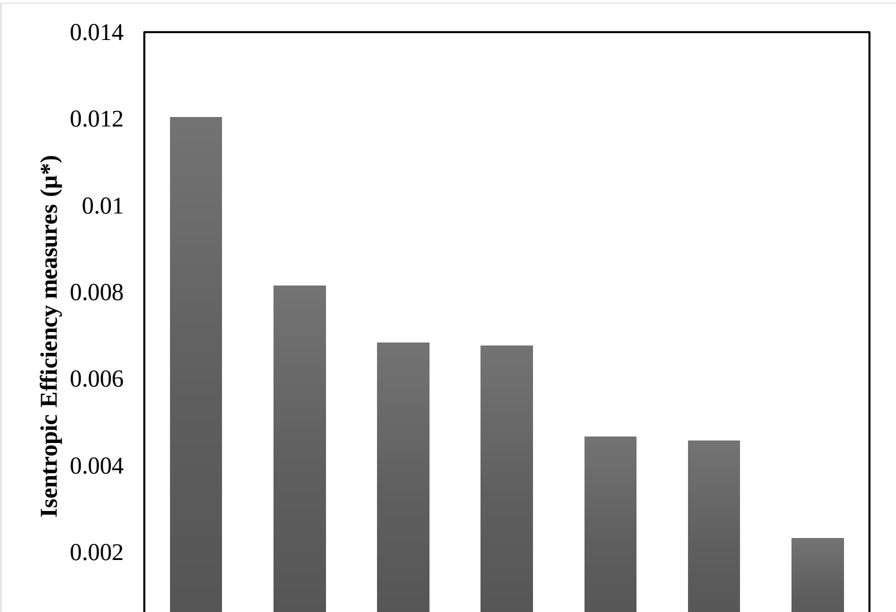

Figure 6 – Elementary Effects method for isentropic efficiency

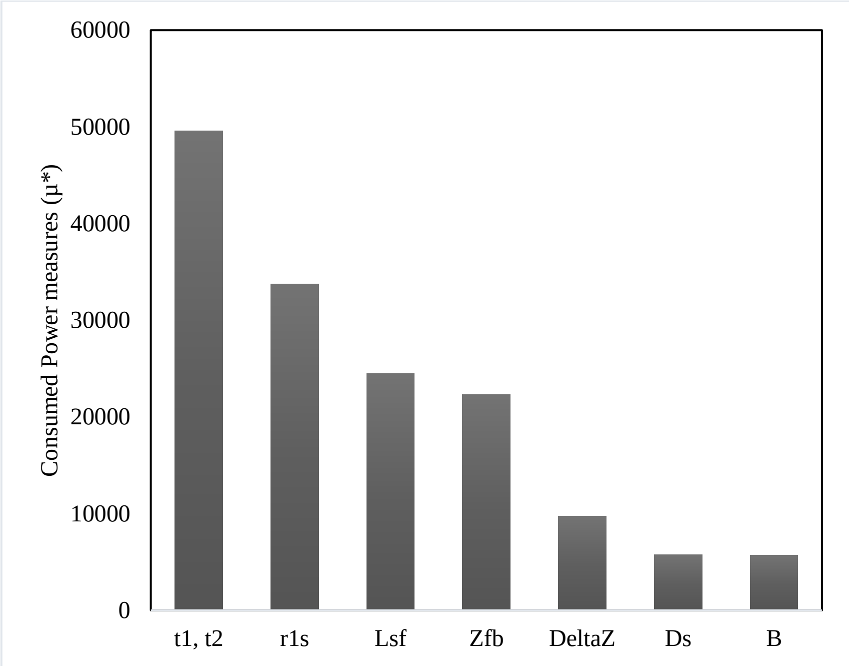

Figure 7 – Elementary Effects method for consumed power

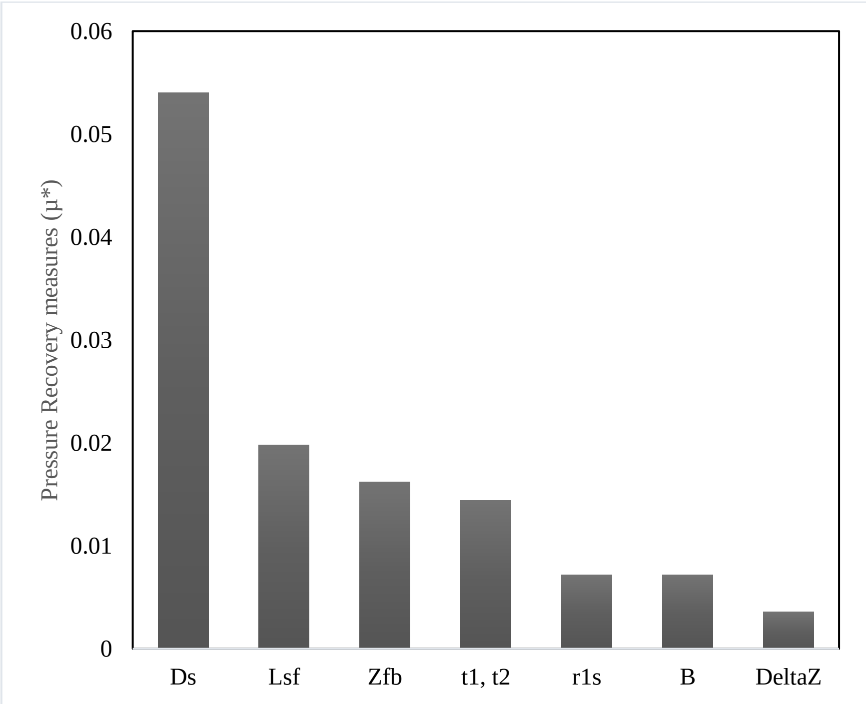

Figure 8 – Elementary Effects method for pressure recovery

3.3 Smoothing Spline ANOVA Results

In order to confirm the combined analyses of the two previous methods, the results of a third Sensitivity Analysis Method (SS-ANOVA), that gives a quantitative measure of the variables relevance, and their two factor interactions, were considered. Thus, the results for isentropic efficiency confirm what was obtained in the other two method: that the first factor (blade thickness) was more relevant than the others, followed by the shroud radius, as shown in Figure 9. Furthermore, the study is able to conclude that the interaction of variables is irrelevant for this specific compressor model and can be disregarded in further analyses, as predicted by the combined screening analysis. Moreover, for consumed power, is observed the same behavior, which confirms the results of the previous methods and the insignificance of the two-factor interactions. In this case, the most important variable was also the blade thickness, as stated in Figure 11. In sequence, for the third interest output (pressure recovery), it was obtained the same expected results based on the other outputs, the confirmation of FF and EE combined analyses. Being the specific diameter (Ds) the expressive dominant variable and confirming that the combination of variables is negligible for the global system, as presented in Figure 10.

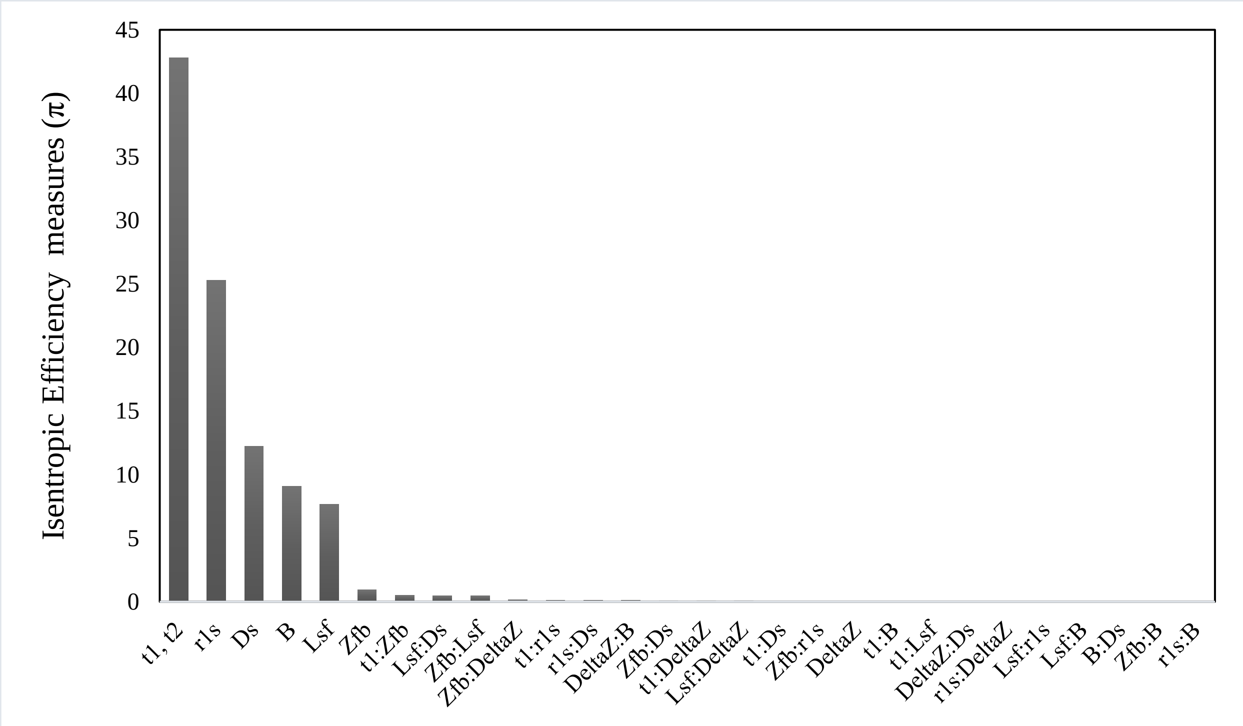

Figure 9 – Smoothing Spline ANOVA results for isentropic efficiency

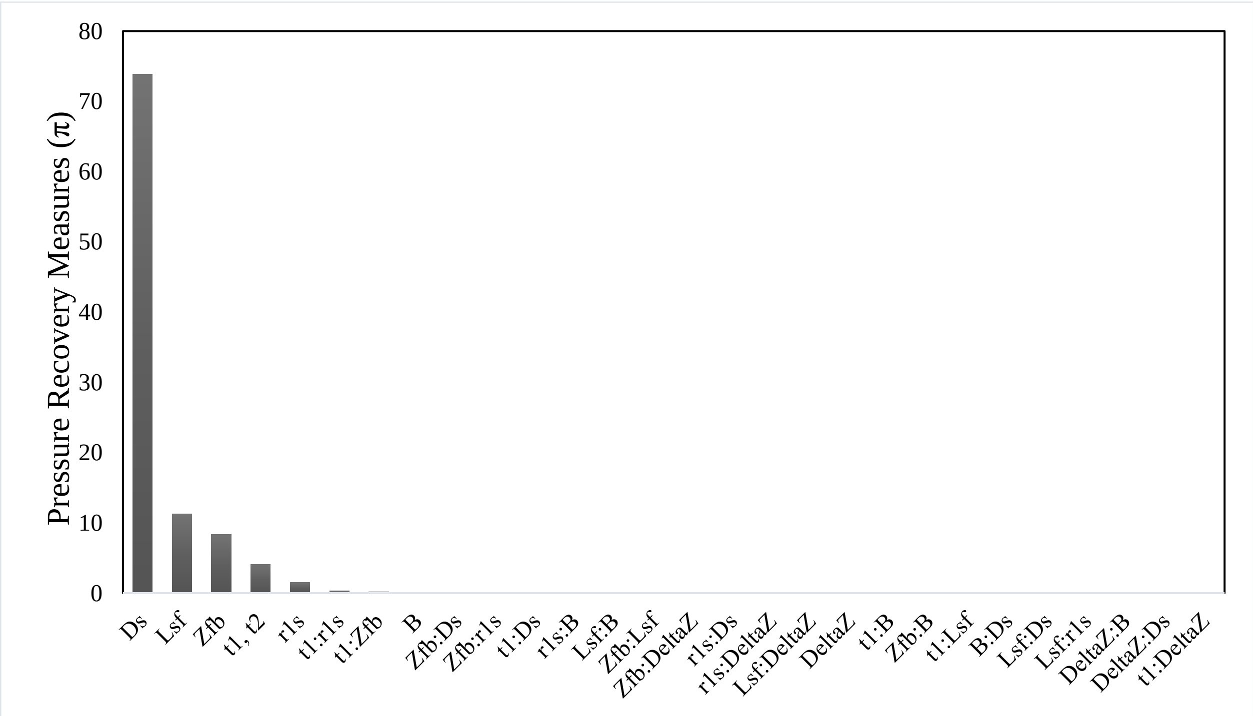

Figure 10 – Smoothing Spline ANOVA results for pressure recovery

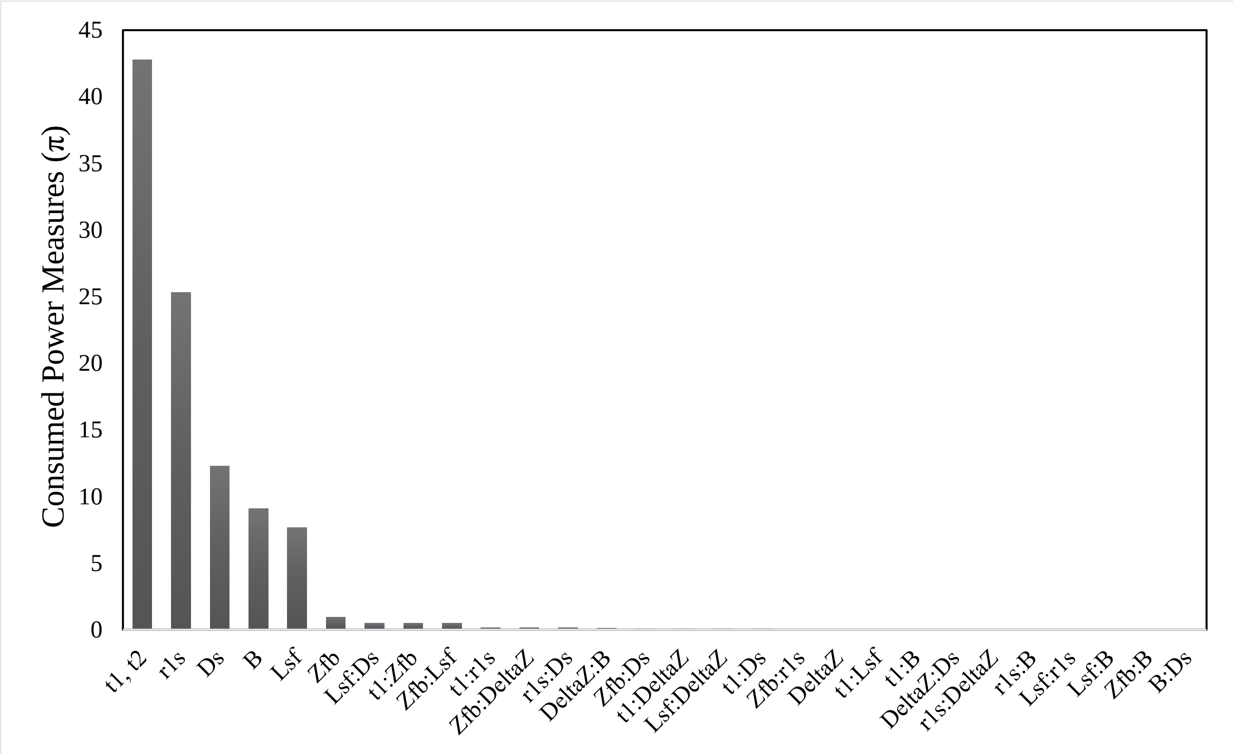

Figure 11 – Smoothing Spline ANOVA results for consumed power

4. CONCLUSIONS

The one-dimensional model was submitted to a screening SA composed by two methods (EE and FF) to evaluate if this qualitative and computationally cheap analysis was able to properly identify the relevance of two-factor interactions to our high-dimensional model. Therefore, the combination of Morris' (Elementary Effects) and the Fractional Factorial methods are reliable to identify which are the most important variables of expensive models, as a high-performance centrifugal compressor CFD model. In addition, the SS-ANOVA method agreed with the results of the other two methods, showing that the two-factor interactions are not important to this model, and they can be disregarded on future design optimization. This result present that the Fractional Factorial two-factor's analyses is trustable, and being implemented with the Elementary Effects method can provide a good ranking of the variables' comportment in the model. Finally, the objective of this research was confirmed, that the combined screening methods of Fractional Factorial and Elementary Effects can describe the relevance of expensive and complex model's main effects, and their interactions becoming a good alternative for turbomachinery SA, with faster, reliable and cheaper results to be used in other methods like CFD. Furthermore, were necessary compute only 16 cases for the FF method, and only 90 for EE, exemplifying how cheap and easy are these methods, in front others like CFD.

5. ACKNOWLEDGEMENTS

This work was supported by the National Agency of Petroleum, Natural Gas and Biofuels (ANP) and Shell Brazil Ltda., through the Investment in Research, Development and Innovation Clause, contained in contracts for Exploration, Development, and Production of Oil and Natural Gas.

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